Inferentialism and natural language semantics

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on going reflections with
Davide CATTA
Alda MARI (and others)

This talk is about "meaning":

"Le sens ne se produit jamais que de la traduction d'un discours en un autre."

"Meaning always results from the translation of one discours into another discourse."

Jacques LACAN L'étourdit 1973

Comments: that is so true.

Montague semantics:
sentence → formula ⟶ models

Model theory:
formula ⟶ models

Proof theoretical semantics
formula ⟶ its proofs

Denotational semantics:
proofs ⟶ continuous functions

From the history of logic

300 BC ⟶ 1900 BC

rules on sentences as formulas

Logic : extension of math reasoning

⟶ to all discourses.

Thales 700 BC Pythagore 500 BC < Aristotle 300 BC

Non contradiction $\neg(A \wedge \neg A)$

Excluded Middle $A \vee \neg A$

(questionnable)

Formulas as sentences

A : All human beings are mammals
O : Not all animals are mammals
O : Not all animals are human beings.

BAROCCO

Aristotle syllogisms
on A E I O statements

Natural Language sentences as Logical Formulas

That's the logical view (Categorial Grammars
Montague Grammar

S	t	o
np	e	ι
n	$e \to t$	$\iota \to o$

Rules on a language
endowed with an implicit
canonical interpretation

Both in math and in linguistics
people think of <u>ONE</u> interpretation
even when they use
axioms / deductions

e.g. Peano arithmetic works for
infinite integers of non standard models

Models, completeness: ~1920
understood: ~1940
still obscure to non-logicians
(standard mathematicians, linguists, ...)

Models (Lowenheim, Skolem, Gödel)

A proof of F make it true
in all models
and there are models
of any cardinality!

Difference: models / proofs

Fermat last theorem:

$\forall n > 2 \, \forall x, y, z \quad xyz \neq 0 \Rightarrow x^n + y^n \neq z^n$

PA statement, proved by Wiles in 1995
using complex analysis

Open question:

can it be proved within PA?

(true in all models of PA)

or does it requires a bigger theory T (*)

(true only in models of $T \supset PA$)

Because of Gödel incompleteness (*) is possible

Semantics of a sentence (standard)

Usually the semantics of a formula F
is the family of models
in which F is true

$[\![F]\!]$ the Ms s.t. $M \models F$

sentence $\longrightarrow$ logical formulaS $\longrightarrow$ models

ambiguous

Computational semantics:

Models ???

$\mathcal{M}$, F : computing

Are truth conditions enough? $\mathcal{M} \models F$??

Even for a clear, simple, mathematical sentences it is very difficult:

4SQ every integer is the sum of the squares of 4 integers

$\mathbb{N} \models 4SQ$ (yes!)

Model theoretic view & computation

The domain is unclear

the interpretation is unclear

$\underbrace{\text{Anna}}_{\text{ref +/- ok}}$ is $\underbrace{\text{tall}}_{??}$ and $\underbrace{\text{wealthy}}_{??}$

The logic is left implicit

"and" means "and"

"for all" means "for all"

"or" means "a"

"exists" means "exists"

but there are nuances...

if I want a for all

that does not implies exists?

Modifying logic: very difficult, from a model theoretic viewpoint

Models of intuitionistic logic?

- Kripke models (54, LJ)
- sheaves of classical models

complex structures, not very intuitive

(a good point for Kripke models is that they adapt easily)

Modifying the logic: quite easy, from a proof theoretical viewpoint

Change rules (e.g. sequent calculus)

$$\text{-----} \vdash \text{.....}$$

ONE formula only

⟶ INTUITIONISTIC rules

controlled weakening / contraction

⟶ Linear Logic

Perhaps it is easier with Hilbert style deductive system (e.g. for modal logic)

adding axioms that rule the modalities

$$\Box(\cancel{\Box} A \supset \cancel{\Box} B) \supset \Box A \supset \Box B$$

rather meaning full

Alternative view:
proof theoretical semantics
(intuitionistic systems,
this will be discussed later on)

BHK

Idea F is interpreted
by the set of its proofs

Brouwer Kolmogorov Heyting

$\longrightarrow$ What is a proof?

A formula F: the set of its proofs

A proof of F (d: as a function from 1 to F

as an individual of type F

Identity are proofs of $A \to A$

$[\![A \& B]\!] = \{(d_1, d_2) \mid d_1 \vdash A, d_2 \vdash B\}$

$[\![A \to B]\!] =$ function that maps proofs of A to proof of B

Denotational semantics

Interesting refinement

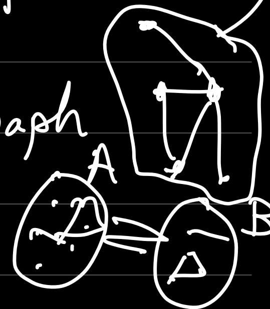
propositional NJ (simply typed λ calculus)

Coherence spaces

atomic formulas : simple graph

$A \& B$: graph on $A \uplus B$

object : cliques



$A \to B$: continuous^{++} functions
stables from A to B
a a coherence space $[\![A \to B]\!]$

$A \vdash B$

$\vdash A \to B$

What are proofs?
When are two proofs equal?

If proofs are the objects that make sense ....
what is a proof?

Hilbert syst
Sequents
ND

Up to
- permutation of rules
- β reduction, cut-elimination
- focusing
- ...

$$\Pi \xrightarrow{\beta\ \text{cut}} \Pi'$$

Denotational semantics $[\![\Pi]\!] = [\![\Pi']\!]$

A difficult question, that goes back to Kreisel

equality of π_1, π_2, with $\vdash^{\pi_1} F$ $\vdash^{\pi_2} F$

syntactic equality
equal normal form (cut elimination)
rules permutation (denotational semantics)

Why intuitionistic logic?

Girard Lafont

$$\dfrac{\dfrac{\dfrac{\pi_1}{\vdash F}}{\vdash F, K}\,w \qquad \dfrac{\dfrac{\pi_2}{\vdash F}}{\vdash F, \neg K}\,w}{\dfrac{\vdash F, F}{\vdash F}\,\mathrm{contr}}\,\mathrm{cut}$$

$$\dfrac{\dfrac{\dfrac{\pi_1}{\vdash F}}{\vdash F, F}\,w}{\vdash F}\,\mathrm{contr} \qquad \Bigg| \qquad \dfrac{\dfrac{\dfrac{\pi_2}{\vdash F}}{\vdash F, F}\,w}{\vdash F}\,\mathrm{contr}$$

Full abstraction?

A question on those proof theoretical interpretation is

is any semantic object is the interpretation of a proof?

proofs / equivalence : yes

interesting structure : difficult

Limits

- intuitionistic proofs only?
- what about non provable formulas?

- unless we impose $[\![\pi]\!] = [\![\pi']\!]$ when $\pi \to \pi'$

We can consider classical and modal logic

- non provable formulas are provable with the right axioms (~ model)

Games

alternate list of moves

asymetric - Proponent (who starts / can come back)

- Opponent

rules

game is won b P:

the last move is by P

and O cannot answer

Games and proof:

proof of F $\sim$ winning strategy for P on an F game

winning strategy for P

a function from begining of games ended by O telling P which move to play next

when P follows the strategy P wins

Proof in (a variant of) LK

$$— \vdash \underline{A \vee B}, A, —$$
$$— \vdash \underline{A \vee B}, —$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A \vee (A \supset \bot), A, \bot}\ \mathrm{Id}}{A \vdash A \vee (A \supset \bot), \bot}\ \vee^R}{\vdash A \vee (A \supset \bot), A \supset \bot}\ \supset^R}{\vdash A \vee (A \supset \bot)}\ \vee^R$$

Dialogical logic (cf Catta's PhD)

P I affirm that $A \vee (A \supset \bot)$

O can you assert one between A and $A \supset \bot$?

P I affirm that $A \supset \bot$

O I grant that A holds, can you show $\bot$?

P I changed my mind : I assert A. Since you granted A, I win.

Limits

- Proofs are not beautiful: games are worse
- Easier for intuitionnistic logic (although it works for classical logic)
  reason: the Proponent/Opponent role are not symmetric, while negation inverts the two.
- What about non provable formulas? axioms, but which ones?
- Does not really correspond to real life debates.

Games are inelegant

moves (heterogenous pairs :
(?/!, formula a connective)

game alternate sequence of moves
satisfying rules (ok)

strategy : function a tree

identity / equivalence of games?
of strategies?

Better/easier for intuitionistic logic

Works for the standard sequent calculus

For classical logic we have to use a sophisticated sequent calculus

For Modal logics? Under discussion.

Non provable formulas

For most formulas G: $\nvdash G \quad \nvdash \neg G$

They do have a model theoretic interpretation
the models $M \models G$

but what proofs do they have?

we need axioms ....

Axioms ?

We can assume some axioms

- word meaning
- knowledge
- beliefs

(for the time being axioms of P, common to O and P)

Axioms

relevant part only of the model (models are back) that the speaker has in mind

Application to textual entailment

(Catta, Moot, Retoré)

$\boxed{\text{very short text}} \xrightarrow{\text{entails ?}} \boxed{\text{Sentence}}$

FRACAS a French FRACAS

Using Grail (Moot)

$$\boxed{\text{short text}} \xrightarrow{\text{Grail}} \text{formula } H$$

$$\boxed{\text{goal}} \xrightarrow{\text{Grail}} C$$

Axioms, (word meaning knowledge)

$$H \overset{?}{\vdash} C$$

in dialogical logic

Limits

do we know the axioms

word meaning, general knowledge

- can be imported from a lexical semantic network (WordNet, JeuxDeMots) with M Lafourcade

speakers knowledge

previous utterances otherwise??

Extensions

- Reasoning IN natural language
  semantics $\Rightarrow$ dialogues
  in natural language

- Emergence of axioms
  during the dialogue
  (this happens in
  ordinary debates
  yieding the speaker's beliefs)

Tricky questions:

Can the argumentative dialogue be "realistic"?

- use current reasoning rules only?

e.g. people use $A \Rightarrow B$ only when A holds

- O and P have different beliefs

- instead of O, P can we have several viewpoints (axioms)

Quantifiers
a proof theoretical refinement
(with Alda MARI)

Proof theoretic interpretation of $\forall$ in French

Singular:

TOUT (every ?)

CHAQUE (each)

Differences

TOUT — imprecise domain: OK
— exceptions: OK

CHAQUE — precise, denumerable domain
— no exception
odd, even when explicitely stated

From a model theoretic viewpoint
we have to view them as $\forall$

Tout only can be used in generic sentences.

(4) a. Tout homme est mortel.
TOUT man is mortal.
b. #Chaque homme est mortel.
CHAQUE man is mortal.

→ *Tout* unrestricted generality.

Tout only can be used if no elements of the class. Which s typical of generic sentences.
If no student got an A:

(5) a. Tout étudiant ayant eu un A a un prix.
TOUT student who got an A has a price.
b. #Chaque étudiant qui a eu un A a un prix.
CHAQUE student who got an A has a price.

(7) a. #Tout homme sur terre est mortel.
TOUT man on earth is mortal. (sounds odd)
b. Chaque homme sur terre est mortel.
CHAQUE man on earth is mortal. (sounds as a weak generalization, but true)

any

a. Prends toute carte qui te fasse gagner.
Pick TOUTE card that allows you to win.

b. Prends chaque carte qui te fasse gagner
Pick CHAQUE card that allows you to win.

all of them

a. Tout chien a quatre pattes.

TOUT dog has four legs/a brain.

b. Sauf le mien, il a eu un accident.

All but mine, he had an accident.

does not wok with "CHAQUE"

Tout

- Compatible with an infinite domain.
- Requires the existence of a law (hence compatible with absence of instances)
- Only compatible with essential properties
- In discourse: it is used prescriptively.

Chaque

- Compatible with both essential and accidental properties
- It requires a well determined domain of quantification (hence incompatible with absence of instances and infinite domains).
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Proof rules for TOUT

TOUT generalisation / abstraction
(Aristotle Rule)
(deductive rule)

$$\frac{A(x)}{\mathrm{TOUT}_x\, A(x)} \quad \text{or} \quad \tau_x\, A(x)$$

without any specific property
not free in any hypothesis

Proof by réasonning (Hilbert Style)
on a generic élement

Proof rule for CHAQUE

The domain D must been known

CHAQUE $\longrightarrow \&_{x \in D} A_x$

$$\frac{\begin{array}{ccc} \pi_1 & \pi_2 & \\ A(d_1) & A(d_2) & \cdots \end{array}}{\text{CHAQUE}_{x\in D} A(x) = \underset{x \in D}{\&} A(x)}$$

Proof rule for CHAQUE

If D infinite ...

USE: ONLY if elements can be enumerated (even in a corpus of math book unless you name it like a skolem constant)

like Gentzen

ω-rule

Conclusion

Proof theoretical semantics

- far from perfect
- but a complementary view → better account of some phenomenas
- models are not far (axioms)

The old dream of a language with a UNIQUE interpretation

Le sens ne
se produit jamais
que de la traduction
d'un discours
en un autre.