Best Heuristic Identification for Constraint Satisfaction

Frederic Koriche, Christophe Lecoutre, Anastasia Paparrizou, **Hugues Wattez** 11 octobre 2022

Présentation à la 6ème journée CAVIAR









Introduction

Multi-Armed Bandit Framework

Adaptive Single Tournament

Experiments

Conclusion

Introduction

Definition (Variable)

A variable x is an entity associated to a value. This value belongs to its domain, denoted dom(x).

Definition (Constraint)

A constraint c is defined by a set of variables, called scope of c and denoted scp(c), and by a mathematical relation which describes the set of tuples allowed by c for the variables of its scope.

Definition (Variable)

A variable x is an entity associated to a value. This value belongs to its domain, denoted dom(x).

Definition (Constraint)

A constraint c is defined by a set of variables, called *scope* of c and denoted scp(c), and by a mathematical relation which describes the set of tuples allowed by c for the variables of its scope.

Definition (CSP)

A Constraint Satisfaction Problem (or Constraint Network) \mathcal{P} is defined by:

- a finite set of **variables**, denoted \mathcal{X}
- a finite set of **constraints**, denoted C, such that $\forall c \in C, \mathtt{scp}(c) \subseteq \mathcal{X}$

Definition (Solution)

A *solution* of a CSP instance \mathcal{P} corresponds to the assignment of a value to each variable of \mathcal{X} such that all the constraints of \mathcal{C} are satisfied.

Definition (CSP)

A Constraint Satisfaction Problem (or Constraint Network) \mathcal{P} is defined by:

- a finite set of variables, denoted ${\cal X}$
- a finite set of **constraints**, denoted C, such that $\forall c \in C, \mathtt{scp}(c) \subseteq \mathcal{X}$

Definition (Solution)

A solution of a CSP instance \mathcal{P} corresponds to the assignment of a value to each variable of \mathcal{X} such that all the constraints of \mathcal{C} are satisfied.

Global scheme : depth first binary tree search with backtracking



 $\mathbf{1^{st}}$ run : root of the tree



Decision : the variable ordering heuristic selects x



Decision : the value ordering heuristic selects a



Propagation : enforcing of the arc-consistency (AC) property



Decision : next selection (variable, valeur)



Propagation : enforcing of the AC property and conflict



Backtracking : parent node



Refutation : we consider $y \neq b$



Propagation : enforcing of the AC property and conflict



Backtracking : parent node (root node)



Refutation : $x \neq a$



Restarting : cutoff reached and nogood extraction



Restarting : backtrack to the root node



 2^{nd} run : root node



 $2^{\mathbf{nd}}\ run$: cutoff reached and restarting



 $\mathbf{t^{th}}\ run$: cutoff reached and restarting



End of solving : satisfiability | unsatisfiability | timeout



lex: lexicographic order

lex: lexicographic order
dom: size of domain

lex:	lexicographic order
dom:	size of <mark>domain</mark>
dom/ddeg:	size of domain and variable degree

lex:	lexicographic order
dom:	size of <mark>domain</mark>
dom/ddeg:	size of domain and variable degree
abs:	activity of variables

lex:	lexicographic order		
dom:	size of domain		
dom/ddeg:	size of domain and variable degree		
abs:	activity of variables		
ibs:	impact of variables		

lex:	lexicographic order
dom:	size of <mark>domain</mark>
dom/ddeg:	size of domain and variable degree
abs:	activity of variables
ibs:	impact of variables
dom/wdeg:	size of domain and constraint weighting

lex:	lexicographic order
dom:	size of <mark>domain</mark>
dom/ddeg:	size of domain and variable degree
abs:	activity of variables
ibs:	impact of variables
dom/wdeg:	size of domain and constraint weighting
chs:	history of constraint conflicts

- lex: lexicographic order
- dom: size of domain
- dom/ddeg: size of domain and variable degree
 - abs: activity of variables
 - ibs: impact of variables
- dom/wdeg: size of domain and constraint weighting
 - chs: history of constraint conflicts
 - cacd: arity and domain of the conflict variables

Problem

Heuristic determines search efficiency...

#instances	dom/wdeg	activity	impact
KnightTour	4	3	5
MultiKnapsack	24	27	25
Subisomorphism	7	2	5

Table 1: Solved instances by heuristic

... but heuristic selection needs expert qualities.

Given a CSP instance and a set of heuristics available in the solver, which heuristic is the best for solving the instance?

Problem

Heuristic determines search efficiency...

#instances	dom/wdeg	activity	impact
KnightTour	4	3	5
MultiKnapsack	24	27	25
Subisomorphism	7	2	5

Table 1: Solved instances by heuristic

... but heuristic selection needs expert qualities.

Given a CSP instance and a set of heuristics available in the solver, which heuristic is the best for solving the instance?

Problem

Heuristic determines search efficiency...

#instances	dom/wdeg	activity	impact
KnightTour	4	3	5
MultiKnapsack	24	27	25
Subisomorphism	7	2	5

Table 1: Solved instances by heuristic

... but heuristic selection needs expert qualities.

Given a CSP instance and a set of heuristics available in the solver, which heuristic is the best for solving the instance?
Multi-Armed Bandit Framework

One-armed bandits with different jackpot probabilities:



The multi-armed bandit problem is characterized by:

• the search for a balance between exploration and exploitation

One-armed bandits with different jackpot probabilities:



The multi-armed bandit problem is characterized by:

• the search for a balance between exploration and exploitation



One-armed bandits with different jackpot probabilities:

The multi-armed bandit problem is characterized by:

• the search for a balance between exploration and exploitation

The bandit problem is described as a game where a player faces the environment. At each trial t:

- the player chooses an action it among a set of actions A (heuristic selection in our case)
- the environment gives a reward $r_t(i_t)$ to the player for the selected action

The player's goal is to minimize his regret after T trials:

$$Regret_T = \max_{i \in A} \sum_{t=1}^T r_t(i) - \sum_{t=1}^T r_t(i_t)$$

The bandit problem is described as a game where a player faces the environment. At each trial t:

- the player chooses an action *i_t* among a set of actions *A* (heuristic selection in our case)
- the environment gives a reward $r_t(i_t)$ to the player for the selected action

The player's goal is to minimize his regret after T trials:

$$Regret_T = \max_{i \in A} \sum_{t=1}^T r_t(i) - \sum_{t=1}^T r_t(i_t)$$

The bandit problem is described as a game where a player faces the environment. At each trial t:

- the player chooses an action *i_t* among a set of actions *A* (heuristic selection in our case)
- the environment gives a reward $r_t(i_t)$ to the player for the selected action

The player's goal is to minimize his regret after T trials:

$$Regret_T = \max_{i \in A} \sum_{t=1}^T r_t(i) - \sum_{t=1}^T r_t(i_t)$$

The bandit problem is described as a game where a player faces the environment. At each trial t:

- the player chooses an action *i_t* among a set of actions *A* (heuristic selection in our case)
- the environment gives a reward $r_t(i_t)$ to the player for the selected action

The player's goal is to minimize his regret after T trials:

$$Regret_T = \max_{i \in A} \sum_{t=1}^T r_t(i) - \sum_{t=1}^T r_t(i_t)$$

The bandit problem is described as a game where a player faces the environment. At each trial t:

- the player chooses an action *i_t* among a set of actions *A* (heuristic selection in our case)
- the environment gives a reward $r_t(i_t)$ to the player for the selected action

The player's goal is to minimize his regret after T trials:

$$Regret_T = \max_{i \in A} \sum_{t=1}^T r_t(i) - \sum_{t=1}^T r_t(i_t)$$

The bandit problem is described as a game where a player faces the environment. At each trial t:

- the player chooses an action *i_t* among a set of actions *A* (heuristic selection in our case)
- the environment gives a reward $r_t(i_t)$ to the player for the selected action

The player's goal is to minimize his regret after T trials:

$$Regret_T = \max_{i \in A} \sum_{t=1}^T r_t(i) - \sum_{t=1}^T r_t(i_t)$$

The bandit problem is described as a game where a player faces the environment. At each trial t:

- the player chooses an action *i_t* among a set of actions *A* (heuristic selection in our case)
- the environment gives a reward $r_t(i_t)$ to the player for the selected action

The player's goal is to minimize his regret after T trials:

$$Regret_T = \max_{i \in A} \sum_{t=1}^T r_t(i) - \sum_{t=1}^T r_t(i_t)$$









Reward function

The reward function is based on the size of the trees pruned during a run t:

$$r_t(i) = \frac{\log_2\left(\sum_{n \in \mathtt{cft}(\mathcal{T}_t)} \prod_{x \in \mathtt{fut}(n)} |\mathtt{dom}(x)|\right)}{\log_2\left(\prod_{x \in \mathtt{vars}(P)} |\mathtt{dom}(x)|\right)}$$

where:

- $cft(\mathcal{T})$: the set of conflictual nodes
- fut(n): the set of unfixed variables
- vars(P): the variables of P
- dom(x): the domain of x



- UCB: simple upper confidence bound (stochastic bandit)
- EXP3: exponential weighting for exploration and exploitation (adversarial bandit)
 - UNI: random and uniform choice (naive policy)
 - VBS: virtual best solver (best policy)

Let the set of choice policies $\mathcal{B} = \{\text{UCB}, \text{EXP3}, \text{UNI}, \text{VBS}, \}$:

- UCB: simple upper confidence bound (stochastic bandit)
- EXP3: exponential weighting for exploration and exploitation (adversarial bandit)
 - UNI: random and uniform choice (naive policy)
 - VBS: virtual best solver (best policy)

Let the set of choice policies $\mathcal{B} = \{\text{UCB}, \text{EXP3}, \text{UNI}, \text{VBS}, \text{AST}\}$:

Adaptive Single Tournament















```
Input: A set of arms [K], a positive integer m \ge 1
```

```
Input: A set of arms [K], a positive integer m \ge 1
1 Set S = [K]
2 for each run t = 1, 2, ... do
```

```
Input: A set of arms [K], a positive integer m \ge 1
1 Set S = [K]
2 for each run t = 1, 2, ... do
       if \sigma_{\text{luby}}(t) = 1 then
3
             Select an arbitrary arm i \in S
4
            Set S = S \setminus \{i\} and if S = \emptyset then set S = [K]
5
```

```
Input: A set of arms [K], a positive integer m \ge 1
1 Set S = [K]
2 for each run t = 1, 2, ... do
        if \sigma_{\text{luby}}(t) = 1 then
3
             Select an arbitrary arm i \in S
4
             Set S = S \setminus \{i\} and if S = \emptyset then set S = [K]
5
6
        else
              Let i_{\text{left}} be the arm played at run t - \sigma_{\text{luby}}(t)
7
              Let i_{\text{right}} be the arm played at run t-1
8
             Choose i \in \{i_{left}, i_{right}\} with best reward r_i
9
```

```
Input: A set of arms [K], a positive integer m \ge 1
1 Set S = [K]
2 for each run t = 1, 2, ... do
        if \sigma_{\text{luby}}(t) = 1 then
 3
              Select an arbitrary arm i \in S
 4
            Set S = S \setminus \{i\} and if S = \emptyset then set S = [K]
 5
 6
        else
              Let i_{\text{left}} be the arm played at run t - \sigma_{\text{luby}}(t)
 7
              Let i_{\text{right}} be the arm played at run t-1
 8
              Choose i \in \{i_{left}, i_{right}\} with best reward r_i
 9
         Play i for m times and set r_i to the mth observed reward at run t
10
```

Experiments

CSP instances (\mathcal{I}_{CSP}): 810 instances XCSP'17/18/19 (83 families)

CSP instances (\mathcal{I}_{CSP}): 810 instances XCSP'17/18/19 (83 families) computation nodes: 3.3 GHz CPU Intel XEON E5-2643 and 32 GB of RAM
CSP instances (\mathcal{I}_{CSP}): 810 instances XCSP'17/18/19 (83 families) 3.3 GHz CPU Intel XEON E5-2643 and 32 GB of RAM computation nodes: timeout: 2,400 seconds solvers: ACE with each heuristics \mathcal{H} and policies \mathcal{B} $u \times luby_t$ where u = 150restarting sequence: cutoff unity: wrong decisions value ordering heuristic: min-dom propagation property: arc-consistency nogoods at the end of runs learning:











Finer-grained analysis



Figure 3: Proportions of heuristics selected by UCB and AST at each cutoff of Luby's sequence, for the CSP instance *Rlfap-scen-11-f01_c18*

Conclusion

In this study, we have:

- focused on the best heuristic identification problem
- presented the non-stochastic bandit algorithm AST

The results have shown a better behaviour than the stochastic and adversarial bandits-based, and are closer to the VBS. In addition, these results are corroborated by a convergence analysis.

In this study, we have:

- focused on the best heuristic identification problem
- presented the non-stochastic bandit algorithm AST

The results have shown a better behaviour than the stochastic and adversarial bandits-based, and are closer to the VBS. In addition, these results are corroborated by a convergence analysis.

In this study, we have:

- focused on the best heuristic identification problem
- presented the non-stochastic bandit algorithm AST

The results have shown a better behaviour than the stochastic and adversarial bandits-based, and are closer to the VBS. In addition, these results are corroborated by a convergence analysis.

Best Heuristic Identification for Constraint Satisfaction

Frederic Koriche, Christophe Lecoutre, Anastasia Paparrizou, **Hugues Wattez** 11 octobre 2022

Présentation à la 6ème journée CAVIAR









As perspectives, we think:

- designing bandit algorithm for others universal restart schemes (*e.g.*, exponential sequence)
- extending learning and autonomy (*e.g.*, branching heuristics and propagation techniques)

As perspectives, we think:

- designing bandit algorithm for others universal restart schemes (*e.g.*, exponential sequence)
- extending learning and autonomy (*e.g.*, branching heuristics and propagation techniques)