Zenon Modulo: When Achilles Outruns the Tortoise using Deduction Modulo

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Proof Search in Axiomatic Theories

Current Trends

- Axiomatic theories (Peano arithmetic, set theory, etc.);
- Decidable fragments (Presburger arithmetic, arrays, etc.);
- Applications of formal methods in industrial settings.

Place of the Axioms?

- Leave axioms wandering among the hypotheses?
- Induce a combinatorial explosion in the proof search space;
- Do not bear meaning usable by automated theorem provers.

Proof Search in Axiomatic Theories

A Solution

- A cutting-edge combination between:
  - First order automated theorem proving method (resolution);
  - Theory-specific decision procedures (SMT approach).

Drawbacks

- Specific decision procedure for each given theory;
- Decidability constraint over the theories;
- Lack of automatability and genericity.

Proof Search in Axiomatic Theories

Use of Deduction Modulo

- Transform axioms into rewrite rules;
- Turn proof search among the axioms into computations;
- Avoid unnecessary blowups in the proof search;
- Shrink the size of proofs (record only meaningful steps).

This Talk

- Introduce the principles of deduction modulo;
- Present the results of an experiment with Zenon;
- Give an overview of the BWare project.

Principles of Deduction Modulo

Inclusion

$$\forall a \forall b\, ((a \subseteq b) \Leftrightarrow (\forall x\, (x \in a \Rightarrow x \in b)))$$

Proof in Sequent Calculus

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\ldots, x \in A \vdash A \subseteq A, x \in A}\,\text{Ax}}{\ldots \vdash A \subseteq A, x \in A \Rightarrow x \in A}\,\Rightarrow\text{R}}{\ldots \vdash A \subseteq A, \forall x\, (x \in A \Rightarrow x \in A)}\,\forall\text{R} \qquad \dfrac{}{\ldots, A \subseteq A \vdash A \subseteq A}\,\text{Ax}}{\ldots, (\forall x\, (x \in A \Rightarrow x \in A)) \Rightarrow A \subseteq A \vdash A \subseteq A}\,\Rightarrow\text{L}}{A \subseteq A \Leftrightarrow (\forall x\, (x \in A \Rightarrow x \in A)) \vdash A \subseteq A}\,\wedge\text{L}}{\forall a \forall b\, ((a \subseteq b) \Leftrightarrow (\forall x\, (x \in a \Rightarrow x \in b))) \vdash A \subseteq A}\,\forall\text{L} \times 2$$

Principles of Deduction Modulo

Inclusion

$$\forall a \forall b\ ((a \subseteq b) \longrightarrow (\forall x\ (x \in a \Rightarrow x \in b)))$$

Rewrite Rule

$$(a \subseteq b) \longrightarrow (\forall x\ (x \in a \Rightarrow x \in b))$$

Proof in Deduction Modulo

$$\dfrac{\dfrac{\dfrac{}{x \in A \vdash x \in A}\ \mathrm{Ax}}{\vdash x \in A \Rightarrow x \in A}\ \Rightarrow\mathrm{R}}{\vdash A \subseteq A}\ \forall\mathrm{R},\ A{\subseteq}A{\longrightarrow}\forall x\ (x{\in}A{\Rightarrow}x{\in}A)$$

From Axioms to Rewrite Rules

Difficulties

- Confluence and termination of the rewrite system;
- Preservation of the consistency;
- Preservation of the cut-free completeness;
- Automation of the transformation.

An Example

- Axiom $A \Leftrightarrow (A \Rightarrow B)$;
- Transformed into $A \longrightarrow A \Rightarrow B$;
- We want to prove: B.

An Example (Continued)

- In sequent calculus, we have a cut-free proof:

$$
\dfrac{
  \dfrac{\dfrac{\sim \Pi}{A \Rightarrow (A \Rightarrow B), A \vdash B, B}}{A \Rightarrow (A \Rightarrow B) \vdash B, A \Rightarrow B}\;{\Rightarrow}\mathrm{R}
  \qquad
  \dfrac{\Pi}{A \Rightarrow (A \Rightarrow B), A \vdash B}
}{
  \dfrac{A \Rightarrow (A \Rightarrow B), (A \Rightarrow B) \Rightarrow A \vdash B}{A \Leftrightarrow (A \Rightarrow B) \vdash B}\;{\Leftrightarrow}\mathrm{L}
}\;{\Rightarrow}\mathrm{L}
$$

Where Π is:

$$
\dfrac{
  \dfrac{}{A \vdash B, A}\;\mathrm{ax}
  \qquad
  \dfrac{\dfrac{}{A \vdash B, A}\;\mathrm{ax} \qquad \dfrac{}{A, B \vdash B}\;\mathrm{ax}}{A, A \Rightarrow B \vdash B}\;{\Rightarrow}\mathrm{L}
}{A \Rightarrow (A \Rightarrow B), A \vdash B}\;{\Rightarrow}\mathrm{L}
$$

From Axioms to Rewrite Rules

An Example (Continued)

- In deduction modulo, we have to cut A to get a proof:

$$\dfrac{\dfrac{\Pi}{A \vdash B} \qquad \dfrac{\dfrac{\Pi}{A \vdash B}}{\vdash A} \Rightarrow\mathrm{R},\ A \longrightarrow A \Rightarrow B}{\vdash B}\ \mathrm{cut}$$

Where Π is:

$$\dfrac{\dfrac{}{A \vdash A}\ \mathrm{ax} \qquad \dfrac{\dfrac{}{A \vdash A}\ \mathrm{ax} \qquad \dfrac{}{A, B \vdash B}\ \mathrm{ax}}{A, A \vdash B} \Rightarrow\mathrm{L},\ A \longrightarrow A \Rightarrow B}{A \vdash B}\ \mathrm{cut}$$

The Zenon Automated Theorem Prover

Features of Zenon

- First order logic with equality;
- Tableau-based proof search method;
- Extensible by adding new deductive rules;
- Certifying, 3 outputs: Coq, Isabelle, Dedukti;
- Used by other systems: Focalize, TLA.

Zenon

- Reference:

  R. Bonichon, D. Delahaye, D. Doligez. *Zenon: An Extensible Automated Theorem Prover Producing Checkable Proofs.* LPAR (2007).
- Freely available (BSD license);
- Developed by D. Doligez;
- Download: `http://focal.inria.fr/zenon/`

The Zenon Automated Theorem Prover

le cnam
Inria
BWare

The Tableau Method

- We start from the negation of the goal (no clausal form);
- We apply the rules in a top-down fashion;
- We build a tree whose each branch must be closed;
- When the tree is closed, we have a proof of the goal.

Closure and Cut Rules

$$\frac{\bot}{\odot}\ \odot_{\bot} \qquad \frac{\neg\top}{\odot}\ \odot_{\neg\top} \qquad \frac{}{P \mid \neg P}\ \text{cut}$$

$$\frac{\neg R_r(t,t)}{\odot}\ \odot_r \qquad \frac{P \quad \neg P}{\odot}\ \odot \qquad \frac{R_s(a,b) \quad \neg R_s(b,a)}{\odot}\ \odot_s$$

Analytic Rules

$$\frac{\neg\neg P}{P}\,\alpha_{\neg\neg} \qquad \frac{P \Leftrightarrow Q}{\neg P, \neg Q \mid P, Q}\,\beta_{\Leftrightarrow} \qquad \frac{\neg(P \Leftrightarrow Q)}{\neg P, Q \mid P, \neg Q}\,\beta_{\neg\Leftrightarrow}$$

$$\frac{P \wedge Q}{P, Q}\,\alpha_{\wedge} \qquad \frac{\neg(P \vee Q)}{\neg P, \neg Q}\,\alpha_{\neg\vee} \qquad \frac{\neg(P \Rightarrow Q)}{P, \neg Q}\,\alpha_{\neg\Rightarrow}$$

$$\frac{P \vee Q}{P \mid Q}\,\beta_{\vee} \qquad \frac{\neg(P \wedge Q)}{\neg P \mid \neg Q}\,\beta_{\neg\wedge} \qquad \frac{P \Rightarrow Q}{\neg P \mid Q}\,\beta_{\Rightarrow}$$

$$\frac{\exists x\ P(x)}{P(\epsilon(x).P(x))}\,\delta_{\exists} \qquad \frac{\neg\forall x\ P(x)}{\neg P(\epsilon(x).\neg P(x))}\,\delta_{\neg\forall}$$

γ-Rules

$$\frac{\forall x\ P(x)}{P(X)}\ \gamma_{\forall M} \qquad \frac{\neg\exists x\ P(x)}{\neg P(X)}\ \gamma_{\neg\exists M}$$

$$\frac{\forall x\ P(x)}{P(t)}\ \gamma_{\forall\text{inst}} \qquad \frac{\neg\exists x\ P(x)}{\neg P(t)}\ \gamma_{\neg\exists\text{inst}}$$

Relational Rules

- Equality, reflexive, symmetric, transitive rules;
- Are not involved in the computation of superdeduction rules.

Example of Proof Search

$$\frac{\forall x\ (P(x) \lor Q(x))\ ,\quad \neg P(a)\ ,\quad \neg Q(a)}{}$$

Example of Proof Search

$$\frac{\forall x\ (P(x) \lor Q(x))\ ,\quad \neg P(a)\ ,\quad \neg Q(a)}{P(X) \lor Q(X)}\ \gamma_{\forall M}$$

The Zenon Automated Theorem Prover

Example of Proof Search

$$\dfrac{\dfrac{\forall x\ (P(x) \lor Q(x))\ ,\quad \neg P(a)\ ,\quad \neg Q(a)}{P(X) \lor Q(X)}\ \gamma_{\forall M}}{P(X) \qquad\qquad Q(X)}\ \beta_{\lor}$$

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Example of Proof Search

$$\dfrac{\dfrac{\forall x\ (P(x) \lor Q(x))\ ,\ \ \neg P(a)\ ,\ \ \neg Q(a)}{P(X) \lor Q(X)}\ \gamma_{\forall M}}{P(X) \qquad\qquad Q(X)}\ \beta_{\lor}$$

Example of Proof Search

$$\dfrac{\forall x\,(P(x) \vee Q(x))\ ,\quad \neg P(a)\ ,\quad \neg Q(a)}{\dfrac{P(X) \vee Q(X)}{\dfrac{P(X)}{P(a) \vee Q(a)}\,\gamma_{\forall\mathrm{inst}} \qquad Q(X)}\,\beta_{\vee}}\,\gamma_{\forall M}$$

Example of Proof Search

$$\dfrac{\forall x\,(P(x)\lor Q(x))\ ,\ \ \neg P(a)\ ,\ \ \neg Q(a)}{\dfrac{P(X)\lor Q(X)}{\dfrac{P(X)}{\dfrac{P(a)\lor Q(a)}{P(a)\qquad Q(a)}\,\beta_{\lor}}\,\gamma_{\forall\mathrm{inst}} \qquad\qquad Q(X)}\,\beta_{\lor}}\,\gamma_{\forall M}$$

Example of Proof Search

$$\cfrac{\forall x\,(P(x)\lor Q(x))\ ,\ \ \neg P(a)\ ,\ \ \neg Q(a)}{\cfrac{P(X)\lor Q(X)}{\cfrac{P(X)}{\cfrac{P(a)\lor Q(a)}{\cfrac{P(a)}{\odot}\quad \odot \quad \cfrac{Q(a)}{}}\ \beta_\lor}\ \gamma_{\forall\mathrm{inst}} \qquad\qquad Q(X)}\ \beta_\lor}\ \gamma_{\forall M}$$

Example of Proof Search

$$\dfrac{\forall x\,(P(x)\lor Q(x))\,,\ \neg P(a)\,,\ \neg Q(a)}{\dfrac{P(X)\lor Q(X)}{\dfrac{P(X)}{\dfrac{P(a)\lor Q(a)}{\dfrac{P(a)}{\odot}\odot \qquad \dfrac{Q(a)}{\odot}\odot}\beta_\lor}\gamma_{\forall\text{inst}} \qquad Q(X)}\beta_\lor}\gamma_{\forall M}$$

Example of Proof Search

$$\dfrac{\forall x\,(P(x)\lor Q(x))\ ,\quad \neg P(a)\ ,\quad \neg Q(a)}{\dfrac{P(X)\lor Q(X)}{\dfrac{P(X)}{\dfrac{P(a)\lor Q(a)}{\dfrac{P(a)}{\odot}\,\odot \qquad \dfrac{Q(a)}{\odot}\,\odot}\,\beta_{\lor}}\,\gamma_{\forall\mathrm{inst}} \qquad Q(X)}\,\beta_{\lor}}\,\gamma_{\forall M}$$

Example of Proof Search

$$\dfrac{\forall x\ (P(x) \vee Q(x))\ ,\ \ \neg P(a)\ ,\ \ \neg Q(a)}{\dfrac{P(a) \vee Q(a)}{\dfrac{P(a)}{\odot}\ \odot \qquad \dfrac{Q(a)}{\odot}\ \odot}\ \beta_{\vee}}\ \gamma_{\forall\mathrm{inst}}$$

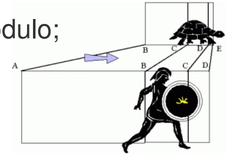
Integrating Deduction Modulo to Zenon

Goals

- Improve the proof search in axiomatic theories;
- Reduce the proof size;
- New tool: Zenon + Deduction Modulo = Zenon Modulo!

Compared to Super Zenon

- Extension of Zenon to superdeduction;
- Superdeduction: variant of deduction modulo;
- Freely available (GPL license);
- Collaboration Cnam and Siemens;
- Download:
  `http://cedric.cnam.fr/~delahaye/super-zenon/`



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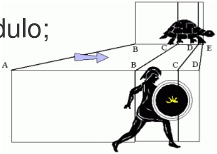
- Reference:

  M. Jacquel, K. Berkani, D. Delahaye, C. Dubois. *Tableaux Modulo Theories Using Superdeduction: An Application to the Verification of B Proof Rules with the Zenon Automated Theorem Prover.* IJCAR (2012).



Integrating Deduction Modulo to Zenon

Goals

- Improve the proof search in axiomatic theories;
- Reduce the proof size;
- New tool: Zenon + Deduction Modulo = Zenon Modulo!

Compared to Super Zenon

- Compare deduction modulo and superdeduction in practice;
- Rewrite rules over propositions and terms;
- Normalization strategies (efficiency);
- Light integration (metavariable management);
- No trace of computation in the proofs.

Class Rewrite System

Definition

A class rewrite system is a pair consisting of:

- $\mathcal{R}$: a set of proposition rewrite rules;
- $\mathcal{E}$: a set of term rewrite rules (and equational axioms).

Rewrite Rules

- Proposition rewrite rule: $l \longrightarrow r$, where l is an atomic proposition and $FV(r) \subseteq FV(l)$;
- Term rewrite rule: $l \longrightarrow r$, where $FV(r) \subseteq FV(l)$.

Congruence

- $=_{\mathcal{RE}} \equiv$ congruence generated by the set $\mathcal{R} \cup \mathcal{E}$.

Rules of Zenon Modulo

Closure and Cut Rules

$$\frac{P \qquad \neg Q}{\odot}\ \odot \ \text{if } P =_{\mathcal{RE}} Q \qquad \frac{}{P \mid \neg Q}\ \text{cut} \ \text{if } P =_{\mathcal{RE}} Q$$

$$\frac{P}{\odot}\ \odot_\bot \ \text{if } P =_{\mathcal{RE}} \bot \qquad \frac{\neg P}{\odot}\ \odot_{\neg\top} \ \text{if } P =_{\mathcal{RE}} \top$$

$$\frac{\neg P}{\odot}\ \odot_r \ \text{if } P =_{\mathcal{RE}} R_r(t,t) \qquad \frac{P \qquad \neg Q}{\odot}\ \odot_s \ \text{if } P =_{\mathcal{RE}} R_s(a,b) \text{ and } Q =_{\mathcal{RE}} R_s(b,a)$$

Where R_r is a reflexive relation, and R_s a symmetric relation.

Rules of Zenon Modulo

α/β-Rules

$$\frac{\neg S}{P}\ \alpha_{\neg\neg}\quad \text{if } S =_{\mathcal{RE}} \neg P$$

$$\frac{S}{P, Q}\ \alpha_{\wedge}\quad \text{if } S =_{\mathcal{RE}} P \wedge Q \qquad \frac{\neg S}{\neg P \mid \neg Q}\ \beta_{\neg\wedge}\quad \text{if } S =_{\mathcal{RE}} P \wedge Q$$

$$\frac{S}{P \mid Q}\ \beta_{\vee}\quad \text{if } S =_{\mathcal{RE}} P \vee Q \qquad \frac{\neg S}{\neg P, \neg Q}\ \alpha_{\neg\vee}\quad \text{if } S =_{\mathcal{RE}} P \vee Q$$

$$\frac{S}{\neg P \mid Q}\ \beta_{\Rightarrow}\quad \text{if } S =_{\mathcal{RE}} P \Rightarrow Q \qquad \frac{\neg S}{P, \neg Q}\ \alpha_{\neg\Rightarrow}\quad \text{if } S =_{\mathcal{RE}} P \Rightarrow Q$$

$$\frac{S}{\neg P, \neg Q \mid P, Q}\ \beta_{\Leftrightarrow}\quad \text{if } S =_{\mathcal{RE}} P \Leftrightarrow Q$$

$$\frac{\neg S}{\neg P, Q \mid P, \neg Q}\ \beta_{\neg\Leftrightarrow}\quad \text{if } S =_{\mathcal{RE}} P \Leftrightarrow Q$$

Rules of Zenon Modulo

δ/γ-Rules

$$\frac{S}{P(\epsilon(x).P(x))}\ \delta_{\exists}\ \text{ if } S =_{\mathcal{RE}} \exists x\ P(x)$$

$$\frac{\neg S}{\neg P(\epsilon(x).\neg P(x))}\ \delta_{\neg\forall}\ \text{ if } S =_{\mathcal{RE}} \forall x\ P(x)$$

$$\frac{S}{P(X)}\ \gamma_{\forall M}\ \text{ if } S =_{\mathcal{RE}} \forall x\ P(x) \qquad \frac{\neg S}{\neg P(X)}\ \gamma_{\neg\exists M}\ \text{ if } S =_{\mathcal{RE}} \exists x\ P(x)$$

$$\frac{S}{P(t)}\ \gamma_{\forall\text{inst}}\ \text{ if } S =_{\mathcal{RE}} \forall x\ P(x) \qquad \frac{\neg S}{\neg P(t)}\ \gamma_{\neg\exists\text{inst}}\ \text{ if } S =_{\mathcal{RE}} \exists x\ P(x)$$

Example of Proof

Example with the Set Inclusion

- With regular rules of Zenon:

$$\dfrac{\dfrac{\forall a \forall b\, ((a \subseteq b) \Leftrightarrow (\forall x\, (x \in a \Rightarrow x \in b))), A \not\subseteq A}{(X \subseteq Y) \Leftrightarrow (\forall x\, (x \in X \Rightarrow x \in Y))}\,\gamma_{\forall M} \times 2}{\dfrac{\dfrac{X \subseteq Y, \forall x\, (x \in X \Rightarrow x \in Y)}{(A \subseteq A) \Leftrightarrow (\forall x\, (x \in A \Rightarrow x \in A))}\,\gamma_{\forall\text{inst}} \times 2}{\dfrac{A \subseteq A, \forall x\, (x \in A \Rightarrow x \in A)}{\odot}\,\odot \qquad \Pi}\,\beta_{\Leftrightarrow} \qquad \Pi'}\,\beta_{\Leftrightarrow}$$

Where Π is:

$$\dfrac{\dfrac{\dfrac{A \not\subseteq A, \neg\forall x\, (x \in A \Rightarrow x \in A)}{\neg(\epsilon_x \in A \Rightarrow \epsilon_x \in A)}\,\delta_{\neg\forall}}{\epsilon_x \in A, \epsilon_x \notin A}\,\alpha_{\neg\Rightarrow}}{\odot}\,\odot$$

with $\epsilon_x = \epsilon(x).\neg(x \in A \Rightarrow x \in A)$

Example of Proof

Example with the Set Inclusion

- With regular rules of Zenon:

$$\dfrac{\forall a \forall b\,((a \subseteq b) \Leftrightarrow (\forall x\,(x \in a \Rightarrow x \in b))), A \not\subseteq A}{\dfrac{(A \subseteq A) \Leftrightarrow (\forall x\,(x \in A \Rightarrow x \in A))}{\dfrac{A \subseteq A, \forall x\,(x \in A \Rightarrow x \in A)}{\odot}\,\odot \qquad \Pi}\,\beta_{\Leftrightarrow}}\,\gamma_{\forall\text{inst}} \times 2$$

Where Π is:

$$\dfrac{A \not\subseteq A, \neg\forall x\,(x \in A \Rightarrow x \in A)}{\dfrac{\neg(\epsilon_x \in A \Rightarrow \epsilon_x \in A)}{\dfrac{\epsilon_x \in A, \epsilon_x \notin A}{\odot}\,\odot}\,\alpha_{\neg\Rightarrow}}\,\delta_{\neg\forall}$$

with $\epsilon_x = \epsilon(x).\neg(x \in A \Rightarrow x \in A)$

Example of Proof

Example with the Set Inclusion

- With the rules of Zenon Modulo:

$$
\dfrac{\dfrac{\dfrac{\dfrac{A \not\subseteq A}{\neg\forall x\ (x \in A \Rightarrow x \in A)}\ A{\subseteq}A \longrightarrow \forall x\ (x{\in}A{\Rightarrow}x{\in}A)}{\neg(\epsilon_x \in A \Rightarrow \epsilon_x \in A)}\ \delta_{\neg\forall}}{\epsilon_x \in A, \epsilon_x \notin A}\ \alpha_{\neg\Rightarrow}}{\odot}\ \odot
$$

with $\epsilon_x = \epsilon(x).\neg(x \in A \Rightarrow x \in A)$

Example of Proof

Example with the Set Inclusion

- With the rules of Zenon Modulo:

$$\cfrac{\cfrac{A \not\subseteq A}{\neg(\epsilon_x \in A \Rightarrow \epsilon_x \in A)}\ \delta_{\neg\forall},\ A{\subseteq}A =_{\mathcal{RE}} \forall x\ (x{\in}A{\Rightarrow}x{\in}A)}{\cfrac{\epsilon_x \in A, \epsilon_x \notin A}{\odot}\ \odot}\ \alpha_{\neg\Rightarrow}$$

with $\epsilon_x = \epsilon(x).\neg(x \in A \Rightarrow x \in A)$

Zenon Modulo over the TPTP Library

For any First Order Theory

- Automated orientation of the theories;
- Not oriented axioms left as axioms.

Heuristic

- $\forall \bar{x}\ (P \Leftrightarrow \varphi)$: $P \longrightarrow \varphi$ is generated if $\mathrm{FV}(\varphi) \subseteq \mathrm{FV}(P)$;
  Otherwise if φ literal and $\mathrm{FV}(P) \subset \mathrm{FV}(\varphi)$ then apply heuristic to $\forall \bar{x}\ (\varphi \Leftrightarrow P)$;
- $\forall \bar{x}\ (\neg P \Leftrightarrow \varphi)$: $P \longrightarrow \neg\varphi$ is generated if $\mathrm{FV}(\varphi) \subseteq \mathrm{FV}(P)$;
  Otherwise if φ literal and $\mathrm{FV}(P) \subset \mathrm{FV}(\varphi)$ then apply heuristic to $\forall \bar{x}\ (\varphi \Leftrightarrow \neg P)$;
- $\forall \bar{x}\ (s = t)$: $s \longrightarrow t$ is generated if $\mathrm{FV}(t) \subseteq \mathrm{FV}(s)$;
  Otherwise $t \longrightarrow s$ if $\mathrm{FV}(s) \subset \mathrm{FV}(t)$;
  In addition, commutativity axioms are excluded.

Experimental Results

Figures

TPTP Category	Zenon	Zenon Mod. (Prop. Rew.)	Zenon Mod. (Term/Prop. Rew.)
FOF 6,659 prob.	1,586	1,626 (2.5%) +114 (7.2%) -74 (4.7%)	1,616 (1.9%) +170 (10.7%) -140 (8.8%)
SET 462 prob.	149	219 (47%) +78 (52.3%) -8 (5.4%)	222 (49%) +86 (57.7%) -13 (8.7%)

- TPTP Library v5.5.0;
- Intel Xeon X5650 2.67GHz;
- Timeout 300 s, memory limit 1 GB.

Experimental Results

Figures

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SET 462 prob.	149	219 (47%) +78 (52.3%) -8 (5.4%)	222 (49%) +86 (57.7%) -13 (8.7%)

- 29 difficult problems (TPTP ranking);
- 29 with a ranking ≥ 0.7;
- 9 with a ranking ≥ 0.8;
- 1 with a ranking ≥ 0.9.

Proof Compression

Experiment

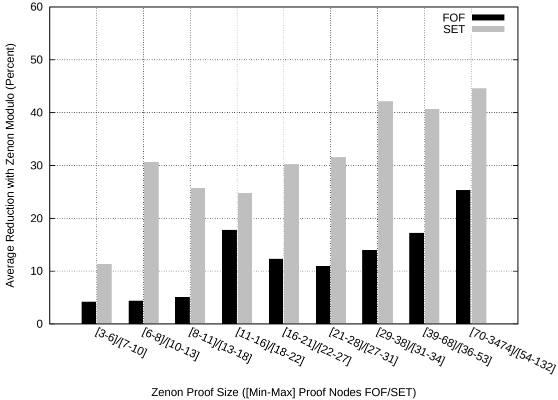
- 1,446 problems proved by both Zenon and Zenon Modulo;
- 624 FOF problems and 110 SET problems;
- Subset of proofs where rewriting occurs;
- Measure: number of proof nodes of the resulting proof.

Figures

TPTP Category	Average Reduction	Maximum Reduction
FOF 624 problems	6.8%	91.4%
SET 110 problems	21.6%	84.6%

Proof Compression

Figures



A Backend for Zenon Modulo

Using the Existing Backends

- Create special inference nodes for rewriting rules;
- Record rewrite steps in the proof traces;
- Extend the existing backends of Zenon;
- Prove the rewriting lemmas in Coq and Isabelle.

Problems of this Approach

- Possible large number of rewrite steps to record;
- May Lead to memory explosion;
- Against the Poincaré principle;
- Loss of deduction modulo benefits.

Using the Dedukti Universal Proof Checker

Features of Dedukti

- Universal proof checker for the $\lambda\Pi$-calculus modulo;
- Propositions/types and proofs/λ-terms (Curry-Howard);
- Native support of rewriting;
- Only need to provide the set of rewrite rules.

Dedukti

- Freely available (CeCILL-B license);
- Developed by Deducteam;
- Download:
  `https://www.rocq.inria.fr/deducteam/Dedukti/`

Using the Dedukti Universal Proof Checker

From Zenon Modulo Proofs to Dedukti

- From classical to intuitionistic logic;
- Based on a double-negation translation;
- Optimized to minimize the number of double-negations;
- 54% of the TPTP proofs already intuitionistic.

Dedukti

- Freely available (CeCILL-B license);
- Developed by Deducteam;
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Experimental Results over the TPTP Library

Figures

FOF 624 prob.	Dedukti Success	Dedukti Failure	Backend Issue
Problems	559	5	60
Rate	89.6%	0.8%	9.6%

Failures

- Dedukti: rewrite system (termination, confluence, etc.);
- Backend: minimization of the double-negations.

References for Zenon Modulo

Rules, Results, and Backend

- LPAR'13 paper:

  D. Delahaye, D. Doligez, F. Gilbert, P. Halmagrand, O. Hermant. *Zenon Modulo: When Achilles Outruns the Tortoise using Deduction Modulo.* LPAR (2013).

Proof Certification and Compression

- IWIL'13 paper:

  D. Delahaye, D. Doligez, F. Gilbert, P. Halmagrand, O. Hermant. *Zenon Modulo: When Achilles Uses Deduction Modulo to Outrun the Tortoise with Shorter Steps.* IWIL (2013).

The BWare Project

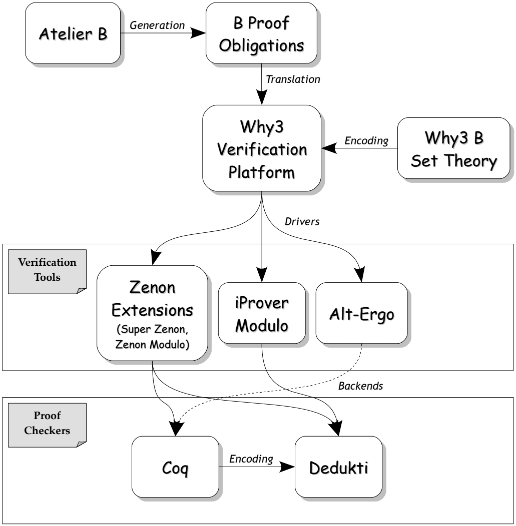
The Project

- INS prog. of the French National Research Agency (ANR);
- Academics: Cnam, LRI, Inria;
- Companies: Mitsubishi, ClearSy, OCamlPro.

Goals

- Mechanized framework for automated verification of B PO;
- Generic platform (several automated deduction tools);
- First order tools and SMT solvers;
- Production of proof objects (certificates).

The BWare Project



Deduction Modulo in the BWare Project

Tools

- Super Zenon, Zenon Modulo (extensions of Zenon);
- iProver Modulo (extension of iProver);
- Backend for these tools: Dedukti.

Adequacy of the Tools

- Build a B set theory modulo (manually);
- Comprehension scheme (higher order) hard-coded;
- Good results of Super Zenon for B proof rules;
- Good results of Zenon Modulo in the SET category of TPTP.

Conclusion

Deduction Modulo in Automated Tools

- Resolution: iProver Modulo (based on iProver);
- Tableaux: Super Zenon, Zenon Modulo (based on Zenon);
- Appropriate backend: Dedukti ($\lambda\Pi$-calculus modulo).

Experimental Results

- Performances increased for generic benchmarks (TPTP);
- Successful use in industrial settings (B method):
  - Collaboration Cnam/Siemens: verification of B proof rules;
  - BWare project: verification of B PO (work in progress).

Automated Deduction

Automated Generation of Theories Modulo

- Generation of theories modulo “on the fly”;
- Preservation of “good” properties (cut-free completeness);
- Difficulties for term rewrite rules (heuristics);
- Use of external tools to study the rewrite system;
- Integration of the equational axioms (rewriting modulo).

Set Theory Modulo

- Good experimental results for set theory;
- Results of Super Zenon (B), Zenon Modulo (TPTP);
- Ability to prove difficult problems in this domain;
- Promising for the BWare project;
- Problem of large formulas, large contexts (PO).

Proof Checking for Automated Tools

- $\lambda\Pi$-calculus modulo appropriate to encode theories;
- Suitable framework to certify deduction modulo proofs;
- High quality proof certificates (size in particular);
- Dedukti as a backend for several automated tools:
  - Zenon Modulo (extension of Zenon);
  - iProver Modulo (extension of iProver).

Interoperability between Proof Systems

- Shallow embeddings of theories;
- Dedukti embeddings:
  - CoqInE (from Coq);
  - Holide (from HOL);
  - Focalide (from Focalize).