

Constraint Programming for Itemset Mining with Multiple Minimum Supports

Mohamed-Bachir Belaid¹, Nadjib Lazaar²

30/11/2021

1

simula



2

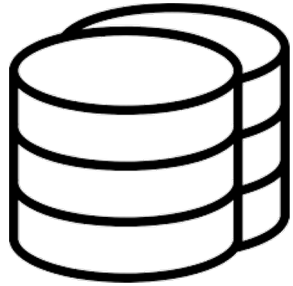


LIRMM

Introduction

- ▶ Aim: show the flexibility of CP to cope with additional dimension (multiple support)
- ▶ Can we do it? How? Is the propagation complete?
- ▶ What is the motivation? How can it be useful? Interesting queries?
- ▶ Accepted at ICTAI 2021

Motivation



Dataset



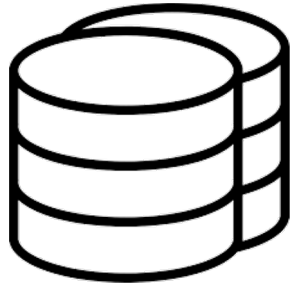
Query



Specialized
Algorithm



Motivation



Dataset



Query

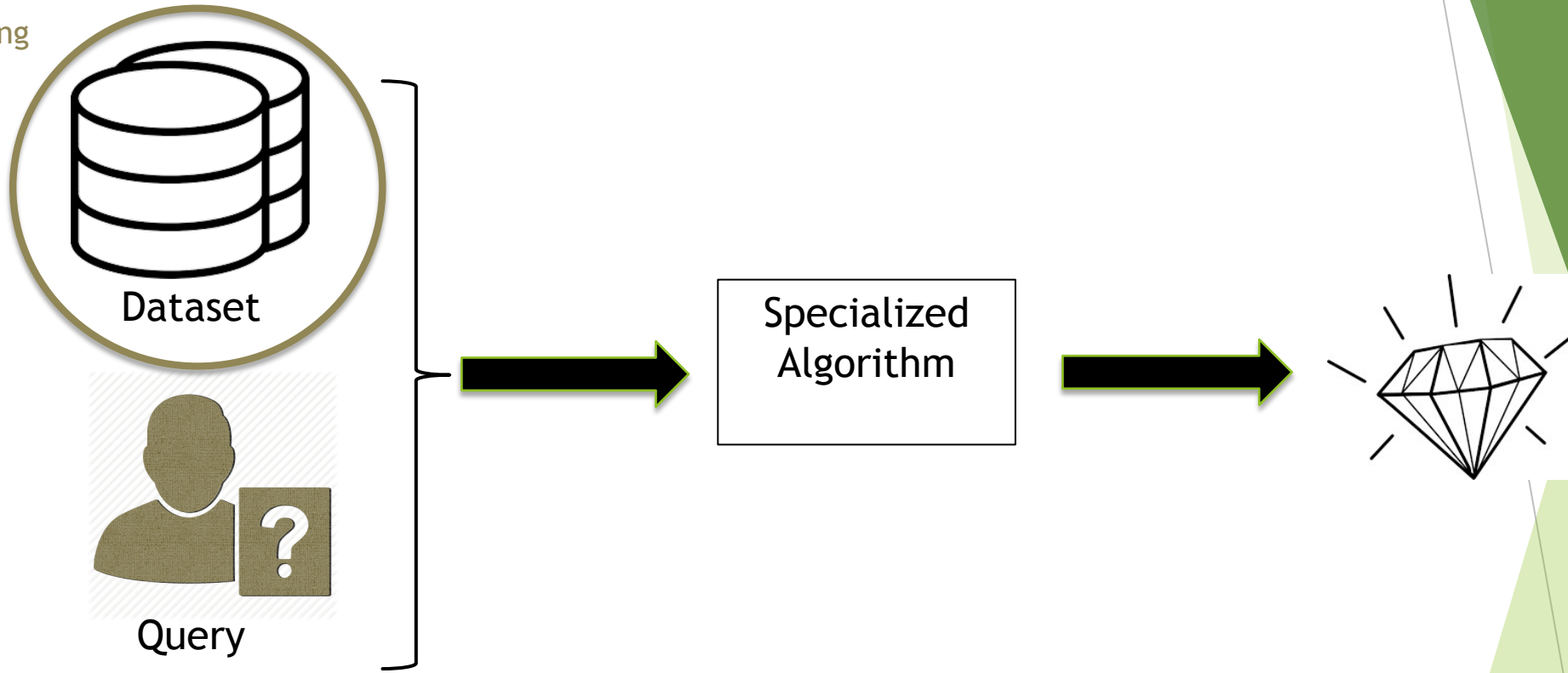


Specialized
Algorithm



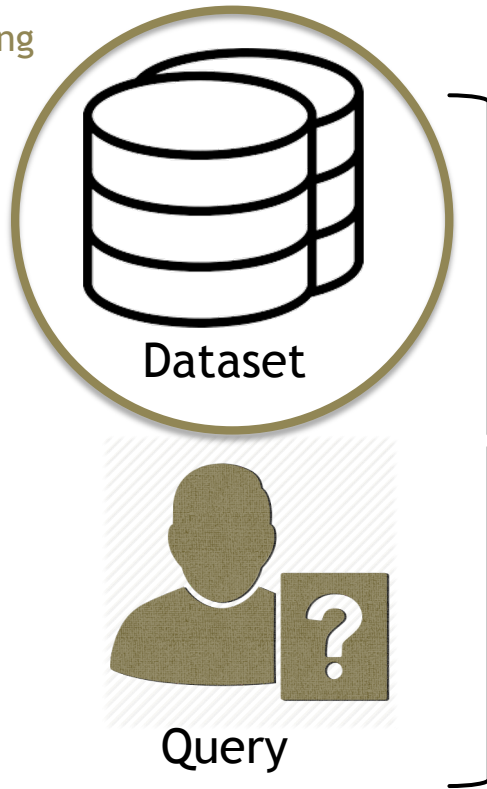
Motivation

1- Pre-processing



Motivation

1- Pre-processing



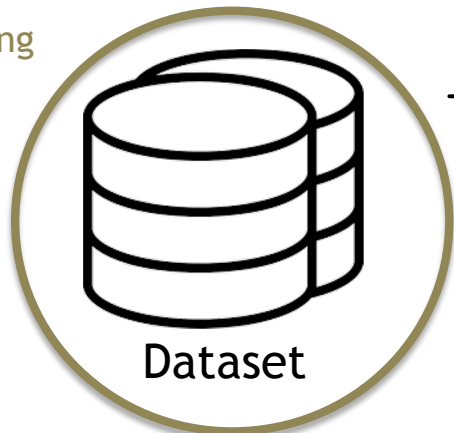
Specialized
Algorithm



2- Post-processing

Motivation

1- Pre-processing

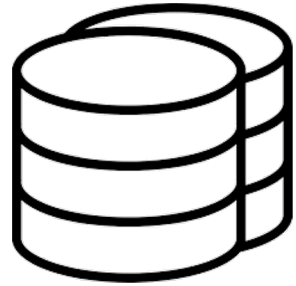


Specialized
Algorithm

3- New algorithm



Motivation



Dataset



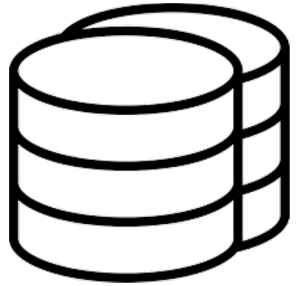
Query



A generic
framework



Motivation



Dataset



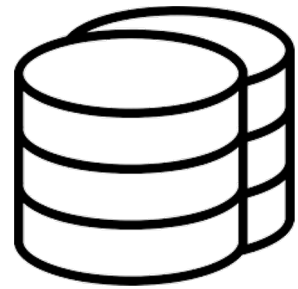
Query



A generic
framework



Motivation



Dataset



Query



A generic
framework



Constraint
Programming
Model



Constraint
Programming
Solver



Solutions

Constraint Programming

Constraint Programming

- ▶ In Constraint Programming (CP) the user declares:

- ▶ A set of variables

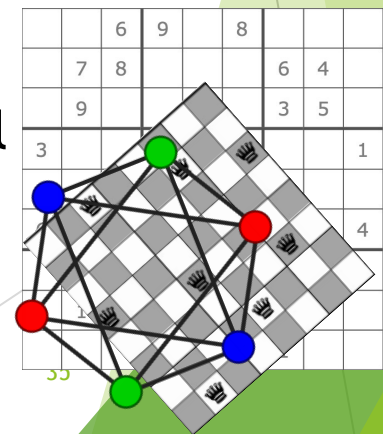
$$X = \{x_1, \dots, x_n\}$$

- ▶ A set of domains (set of possible values)

$$\text{dom} = \{\text{dom}(x_1), \dots, \text{dom}(x_n)\}$$

- ▶ A set of constraints C on variables where c is a relation between set of variables

- ▶ The constraint solver finds **solutions** (assignments on X satisfying all constraints)



Constraint Programming

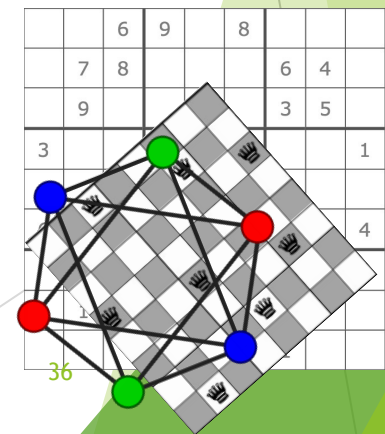
- ▶ A filtering algorithm (aka propagator)

$$\text{dom}(X_1) = \{1,2,4\}$$

$$\text{dom}(X_2) = \{2,3,5\}$$

$$\text{dom}(X_3) = \{3,8,9\}$$

$$X_1 + X_2 = X_3$$



Constraint Programming

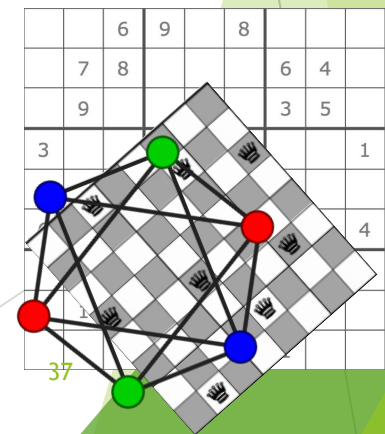
- ▶ A filtering algorithm (aka propagator)

$$\text{dom}(X_1) = \{1, \cancel{2}, 4\}$$

$$\text{dom}(X_2) = \{2, 3, 5\}$$

$$\text{dom}(X_3) = \{3, 8, 9\}$$

$$X_1 + X_2 = X_3$$



Constraint Programming

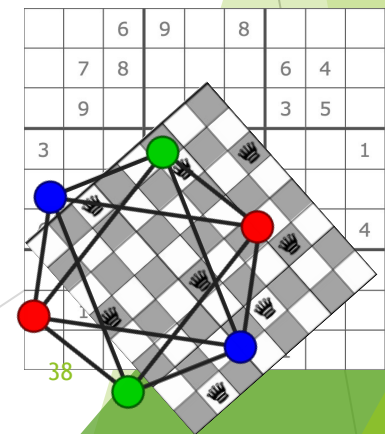
- ▶ A filtering algorithm (aka propagator)

$$\text{dom}(X_1) = \{1, \cancel{2}, 4\}$$

$$\text{dom}(X_2) = \{2, \cancel{3}, 5\}$$

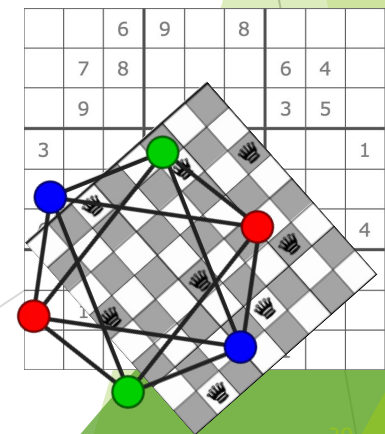
$$\text{dom}(X_3) = \{3, \cancel{8}, 9\}$$

$$X_1 + X_2 = X_3$$



Global Constraints

- ▶ Constraints defined by a relation on any number of variables
- ▶ Example: **AllDifferent**(x_1, \dots, x_n) specifies that all its variables must take different values



Itemset Mining

Itemset Mining

- ▶ Find useful patterns from transaction databases



Itemset Mining

- Find useful patterns from transaction databases



\mathcal{T}

\mathcal{I}

TD	Items				
1					
2					
3					
4					
5					
6					

Frequent/Infrequent Itemsets

- ▶ Itemset = set of items
- ▶ Cover: $\text{cover}(AB) = \{t_1, t_4, t_5\}$
- ▶ Frequency: $\text{freq}(AB) = |\text{cover}(AB)| = 3$
- ▶ Given a frequency threshold $s = 3$:
 - ▶ AB is frequent ($\text{freq}(AB) = 3 \geq 3$)
 - ▶ AD is infrequent ($\text{freq}(AD) = 2 < 3$)

trans.	Items				
t_1	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
t_2		<i>B</i>	<i>C</i>		
t_3		<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>
t_4	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	
t_5	<i>A</i>	<i>B</i>	<i>C</i>		<i>E</i>
t_6		<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>

Frequent/Infrequent Itemsets

- ▶ Itemset = set of items
- ▶ Cover: $\text{cover}(AB) = \{t_1, t_4, t_5\}$
- ▶ Frequency: $\text{freq}(AB) = |\text{cover}(AB)| = 3$
- ▶ Given a frequency threshold $s = 3$:
 - ▶ AB is frequent ($\text{freq}(AB) = 3 \geq 3$)
 - ▶ AD is infrequent ($\text{freq}(AD) = 2 < 3$)

trans.	Items				
t_1	A	B	C	D	E
t_2		B	C		
t_3		B	C	D	E
t_4	A	B	C	D	
t_5	A	B	C		E
t_6		B	C	D	E

Frequent/Infrequent Itemsets

- ▶ Itemset = set of items
- ▶ Cover: $\text{cover}(AB) = \{t_1, t_4, t_5\}$
- ▶ Frequency: $\text{freq}(AB) = |\text{cover}(AB)| = 3$
- ▶ Given a frequency threshold $s = 3$:
 - ▶ AB is frequent ($\text{freq}(AB) = 3 \geq 3$)
 - ▶ AD is infrequent ($\text{freq}(AD) = 2 < 3$)

trans.	Items				
t_1	A	B	C	D	E
t_2		B	C		
t_3		B	C	D	E
t_4	A	B	C	D	
t_5	A	B	C		E
t_6		B	C	D	E

Single threshold problem (example)



\mathcal{T}

\mathcal{I}

TD	Items	
1		
2		
3		
4		
5		
6		
7		
8		

Single threshold problem (example)



$$S = 4$$

\mathcal{T}

\mathcal{I}

TD	Items	
1		
2		
3		
4		
5		
6		
7		
8		

Single threshold problem (example)



$$S = 4$$

\mathcal{T}

\mathcal{I}

TD	Items				
1					
2					
3					
4					
5					
6					
7					
8					

Single threshold problem (example)



$S = 1$

\mathcal{T}

\mathcal{I}

TD	Items	
1		
2		
3		
4		
5		
6		
7		
8		

Single threshold problem (example)



$S = 1$

\mathcal{T}

\mathcal{I}

TD	Items
1	
2	
3	
4	
5	
6	
7	
8	

Single threshold problem (example)



\mathcal{T}

\mathcal{I}

TD	Items									
1										
2										
3										
4										
5										
6										
7										
8										
MIS	3	4	5	3	3	2	2	1	1	

Single threshold problem (example)



\mathcal{T}

\mathcal{I}

TD	Items									
1										
2										
3										
4										
5										
6										
7										
8										
MIS	3	4	5	3	3	2	2	1	1	

Single threshold problem (example)



\mathcal{T}

\mathcal{I}

TD	Items									
1										
2										
3										
4										
5										
6										
7										
8										
MIS	3	4	5	3	3	2	2	1	1	

A constraint?



Basic CP model for mining frequent itemsets (Luc De Raedt et.al, 2008)

- ▶ Variables:
 - ▶ A binary variable for every item i : the presence of the item i in the searched itemset (P)
 - ▶ A binary variable for every transaction t : the presence of the searched itemset (P) in the transaction t
- ▶ Constraints (reified):
 - ▶ Cover constraint
 - ▶ Threshold constraint ($\text{freq}(P) \geq s$)

With multiple minimum supports (MIS)

- ▶ Extend the model:
 - ▶ Replace “ $\text{freq}(P) \geq s$ ” by “ $\text{freq}(P) \geq \min(\text{MIS}_k | k \text{ in } P)$ ”
 - ▶ Does not scale!
- ▶ Define a global constraint “FreqRare”:
 - ▶ Only item variables (no need for transaction variables)
 - ▶ Dedicated propagator

FreqRare

- ▶ Holds if the searched itemset ($P=\{i \mid x_i=1\}$) is frequent w.r.t the list MIS
- ▶ Propagator \rightarrow remove 1 from x_i if including i results a frequency less than the minimum of remaining MIS values
- ▶ Time complexity: $O(|\text{items}| * |\text{transactions}|)$
- ▶ Result \rightarrow Backtrack-free using minimum MIS as variable ordering heuristic

User queries

- ▶ In CP → simply extend the model
- ▶ Specialized methods → a post processing step (checker)

User queries

- ▶ Return itemsets including items of the same type (distance between MISs is bounded above):
 - ▶ $|MIS_i - MIS_j| \leq ub$
- ▶ Size of the itemset is bounded below:
 - ▶ $|P| \geq c$
- ▶ K-pattern mining [Guns et al., 2011] (K patterns with constraints between them):
 - ▶ K vectors of Boolean variables
 - ▶ K distinct itemsets satisfying both constraints

User queries

\mathcal{I}

TD	Items									
1										
2										
3										
4										
5										
6										
7										
8										
MIS	3	4	5	3	3	2	2	1	1	

User queries

- ▶ Return itemsets including items of the same type (distance between MISs is bounded above):
 - ▶ $|MIS_i - MIS_j| \leq ub$
- ▶ Size of the itemset is bounded below:
 - ▶ $|P| \geq c$
- ▶ K-pattern mining [Guns et al., 2011] (K patterns with constraints between them):
 - ▶ K vectors of Boolean variables
 - ▶ K distinct itemsets satisfying both constraints

Experiments

- ▶ We selected several real-sized datasets from the FIMI repository
- ▶ Our approach (**CP4MIS**) compared with: 1) CPFGrowth++ (SPMF implementation) 2) Basic CP Model (Rmodel)
- ▶ For CP we have used **Oscar solver** within Scala
- ▶ $MIS_i = \max(\text{Beta} * \text{freq}(i), \text{Min})$ as in [Bing Liu et.al, 1999]
- ▶ Machine = Intel core i7, 2.8Ghz with a RAM of **16GB**
- ▶ Time limit = one hour

Results (Mining frequent itemsets)

Q0:	CFPG	Rmodel		CP4MIS		#sol
	(a) Time	(b) Time	(b) Memory	(c) Time	(c) Memory	
Zoo	0.81	12.00	3,760	1.34	20	1.3M
Vote	1.56	196.17	2,164	2.23	8	2.1M
Anneal	30.91	134.74	3,095	64.82	49	71.7M
Chess	11.64	305.03	3,153	28.20	67	22.6M
Mushroom	45.53	TO	–	106.00	48	105.2M
Connect	48.45	TO	–	854.59	218	91.7M
T40	409.55	–	OOM	91.70	2,304	15.8M
Pumsb	38.60	–	OOM	115.67	916	13.5M

Results (Mining frequent itemsets)

Q0:	CFPG	Rmodel		CP4MIS		#sol
	(a)	(b)		(c)		
	Time	Time	Memory	Time	Memory	
Zoo	0.81	12.00	3,760	1.34	20	1.3M
Vote	1.56	196.17	2,164	2.23	8	2.1M
Anneal	30.91	134.74	3,095	64.82	49	71.7M
Chess	11.64	305.03	3,153	28.20	67	22.6M
Mushroom	45.53	TO	–	106.00	48	105.2M
Connect	48.45	TO	–	854.59	218	91.7M
T40	409.55	–	OOM	91.70	2,304	15.8M
Pumsb	38.60	–	OOM	115.67	916	13.5M

Results (Mining frequent itemsets)

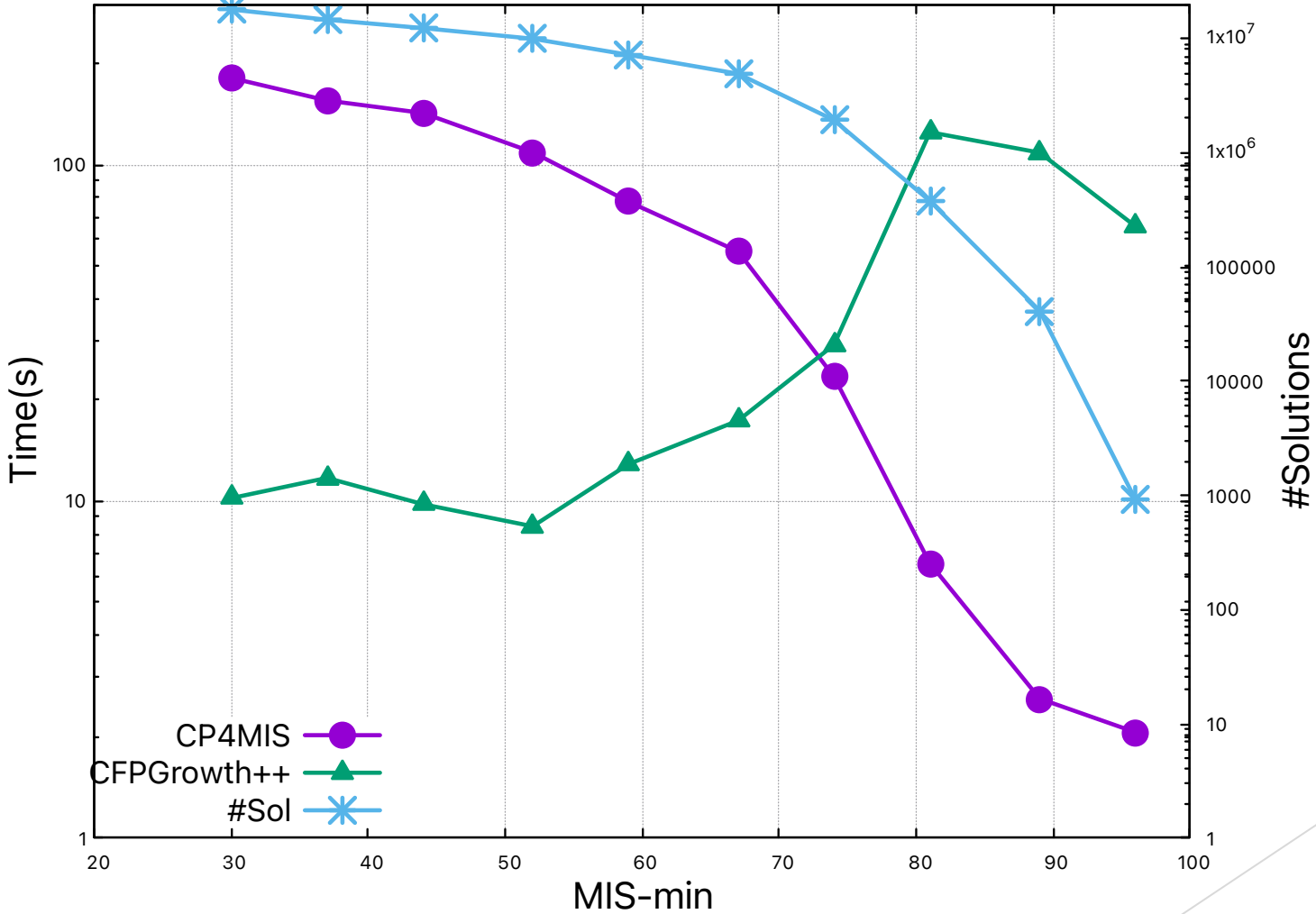
Q0:	CFPG	Rmodel		CP4MIS		#sol
	(a)	(b)		(c)		
	Time	Time	Memory	Time	Memory	
Zoo	0.81	12.00	3,760	1.34	20	1.3M
Vote	1.56	196.17	2,164	2.23	8	2.1M
Anneal	30.91	134.74	3,095	64.82	49	71.7M
Chess	11.64	305.03	3,153	28.20	67	22.6M
Mushroom	45.53	TO	–	106.00	48	105.2M
Connect	48.45	TO	–	854.59	218	91.7M
T40	409.55	–	OOM	91.70	2,304	15.8M
Pumsb	38.60	–	OOM	115.67	916	13.5M

Results (Mining frequent itemsets)

Q0:	CFPG	Rmodel		CP4MIS		#sol
	(a)	(b)		(c)		
	Time	Time	Memory	Time	Memory	
Zoo	0.81	12.00	3,760	1.34	20	1.3M
Vote	1.56	196.17	2,164	2.23	8	2.1M
Anneal	30.91	134.74	3,095	64.82	49	71.7M
Chess	11.64	305.03	3,153	28.20	67	22.6M
Mushroom	45.53	TO	–	106.00	48	105.2M
Connect	48.45	TO	–	854.59	218	91.7M
T40	409.55	–	OOM	91.70	2,304	15.8M
Pumsb	38.60	–	OOM	115.67	916	13.5M

Results (CFPG vs CP4MIS)

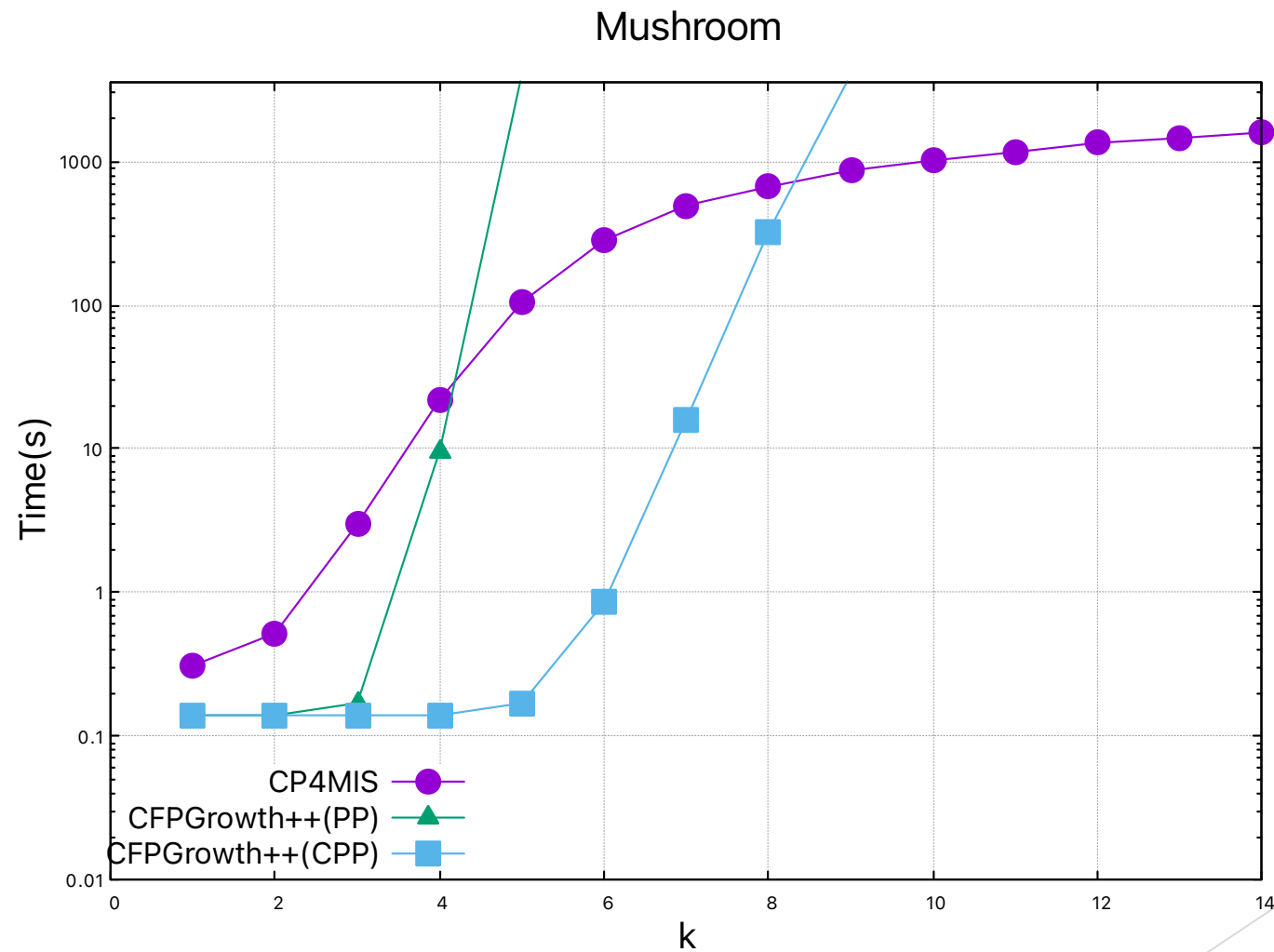
Connect ($\beta = 0.8$)



Results (Constrained itemsets)

Q2:	<i>ub</i>	<i>c</i>	CFPG +Checker	CP4MIS	#sol
			(d)	(c)	
Zoo	2	10	0.62	0.10	14
Vote	1	10	1.14	0.13	12
Anneal	30	8	12.78	0.18	7
Chess	80	8	6.49	0.23	30
Mushroom	50	8	19.19	0.36	27
Connect	1000	10	20.88	2.03	2
T40	100	6	389.80	54.31	14
Pumsb	1000	8	27.91	2.86	17

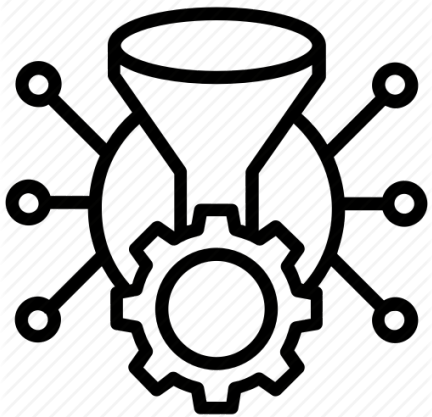
Results (K-pattern mining)



Conclusion

- ▶ We have introduced a CP-based approach for mining frequent itemsets with multiple minimum supports
- ▶ We have provided a **propagator** and showed that, using minMIS heuristic, the propagation is **backtrack-free** (0 fails)
- ▶ Our CP approach have shown the flexibility and the performance in taking in consideration additional user constraints
- ▶ Future: use the expressiveness of CP to solve problems that involve more complex constraints on MISs

t1			T			S	I	O		U	
t2	A	N	T	H	K	S			E		
t3	A				K	S	I				Q



bachir@simula.no



t1			T			S	I	O		U	
t2	A	N	T	H	K	S			E		
t3	A				K	S	I				Q

THANKS

bachir@simula.no

