



## **Towards argumentative semantics**

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## **A Foreword: semantics, argumentation, coherence**



## A.1. An interdisciplinary research program

Mainly:

- Mathematical logic (proof theory, type theory)
- Formal semantics, philosophy of language

and to a lesser extent:

- Formal syntax
- Symbolic Natural Language Processing (e.g. Text entailment)
- Cognitive sciences

Our first steps within this approach:

- Tout / chaque
- Peu / un peu



## A.2. Little / A little

Consider the following dialogue:

- (1) Alda — Could you lend me some money?
- (2) Bob — Sorry, I can't.
- (3) Alda — Why?
- (4) a. Bob — \* I have a little money.  
b. Bob — I have little money.

Although both *little* and *a little* both mean *not much*.



### A.3. Tout / chaque

- (5) Alda — Tout chien a 4 pattes.
- (6) Bob — Pas Rex.
- (7) Alda — Il a eu un accident.

*The exception does not refute the "tout" sentence.*

- (8) Alda — Chaque chien de l'élevage d'à côté aboie jour et nuit.
- (9) Bob — Pas Rex.
- (10) a. Alda — Ah oui j'oubliais Rex, tu as raison.  
b. Alda — Mais non, Rex est mon chien.

*The "chaque" sentence is refuted — or the domain was inappropriate.*



## A.4. Moral

Examples above show at least two things:

- the argumentative aspect of a sentence participates in the coherence of a discourse or dialogue,
- two expressions may have similar denotations but different argumentative uses



**B A welcome part of of formal semantics:  
from sentences to formulas**



## B.1. Formal semantics and logic

Two sides of semantics, both contributing to meaning:

- **lexical** semantics: interpreting terms (words, noun phrases, even **quantified** nouns phrases)
- **formal/compositional** semantics: interpreting propositions, reasoning





## B.2. Computing logical forms à la Montague

Mind that there are TWO logics: composition / logical form:

One for expressing meanings:

**formulae** of first or higher order logic, single or multi sorted. Meaning postulates, relations between predicates account for existential semantics.

One for meaning assembly:

**proofs** in intuitionistic propositional logic,  $\lambda$ -terms expressing the well-formedness of formulae.



### B.3. Representing formulae within lambda calculus — connectives

Assume that the base types are

$e$  (individuals, often there is just one) and  
 $t$  (propositions)

and that the only constants are

the logical ones (below) and  
the relational and functional symbols of the  
specific logical language (on the next slide).

Logical constants:

- $\sim$  of type  $t \rightarrow t$  (negation)
- $\supset, \&, +$  of type  $t \rightarrow (t \rightarrow t)$   
(implication, conjunction, disjunction)
- two constants  $\forall$  and  $\exists$  of type  $(e \rightarrow t) \rightarrow t$



## B.4. Representing formulae within lambda calculus — language constants

The language constants in many-sorted Logic:

- $R_q$  of type  $\mathbf{e} \rightarrow (\mathbf{e} \rightarrow (\dots \rightarrow \mathbf{e} \rightarrow \mathbf{t}))$
- $f_q$  of type  $\mathbf{e} \rightarrow (\mathbf{e} \rightarrow (\dots \rightarrow \mathbf{e} \rightarrow \mathbf{e}))$

two-place predicates	
<i>likes</i>	$\lambda x^e \lambda y^e (\underline{\text{likes}}^{e \rightarrow (e \rightarrow t)} y) x$
one-place predicates	
<i>cat</i>	$\lambda x. \underline{\text{cat}}^{e \rightarrow t}$
<i>sleeps</i>	$\lambda x. \underline{\text{sleep}}^{e \rightarrow t}$
two proper names	
<i>Evora</i>	$\underline{\text{Evora}} : \mathbf{e}$
<i>Anne—Sophie</i>	$\underline{\text{Anne—Sophie}} : \mathbf{e}$

possibly  $(\mathbf{e} \rightarrow \mathbf{t}) \rightarrow \mathbf{t}$

**Normal terms of type  $\mathbf{t}$ : formulae / of type  $\mathbf{e}$ : terms.**



## B.5. Montague: Syntax/semantics.

(Syntactic type)*	=	Semantic type
$S^*$	=	$t$ a sentence is a proposition
$np^*$	=	$e$ a noun phrase is an entity
$n^*$	=	$e \rightarrow t$ a noun is a subset of the set of entities
$(A \setminus B)^* = (B/A)^*$	=	$A \rightarrow B$ extends easily to all syntactic categories of a Categorical Grammar e.g. a Lambek CG



## B.6. Montague semantics. Algorithm

1. Replace in the lambda-term issued from the syntax the words by the corresponding term of the lexicon.
2. Reduce the resulting  $\lambda$ -term of type  $t$  its normal form corresponds to a formula, the "meaning".



## B.7. Ingredients: a parse structure & a lexicon

### Syntactical structure

(some (club)) (defeated Leeds)

### Semantical lexicon:

<b>word</b>	<b><i>semantics</i></b> : $\lambda$ - <b>term of type (sent. cat.)*</b> $x^v$ the variable or constant $x$ is of type $v$
<b>some</b>	$(e \rightarrow t) \rightarrow ((e \rightarrow t) \rightarrow t)$ $\lambda P^{e \rightarrow t} \lambda Q^{e \rightarrow t} (\exists^{(e \rightarrow t) \rightarrow t} (\lambda x^e (\wedge^{t \rightarrow (t \rightarrow t)} (P x)(Q x))))$
<b>club</b>	$e \rightarrow t$ $\lambda x^e (\text{club}^{e \rightarrow t} x)$
<b>defeated</b>	$e \rightarrow (e \rightarrow t)$ $\lambda y^e \lambda x^e ((\text{defeated}^{e \rightarrow (e \rightarrow t)} x)y)$
<b>Leeds</b>	$e$ Leeds



## B.8. Computing semantic representations

- 1) Insert the semantics terms into the parse structure
- 2)  $\beta$  reduce the resulting term

$$\begin{aligned} & \left( (\lambda P^{e \rightarrow t} \lambda Q^{e \rightarrow t} (\exists^{(e \rightarrow t) \rightarrow t} (\lambda x^e (\wedge (P x) (Q x)))) (\lambda x^e (\text{club}^{e \rightarrow t} x))) \right) \\ & \quad \left( (\lambda y^e \lambda x^e ((\text{defeated}^{e \rightarrow (e \rightarrow t)} x) y)) \text{Leeds}^e \right) \\ & \quad \quad \quad \downarrow \beta \\ & (\lambda Q^{e \rightarrow t} (\exists^{(e \rightarrow t) \rightarrow t} (\lambda x^e (\wedge^{t \rightarrow (t \rightarrow t)} (\text{club}^{e \rightarrow t} x) (Q x)))))) \\ & \quad (\lambda x^e ((\text{defeated}^{e \rightarrow (e \rightarrow t)} x) \text{Leeds}^e)) \\ & \quad \quad \quad \downarrow \beta \\ & (\exists^{(e \rightarrow t) \rightarrow t} (\lambda x^e (\wedge (\text{club}^{e \rightarrow t} x) ((\text{defeated}^{e \rightarrow (e \rightarrow t)} x) \text{Leeds}^e)))) \end{aligned}$$

Usually human beings prefer to write it like this:

$$\exists x : e (\text{club}(x) \wedge \text{defeated}(x, \text{Leeds}))$$





## B.9. Formulas computed à la Montague: good architecture

Good trick (Church):

a propositional logic for meaning assembly  
(proofs/ $\lambda$ -terms)

**computes**

HOL / FOL formulas  
(formulas/meanings; no proofs)





## B.10. Integrating lexical semantics

As such no account of lexical semantics (restriction of selection, facets, meaning transfers, (in)felicitous copredication, etc.)

Extension: the Montagovian Generative Lexicon:

- richer type system with sorts for selection restriction expressed as type mismatch and quantification over types
- word: one principal meaning, with possible transformations
- transformations may fix type mismatch
- transformation may be compatible or not for copredication



## B.11. Fixing syntax/semantics problems, plurals, count nouns,...

normal *categorial analyses*

→ no more *spurious ambiguities*

Extension: the Montagovian Generative Lexicon:

- **generalised quantifiers:**  
compositionality problem,  
wrong syntactic structure  
→ typed subnectors  $\varepsilon$  and  $\tau$  and  $\iota$  →  
(underspecified semantics)
- **plurals, mass nouns:** usually left out  
→ integers, groups and float quantities  
can be encoded within the system,



## B.12. Standard semantics

Sentence  $\rightarrow$  formula  $S \rightarrow$  interpretation  $S = ???$

The meaning of a statement  $S$  is  
the collection of the models in which  $S$  is true.

*models: possible worlds in a Kripke structure*

Cognitive / computational problems:

- Infinite non enumerable set of possible worlds.
- Every model is itself infinite non enumerable.
- Leaves out argumentative aspect of meaning.



## C Proof theoretical semantics

Natural language sentences  $\sim$  logical formulas  
(this sometimes makes sense, e.g. in maths)

So: what exists in (mathematical) logic?



## C.1. Vonconstructivism / Intuitionism / inferentialism

A tradition in the philosophy of mathematics:  
BHK Curry-Howard categorical interpretations  
Martin-Löf Type Theory

Meaning of a formula : its (formal) proofs.

Proofs have a computational content.

Proof reduction or cut-elimination:  
→ only normal proofs proofs in  $[[A]]$  ?

*Impossible for classical logic.  
All proofs of a formula reduce one to another.*



## C.2. Limitations

- 1) First order logic and
- 2) extra logical axioms  
easily express theories (beyond arithmetic)

but some formal properties of proofs are lost:

1. First order: subformula property is weaker.
2. Axioms: normalisation is weaker.



## **D Formal and informal justifications for natural language sentences**



## D.1. Meaning as justifications

Transposition of proof theoretical semantics.

Formula  $F \rightarrow$  formal proofs of  $F$   
Sentence  $S \rightarrow$  justifications of  $S$

Related to text entailment: is a sentence or paragraph consequence of another?

Mathematical practice use natural language.

Observe that when learning to read children are asked text entailment tasks.





## D.2. Justification as logical proofs

(if) a sentence  $\sim$  logical formulas,  
(then) justifications  $\sim$  proofs of those formulas.

### Which logic (deductive system)?

- Intuitionistic logic (for having several non equivalent justification) or modal logic (?)
- First order / many sorted / Type Theory
- *Axioms* not really part of the logic.

Words with a logical contents can be represented by logical connectives and rules.



### D.3. Axioms

Axioms describes extra logical lexical meaning, world knowledge, observations, belief:

- for word meaning  
(dictionary definition, meaning postulates)
- for observations
- for opinions

**Subjective** axioms and justifications.

Even lexical meaning might be speaker dependent.



## D.4. Better than standard semantics?

Proofs: finitely generated from finitely many rules and axioms; proof-correctness is linear.

Even if not all axioms are known, correct proofs from the known axioms exist.

*Axioms can be learnt  
from interactions between proofs, cf. infra.*

Includes argumentative aspects of semantics.



## D.5. Limitations

Not all axioms are known.

We do not include justification of why saying rather than not saying, lying etc.

*A justification for saying That's not a big deal. might be that the speaker wants to minimise a mistake, although actually That IS a big deal.*

Negation is an obstacle to compositionality:

At most one of  $A$  and  $\neg A$  is provable.

At most one of  $[[A]]$  or  $[[\neg A]]$  is not empty.

At least one of  $[[A]]$  or  $[[\neg A]]$  is empty.



## D.6. Proofs and refutations

Problem with negation

→ proofs and refutations on a par ?

Pseudo proofs like in Ludics  
(daimon, circular proofs,...)

Interaction between proofs and refutations:  
proof normalisation reveals axioms.



## D.7. Justifications as informal proofs

Justifications/proofs in natural language?  
Need for an unambiguous language.

Mathematical practice. Natural language (especially for reasoning rules) with some computations and formulas when needed.

Natural logic: Aristotle syllogisms today !  
Sentences with several quantifiers, numbers using fixed grammatical patterns.  
For simple maths and every day reasoning.



## E Examples



## E.1. Quantifier scope —A. Lecomte

Every linguist speak some African languages.

$$(1) \quad \forall x(L(x) \rightarrow \exists y(A(y) \wedge P(x, y)))$$

$$(2) \quad \exists y(A(y) \wedge \forall x(L(x) \rightarrow P(x, y)))$$

$$\frac{\frac{\frac{\text{:}\mathfrak{D}_1}{L(x) \vdash (A(y) \wedge P(x, y))}}{L(x) \vdash \exists y(A(y) \wedge P(x, y))} \exists R}{\vdash L(x) \rightarrow \exists y(A(y) \wedge P(x, y))} \rightarrow R}{\vdash \forall x(L(x) \rightarrow \exists y(A(y) \wedge P(x, y)))} \forall R$$
$$\frac{\frac{\frac{\text{:}\mathfrak{R}_2}{L(x) \vdash P(x, t)}}{\vdash L(x) \rightarrow P(x, t)} \rightarrow R}{\vdash \forall x(L(x) \rightarrow P(x, t))} \forall R}{\frac{\frac{\text{:}\mathfrak{R}_1}{\vdash A(t)}}{\vdash (A(t) \wedge \forall x(L(x) \rightarrow P(x, t)))} \wedge R}{\vdash \exists y(A(y) \wedge \forall x(L(x) \rightarrow P(x, y)))} \exists R$$





## E.2. Peu / un peu — with Davide Catta and Alda Mari

Justification for "having a little money"  
having  $m$  money units,  $m$  is not much

Justification for "having a little money"  
having  $m$  money,  $m$  is not much  
there is an event  $e$  in the context for which  $m$  is enough.

In a simple analysis those simpler assertion do not have to be justified.

"little" can be understood as not enough for some event  $e$ , but this can be viewed as a maxime à la Grice.

*the analysis can be refined, if one ask the justifications to be themselves justified.*



### E.3. Tout / chaque — with Alda Mari

**Chaque (observation):** no exceptions

domain: well defined, enumerable, non empty, possibly contingent

*Chaque*  $x$ .  $P(x)$ : conjunction of  $P(x)$  for  $x \in D$ .

**Tout (rule):** possible exceptions

domain: non contingent, possibly implicit and empty

*Tout*  $x$ .  $P(x)$ : Aristotle abstraction rule, i.e.  $\forall$  rule.

(exceptions: modal logic with D. Catta, M. Parigot).

→ *Certains / quelques* by Audrey Bedel.



## F Perspectives



## F.1. Natural language processing

Categorial grammar: semantic-oriented syntax  
sometimes unwanted syntactic trees (e.g. generalised quantifiers)

Grail large parser producing logical formulas (DRS)

coercions for lexical semantics can be extracted from the lexical network JeuxDeMots

Machine learning cannot recognise Text entailment (e.g. negation? quantifiers? scope?)

Automated debate analysis:  
A refutes B, A justifies B.

Corpus maths (cf. supra). Trials with Michel Parigot.



## F.2. Formal semantics and philosophy of language

— NEGATION

Rex is not a dog.

The chair did not bark.

— IDENTITY

J'ai lu ce livre.

? J'ai lu le même livre.

\*\* Je n'ai pas lu le même livre.

*(unless it means we had different readings)*



### F.3. Mathematics

Mathematics is a corpus: people do mathematics in natural language except equations, computations, complicated quantifier alternations... Reasoning takes place in natural language.

We could analyse mathematical practice, especially in teaching, didactics

Our semantic framework requires some maths:  
Type Theory, Topological Models,...  
Identity? Quotient?  
Negation?

Justifications : what about consequences? (less computable).

Debates proponent/opponent cf. Dialogical Logic.



## F.4. Cognitive sciences

Desperately looking for cognitive scientists, psycholinguists

Logic: many measure of complexities:

- depth, quantifier alternation
- model checking
- provability


Can we measure the complexity of human processing?

Could we define relevant experiments to test those measures?



## G Conclusion





## G.1. *Proofs as meanings of natural language assertions*

Natural outcomes:

- *Computational* model of meaning.
- **argumentative** aspects of meaning.
- includes **coherence** of discourse and dialogue (proving there is no model is difficult)



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