UM. Autumn 2019. Homework 7 to the course «Information theory».
[should be returned by Dec 17 to be counted in *contrôle continu*]

Problem 1. Prove that there exists an integer number Const such that for every pair of binary strings x, y

$$C(x,y) \le |x|+|y|+\log|x|+\log\log|x|+\log\log\log|x|+2\log\log\log\log|x|+\text{Const}.$$

Problem 2. Let x be a binary string of length n with pn *zeros* and $(1-p)n$ *ones*. Prove that

$$C(x) \le \left(p\log\frac{1}{p} + (1-p)\log\frac{1}{1-p}\right)n + O(\log n).$$

Problem 3. Most statements about Kolmogorov complexity are independent of the choice of the optimal decompressor. In this exercise we ask about properties that do depend on the optimal decompressor.

(a) Does there exist an optimal decompressor L_{opt} such that for all x the value of $C_{L_{opt}}(x)$ is *even*?

(b) Does there exist an optimal decompressor L_{opt} such that for all x the value of $C_{L_{opt}}(x)$ is *a power of* 2?