«Calcul formel avancé et application».

28.09.2023. Exercises for lecture 3.

Exercise 1. (a) is it possible to find three binary strings x, y, and z of length 5 such that the Hamming distance between every two of them is equal to 2 ?

(b) The same question for the distance 3.

Exercise 2. Find the checksum matrix of the Hamming code with codewords of length

(a) $n = 3$,
(b) $n = 7$,
(c) $n = 31$.

Exercise 3. Find the list of all codewords of the Hamming code (a) of length $n = 3$ and (b) of length $n = 7$.

Exercise 4. The checksum matrix of of the Hamming code with codewords of length 15 is

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

(a) Compute the number of codewords in this code.

(b) Is the bit string $x = (001000000000000)$ a codeword of this code ? If no, which bit should be inverted to obtain a codeword ?

(c) Is the bit string $x' = (001101011101110)$ a codeword of this code ? If no, which bit should be inverted to obtain a codeword ?

(d) Is the bit string $x'' = (001111011100110)$ a codeword of this code ? If no, which bit should be inverted to obtain a codeword ?

(e) In a codeword $x''' = (00000000000 \star 111)$ one digit is lost (replaced by $\star$). Reconstruct the lost bit.

Exercise 5. Professor chooses a name of one of the 8 students in the class. How many binary questions (with answers "yes" or "no") you need to ask to find out the chosen name if the professor can give the wrong answer at most once ? [Explain your answer : why this number of questions is necessary and why it is sufficient.]