Towards an MDD-based representation of preferences

Noureddine Aribi[1], Souhila Kaci[2], Nadjib Lazaar[2]
[1] University of Oran 1, Lab. LITIO, Oran, Algeria
[2] CNRS, University of Montpellier, LIRMM, Montpellier, France
aribi.noureddine@univ-oran.dz, {kaci,lazaar}@lirmm.fr

The last decade witnessed a considerable number of compact representations of knowledge (beliefs or preferences) which encode an ordering relation (partial/total (pre)orders) over a set of outcomes. The need for such representations is motivated by two main reasons: (1) the set of outcomes is generally exponential which causes problems from representational and spacial points of view, and (2) it is cognitively hard for users to provide an ordering over the whole set of outcomes. Rather, they are more willing to express knowledge over partial descriptions of outcomes. The state of the art fully addressed item (2) covering various ways of expressing partial descriptions of outcomes. Therefore we distinguish between weighted logics, conditional logics and Bayesian-like networks. Given a compact representation of knowledge, several queries can be answered: comparing two outcomes, computing (constrained) optimal outcomes or rank-ordering a set of outcomes. Nevertheless, although existing representations make important advances in knowledge representation and reasoning, we do believe that item (1) is still not fully addressed. More precisely, when it comes to manipulate the whole set of outcomes the problem of its representation is posed again. In particular when we need to show the explicit set of outcomes to users during an interaction process.

In a purely constraint programming (CP) context, Andersen et al. [Andersen et al., 2007] proposed to use the *Multivalued Decision Diagram* structure (MDD) to replace the domain store where constraints have an MDD-Based presentation. An MDD is graphically represented by a (rooted) directed acyclic graph of an ordered list of variables, and can be exponentially smaller than the extensional version of feasible outcomes. Each outcome is encoded as a path in the graph, and each edge in the path encodes a variable assignment. Additionally, an MDD comes with a fast and effective GAC algorithm [Cheng and Yap, 2010], that has time complexity linear to the size of the MDD, and achieves full incrementality in constant time.

To take advantage of MDDs we consider the case of preference constrained problems. That is, not all possible outcomes are feasible. In this proposal, we attempt to address the problem of outcomes representation using MDDs where, in our context, domain store represents all possible outcomes and constraints are constraints restricting the feasibility of outcomes.

Example 1. *Consider three variables X_1 (Color), X_2 (Size) and X_3 (Print) with finite domains:* $D_1 = \{0 : black, 1 : white, 2 : red, 3 : blue\}$, $D_2 = \{0 : small, 1 : medium, 2 : large\}$ *and* $D_3 = \{0 : MIB, 1 : STW\}$.
Suppose we have the following constraints:
$C_1 : X_3 = 0 \Rightarrow X_1 = 1$ *and* $C_2 : X_2 = 0 \Rightarrow X_1 \neq 1$.
Therefore we have 11 feasible outcomes. Fig.1.(a) depicts an MDD Compiling the two constraints. Notice that this is an non-reduced *MDD. A nice property (among many others) of MDD is the ability to merge isomorphic subgraphs (Fig.1.(b) and (c)).*
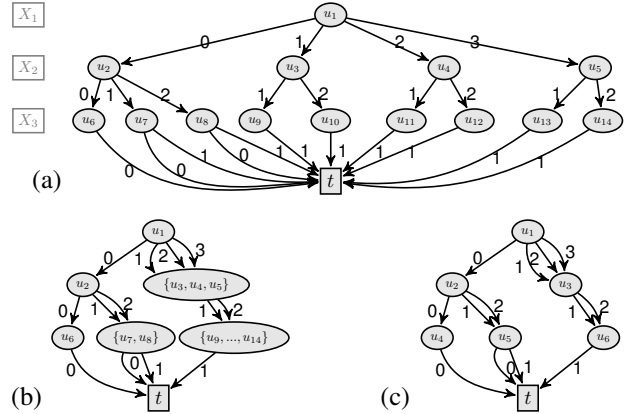

Figure 1: Reduced MDDs induced from Figure (a); (b) merging isomorphic subgraphs (c) the final MDD. t is a node used to regonize a solution.

In an ongoing work, we intend to address the following research questions:

i) Given a set of constraints, how do we determine the order over variables that leads to a compact MDD?

ii) An MDD is close to a CP-net representation [Boutilier et al., 2004], investigate whether an MDD can be extended with preferential information. This allows us to: (1) improve the search for the set of non-dominated outcomes, (2) find the shortest flipping sequence that can be used to answer dominance queries, (3) Negate preference statements.

iii) How can we go beyond CP-nets and extend MDDs with preferential information expressed in other preference representation languages (e.g. conditional logics and weighted logics).