UM. Autumn 2020. Homework 3 to the course «Information theory».
[should be returned by Sep 29 to be counted in *contrôle continu*]

Problem 1. (a) Let $S \subset \{a, b, c\}^n$ be the set of all strings of length n with 50% of letters a, 25% of letters b and 25% of letters c. Prove that there exists an injective mapping

$$F : S \to \{0,1\}^{3n/2}.$$

(b) Prove that there is no injective mapping

$$G : \{a, b, c\}^n \to \{0,1\}^{3n/2}.$$

Problem 2. (a) Prove that for every real number $t \geq 0$ there exists a random variable α such that $H(\alpha) = t$ (i.e., the value of Shannon's entropy can be any non-negative real number.)

(b) Prove that for all real numbers $t_0 \geq 0, t_1 \geq 0, t_2 \geq 0$ there exists a pair of jointly distributed random variables (α, β) such that

$$\begin{array}{rcl} H(\alpha) & = & t_0 + t_1, \\ H(\beta) & = & t_0 + t_2, \\ H(\alpha, \beta) & = & t_0 + t_1 + t_2. \end{array}$$