Some Contributions to Parameterized Complexity

Habilitation à Diriger des Recherches (HDR)

Montpellier, June 25, 2018

Ignasi Sau

CNRS, LIRMM, Université de Montpellier

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	Rolf Niedermeier	-	Technische Universität Berlin
	Fedor V. Fomin	-	University of Bergen
***Examinators*:**	Jean-Claude Bermond	-	CNRS, U. de Nice-Sophia Antipolis
	Marc Noy	-	Univ. Politècnica de Catalunya
	Dimitrios M. Thilikos	-	CNRS, Université de Montpellier
	Gilles Trombettoni	-	Université de Montpellier

Outline of the talk

1. Introduction
   - Career path
   - Scientific context: parameterized complexity
   - A relevant parameter: treewidth
2. Some of my contributions (related to treewidth)
   - The number of graphs of bounded treewidth
   - Linear kernels on sparse graphs
   - Fast FPT algorithms parameterized by treewidth
3. Conclusions

Next section is...

My (scientific) life in one slide

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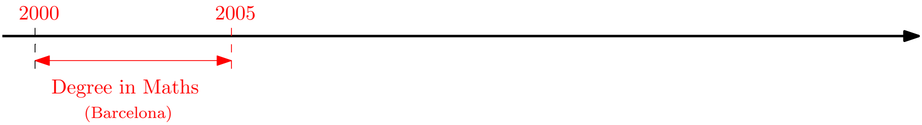

- Maths + Telecom at Universitat Politècnica de Catalunya (UPC).

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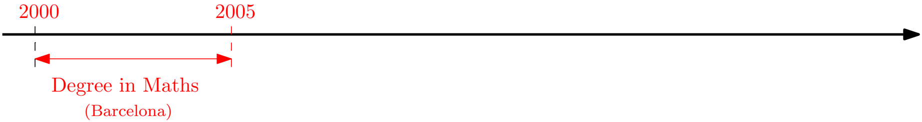

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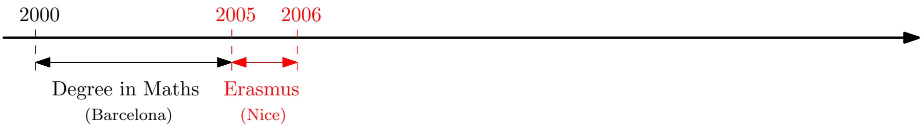

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- **Topic**: traffic grooming in optical networks.

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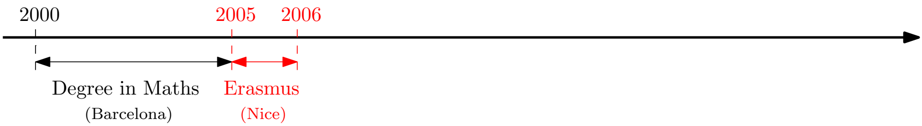

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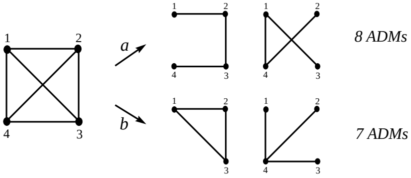

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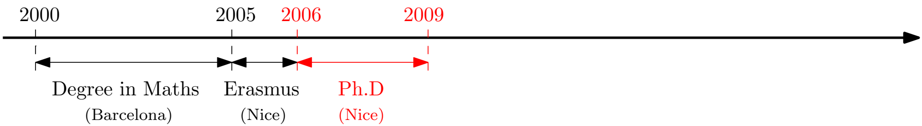

- Advisors: X. Muñoz (Barcelona) + D. Coudert, J.-C. Bermond (Sophia).
- **Topic**: optimization in graphs under degree constraints.

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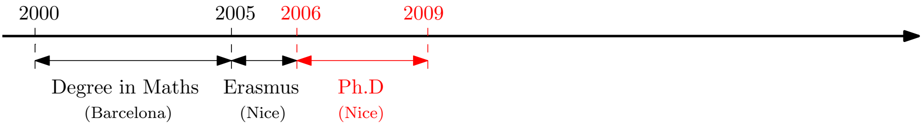

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- I converted to parameterized complexity.

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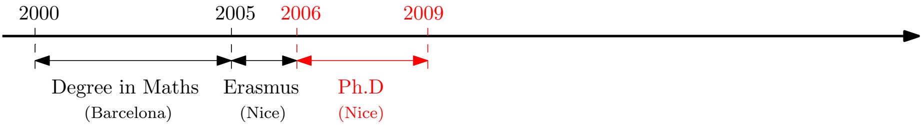

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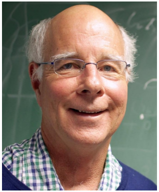

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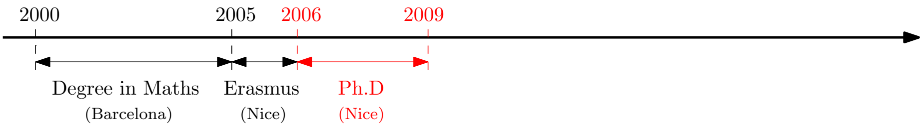

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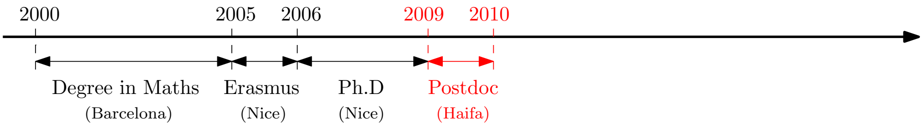
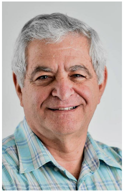


- Postdoc at the Computer Science Department of the Technion.
- With Shmuel Zaks and Mordechai Shalom.



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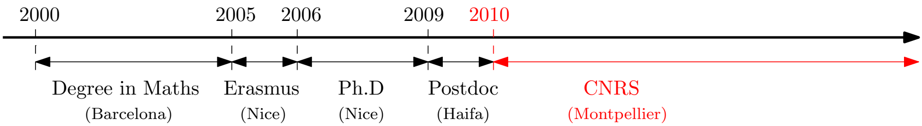

- Since October 2010, I joined the CNRS at LIRMM, Montpellier.
- AIGCo group: Algorithmes, Graphes et Combinatoire.

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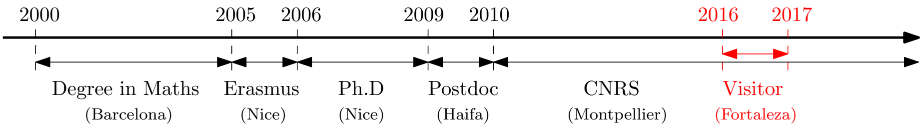

- Visiting professor at Universidade Federal do Ceará, Fortaleza, Brazil.
- ParGO group: Paralelismo, Grafos e Otimização combinatòria.

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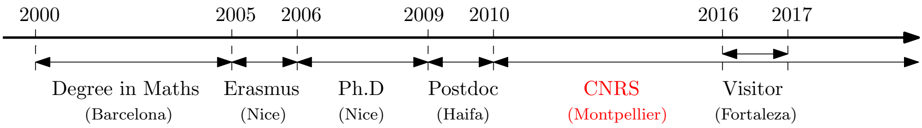

- Since August 2017, back to Montpellier.
- AIGCo group: Algorithmes, Graphes et Combinatoire.

Supervised students

- 03/**2012**-08/**2012** Valentin Garnero (internship M2)
  Polynomial kernels for variants of domination problems on planar graphs
- 02/**2013**-07/**2013** Julien Baste (internship M2)
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- 09/**2018**-08/**2019** Raul Wayne (Ph.D internship, Brazil)
  Fixed-parameter tractability of the Directed Grid Theorem
- 09/**2018**-03/**2019** Guilherme Gomes (Ph.D internship, Brazil)
  Cliques, bicliques and colorings

Next subsection is...

Some history of complexity: NP-completeness

- Cook-Levin Theorem (1971): the SAT problem is NP-complete.
- Karp (1972): list of 21 *important* NP-complete problems.
- Nowadays, literally thousands of problems are known to be NP-hard: unless $\mathsf{P} = \mathsf{NP}$, they cannot be solved in polynomial time.

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- Nowadays, literally thousands of problems are known to be NP-hard: unless $\mathrm{P} = \mathrm{NP}$, they cannot be solved in polynomial time.
- But what does it mean for a problem to be NP-hard?

  No algorithm solves all instances optimally in polynomial time.

Are all instances really hard to solve?

Maybe there are relevant subsets of instances that can be solved efficiently.

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- VLSI design: the number of circuit layers is usually ≤ 10.
- Computational biology: Real instances of DNA chain reconstruction usually have treewidth ≤ 11.
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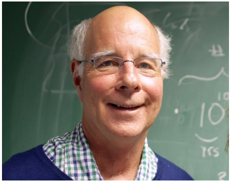
Message In many applications, not only the total size of the instance matters, but also the value of an additional parameter.

The area of parameterized complexity

Idea Measure the complexity of an algorithm in terms of the input size and an additional parameter.

This theory started in the late 80's, by Downey and Fellows:



Today, it is a well-established area with hundreds of articles published every year in the most prestigious TCS journals and conferences.

Parameterized problems

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- k-Vertex Cover: Does a graph G contain a set $S \subseteq V(G)$, with $|S| \leq k$, containing at least an endpoint of every edge?
- k-Independent Set: Does a graph G contain a set $S \subseteq V(G)$, with $|S| \geq k$, of pairwise non-adjacent vertices?
- Vertex k-Coloring: Can the vertices of a graph be colored with $\leq k$ colors, so that any two adjacent vertices get different colors?

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These three problems are NP-hard, but are they equally hard?

They behave quite differently...

- k-Vertex Cover: Solvable in time $\mathcal{O}(2^k \cdot (m+n))$

- k-Independent Set: Solvable in time $\mathcal{O}(k^2 \cdot n^k)$

- Vertex k-Coloring: NP-hard for fixed $k = 3$.

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Why k-CLIQUE may not be FPT?

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Working hypothesis of parameterized complexity: k-Clique is not FPT

(in classical complexity: 3-SAT cannot be solved in poly-time)

How to transfer hardness among parameterized problems?

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W[1]-hard problem: $\exists$ parameterized reduction from k-CLIQUE to it.

W[2]-hard problem: $\exists$ param. reduction from k-DOMINATING SET to it.

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W[i]-hard: strong evidence of not being FPT. Hypothesis: FPT $\neq$ W[1]

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Do all FPT problems admit polynomial kernels? NO!

Theorem (Bodlaender, Downey, Fellows, Hermelin, 2009)

Deciding whether a graph has a PATH *with* $\geq k$ *vertices* is FPT *but does not admit a polynomial kernel, unless* NP $\subseteq$ coNP/poly.

Typical approach to deal with a parameterized problem

Parameterized problem L

k-Clique

k-Vertex Cover

k-Path

Vertex k-Coloring

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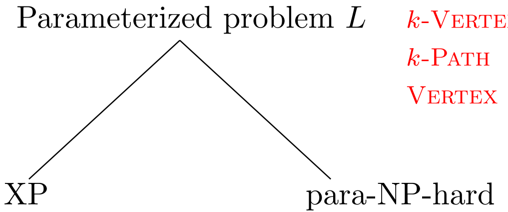

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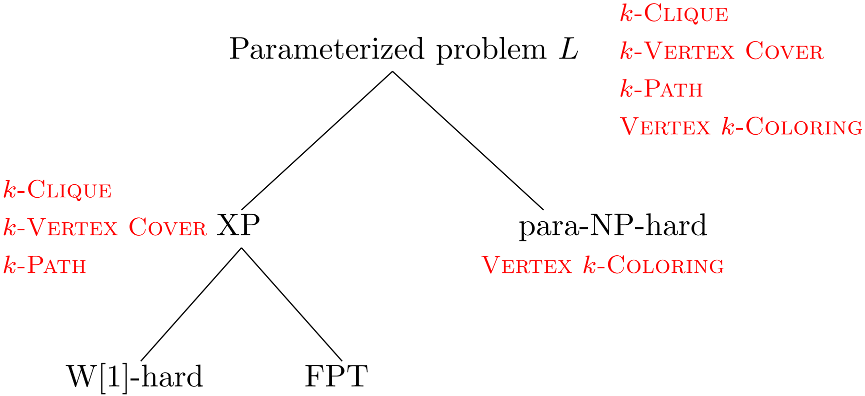

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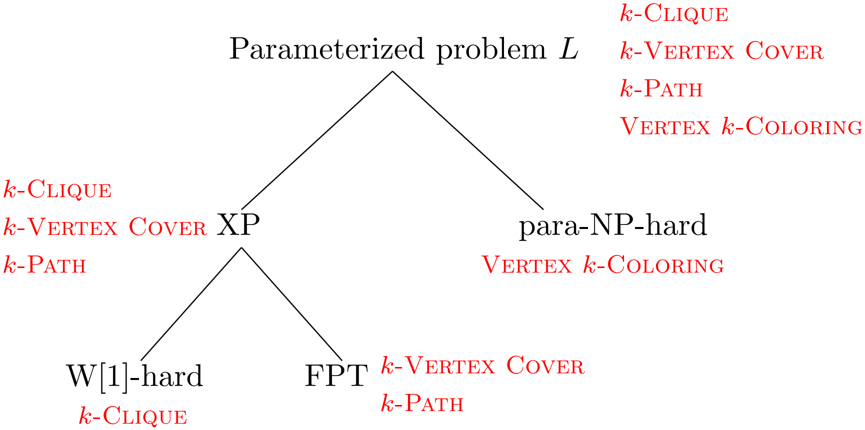

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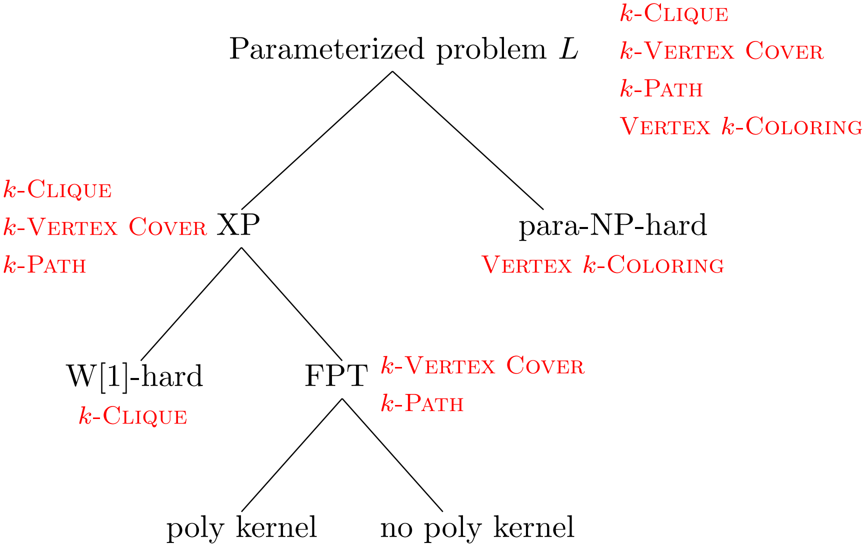

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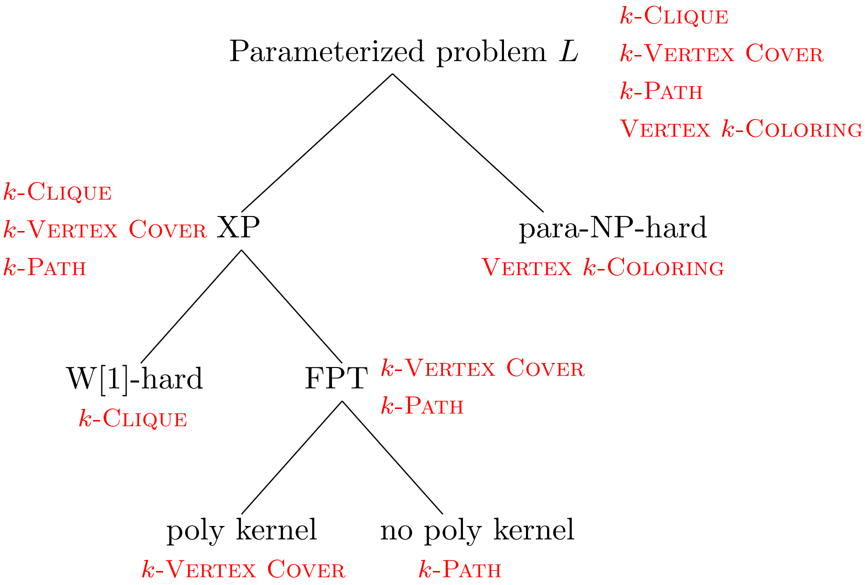

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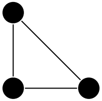
   - Linear kernels on sparse graphs
   - Fast FPT algorithms parameterized by treewidth

3. Conclusions

Treewidth via k-trees

A k-tree is a graph that can be built starting from a $(k+1)$-clique and then iteratively adding a vertex connected to a k-clique.

Example of a 2-tree:



[Figure by Julien Baste]

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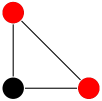

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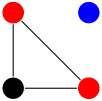

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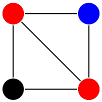

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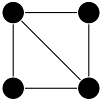

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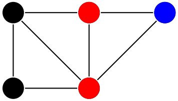

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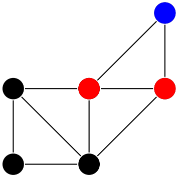

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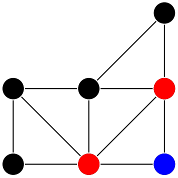

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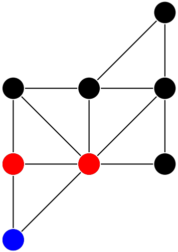

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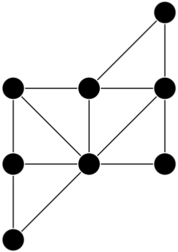

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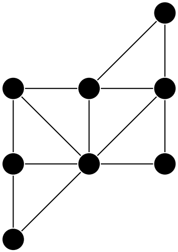

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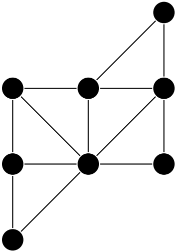

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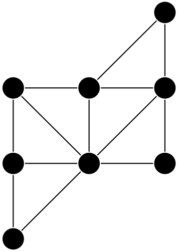

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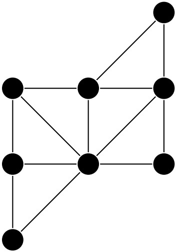

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Construction suggests the notion of tree decomposition: small separators.

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2. Treewidth behaves very well algorithmically, and algorithms parameterized by treewidth appear very often in FPT algorithms.

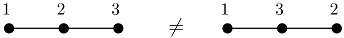
3. In many practical scenarios, it turns out that the treewidth of the associated graph is small (programming languages, road networks, ...).

Next section is...

1. Introduction
   - Career path
   - Scientific context: parameterized complexity
   - A relevant parameter: treewidth
2. Some of my contributions (related to treewidth)
   - The number of graphs of bounded treewidth
   - Linear kernels on sparse graphs
   - Fast FPT algorithms parameterized by treewidth
3. Conclusions

Next subsection is...

What is known about the number of (partial) k-trees?



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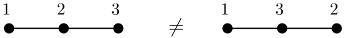

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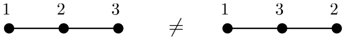

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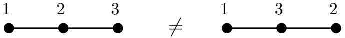

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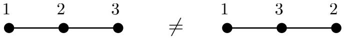

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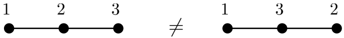

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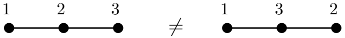

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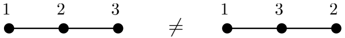

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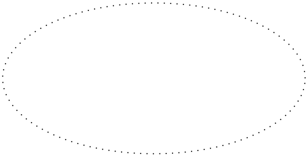

Take a forest on $n - (k - 1)$ vertices:
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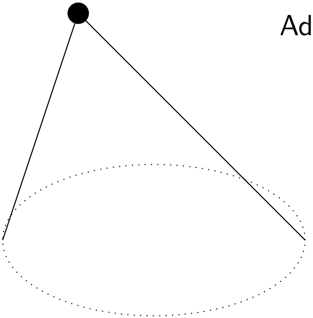

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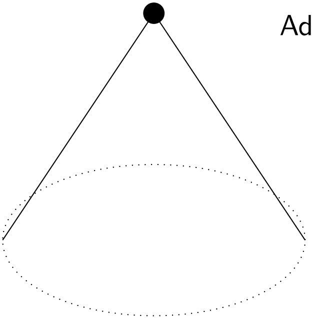

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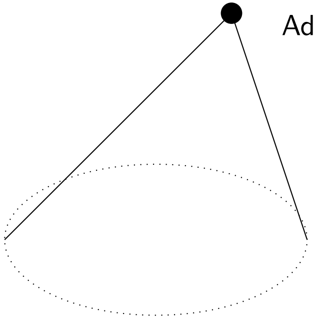

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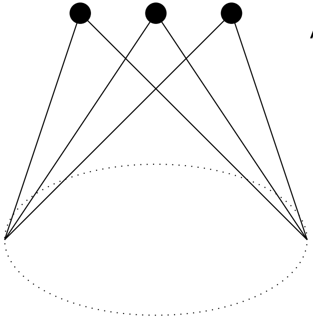

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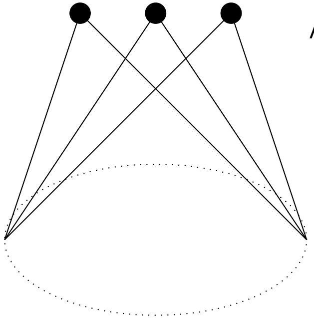

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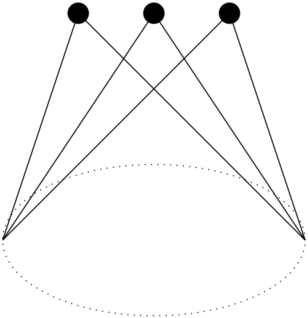

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An improved lower bound

Summarizing, so far we have:

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For any two integers n, k with $1 < k \leq n$, the number $T_{n,k}$ of n-vertex labeled graphs with treewidth at most k satisfies

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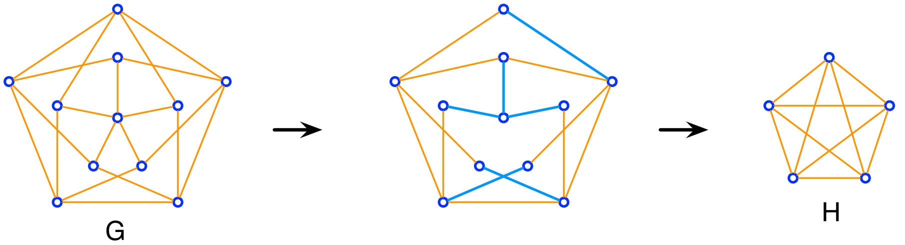
This has been also a very active area in parameterized complexity, specially on sparse graphs: planar graphs, graphs on surfaces, minor-free graphs, ...

Minors and topological minors



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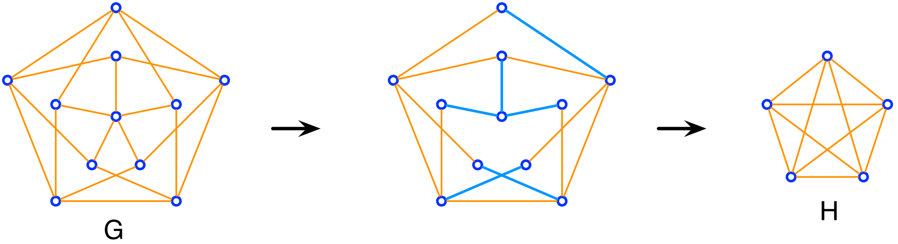
Minors and topological minors



- H is a minor of a graph G if H can be obtained from a subgraph of G by contracting edges.
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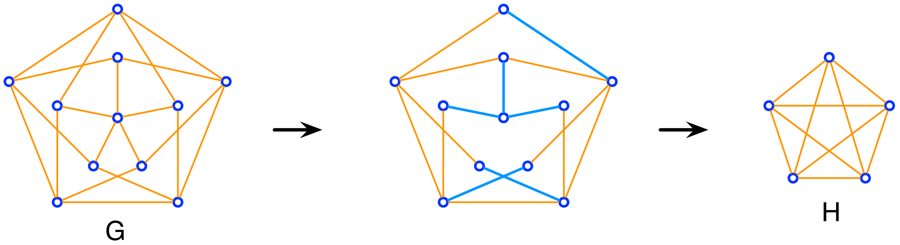

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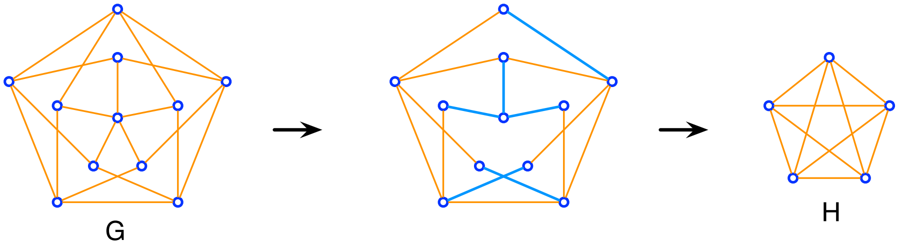

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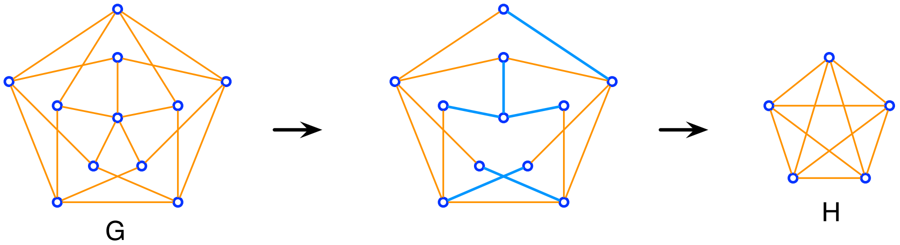

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- Fixed H: H-minor-free graphs $\subseteq$ H-topological-minor-free graphs

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★ Meta-kernelization for topological-minor-free graphs.

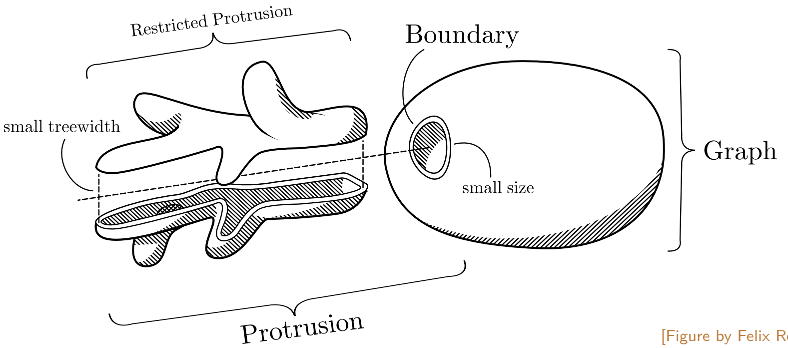
[Kim, Langer, Paul, Reidl, Rossmanith, **S**., Sikdar. 2013]

Protrusions

[Bodlaender, Fomin, Lokshtanov, Penninkx, Saurabh, Thilikos. 2009]

- Given a graph G, a set $W \subseteq V(G)$ is a t-protrusion of G if

$$|\partial_G(W)| \leq t \quad \text{and} \quad tw(G[W]) \leq t.$$



[Figure by Felix Reidl]

- We call $\partial_G(W)$ the boundary and $|W|$ the size of W.

Meta-kernelization on graphs without topological minors

Theorem (Kim, Langer, Paul, Reidl, Rossmanith, S., Sikdar, 2013)

Fix a graph H. Let P be a parameterized graph problem on the class of H-topological-minor-free graphs that is treewidth-bounding and has finite integer index (FII). Then P admits a linear kernel.

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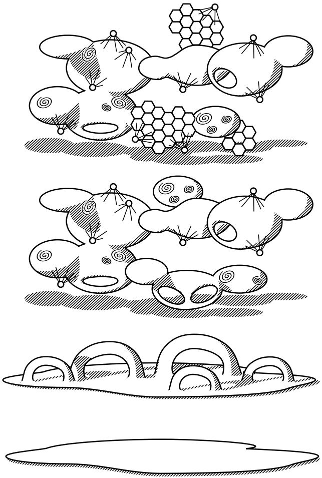
Some problems affected by our result:

TREEWIDTH-t VERTEX DELETION, CHORDAL VERTEX DELETION, INTERVAL VERTEX DELETION, EDGE DOMINATING SET, FEEDBACK VERTEX SET, CONNECTED VERTEX COVER, ...

Linear kernels on sparse graphs – the conditions



H-topological-minor-free — treewidth-bounding

$\cup$

H-minor-free — bidimensional, separation property

$\cup$

bounded genus — quasi-compact

$\cup$

planar — "distance-property"

[Figure by Felix Reidl]

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- Our results imply the linear kernels of [Fomin, Lokshtanov, Saurabh, Thilikos. 2010]

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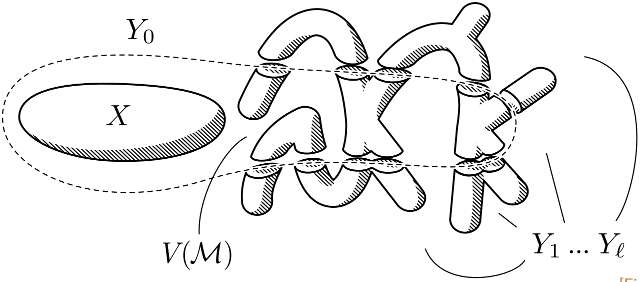

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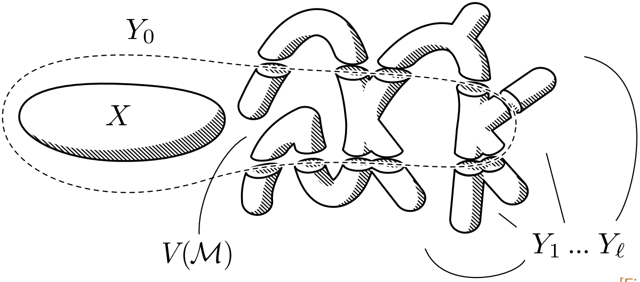

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Idea: Find a linear protrusion decomposition in polynomial time, and replace each of the $\mathcal{O}(k)$ protrusions with a constant-sized gadget.

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There are some techniques to actually construct the kernels (CMSO logic), but it is hard to extract explicit constants on the size of the kernels...

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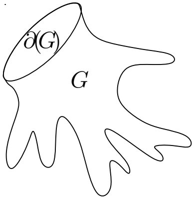
  DP-friendly encoder: we can safely replace equivalent "protrusions".

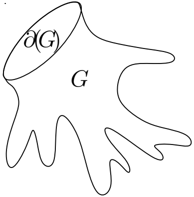
An explicit meta-kernelization result



[By Valentin Garnero]

An explicit meta-kernelization result

$|\partial(G)| \leq \mathsf{tw}(G)$



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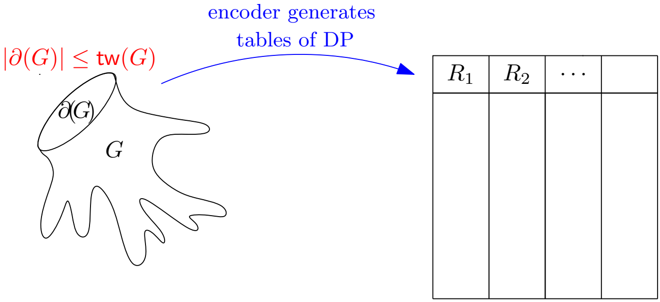

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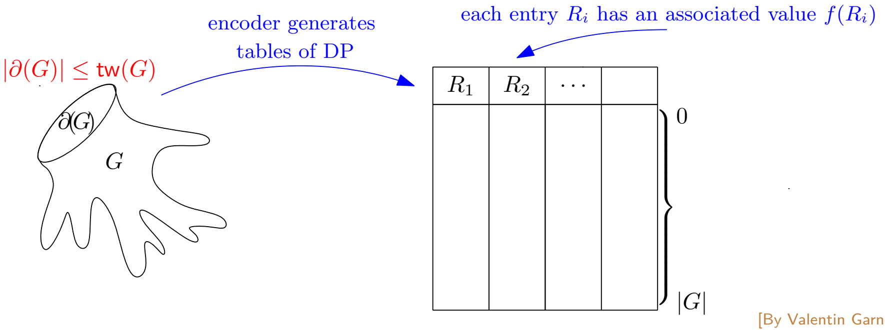

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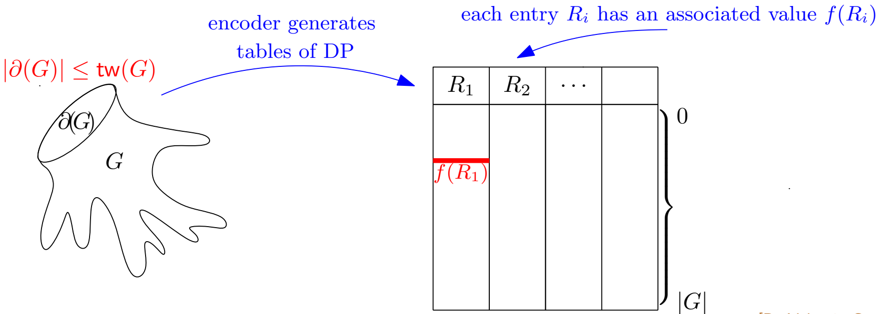

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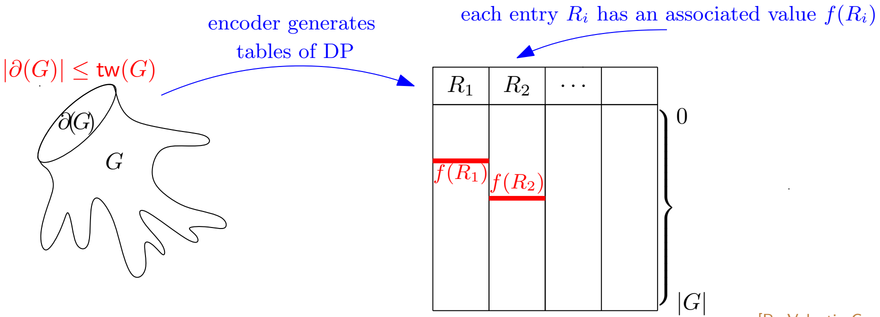

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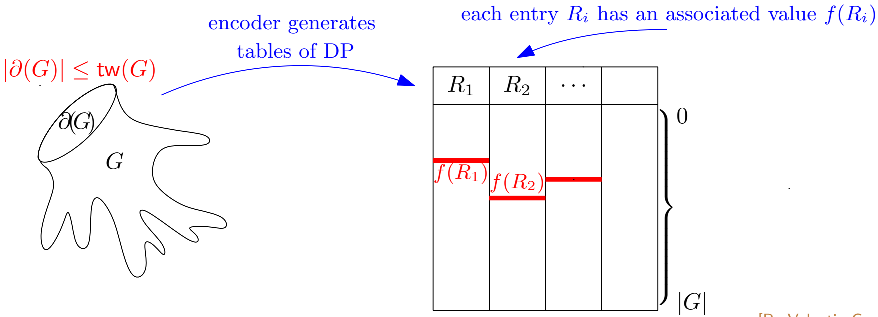

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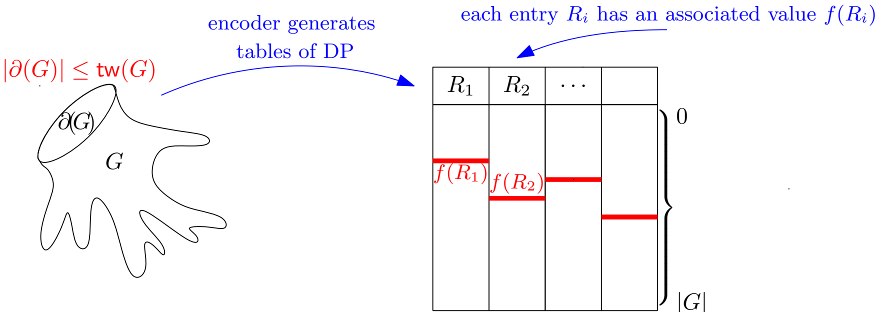

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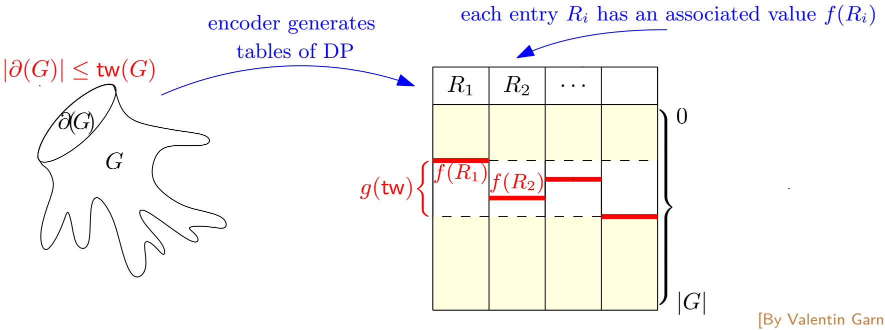

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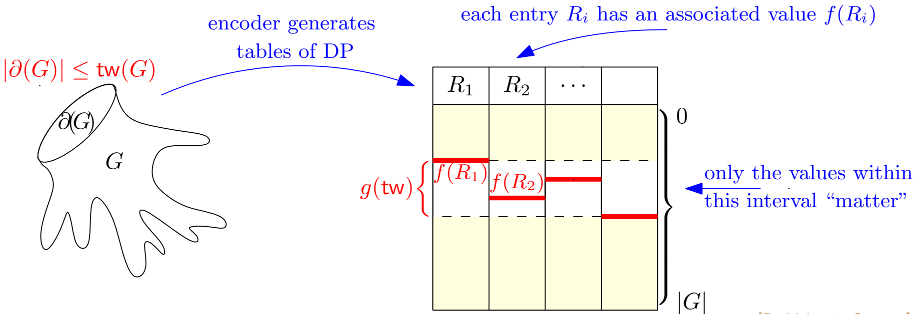

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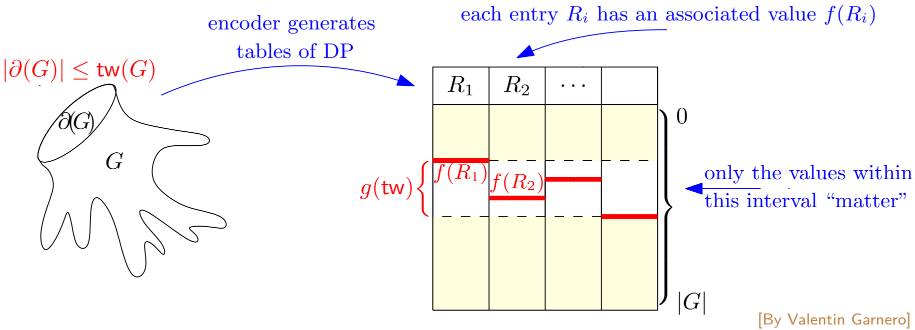

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Theorem (Garnero, Paul, S., Thilikos, 2014)

Linear protrusion decomp. $+$ *confined DP-friendly encoder* $=$ *linear kernel.*

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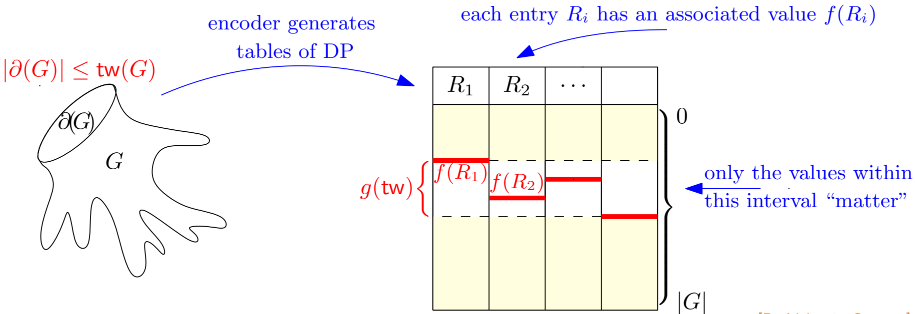

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Some problems affected by our result:

r-Dominating Set and r-Scattered Set on apex-minor-free graphs,
Planar-$\mathcal{F}$-Deletion on (topological)-minor-free graphs,
several generalizations of $\mathcal{F}$-Packing, ...

Next subsection is...

1. Introduction
   - Career path
   - Scientific context: parameterized complexity
   - A relevant parameter: treewidth
2. Some of my contributions (related to treewidth)
   - The number of graphs of bounded treewidth
   - Linear kernels on sparse graphs
   - Fast FPT algorithms parameterized by treewidth
3. Conclusions

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Theorem (Courcelle, 1990)

Every problem expressible in MSOL can be solved in time $f(\text{tw}) \cdot n$ on graphs on n vertices and treewidth at most tw.

Examples: Vertex Cover, Dominating Set, Hamiltonian Cycle, Clique, Independent Set, k-Coloring for fixed k, ...

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Major goal: find the smallest possible function $f(\text{tw})$.

This is a very active area in parameterized complexity.

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Typical statements:

ETH $\Rightarrow$ k-Vertex Cover cannot be solved in time $2^{o(k)} \cdot n^{O(1)}$.

ETH $\Rightarrow$ Planar k-Vertex Cover cannot in time $2^{o(\sqrt{k})} \cdot n^{O(1)}$.

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But for the so-called connectivity problems, like Longest Path or Steiner Tree, the "natural" DP algorithms provide only time

$$2^{\mathcal{O}(\mathrm{tw} \cdot \log \mathrm{tw})} \cdot n^{\mathcal{O}(1)}.$$

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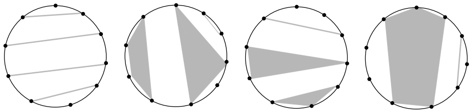
Main idea special type of decomposition with nice topological properties:

partial solutions $\Longleftrightarrow$ non-crossing partitions



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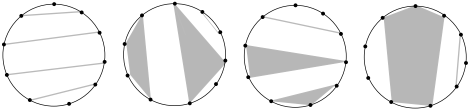
- Minor-free graphs: [Dorn, Fomin, Thilikos. 2008]
  [Rué, **S.**, Thilikos. 2012]

Main idea special type of decomposition with nice topological properties:

partial solutions $\Longleftrightarrow$ non-crossing partitions



$$\mathrm{CN}(k) = \frac{1}{k+1}\binom{2k}{k} \sim \frac{4^k}{\sqrt{\pi}k^{3/2}} \leq 4^k.$$

The revolution of single-exponential algorithms

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Representative sets in matroids: [Fomin, Lokshtanov, Saurabh. 2014]

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There are other examples of such problems...

The $\mathcal{F}$-DELETION problem

Let $\mathcal{F}$ be a fixed finite collection of graphs.

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$\mathcal{F}$-Deletion
Input: A graph G and an integer k.
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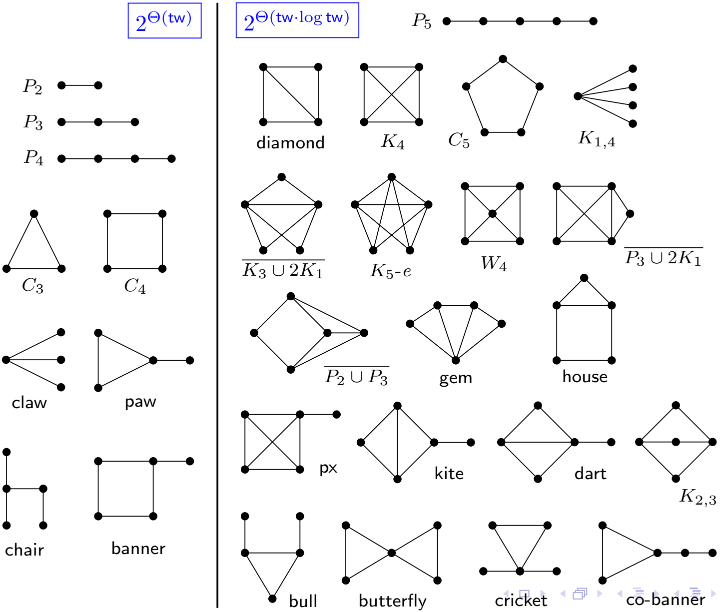
With Julien Baste and Dimitrios M. Thilikos we proved the following...

[arXiv:1704.07284. 2018]

Complexity of $\{H\}$-DELETION for small planar graphs H



Next section is...

1. Introduction
   - Career path
   - Scientific context: parameterized complexity
   - A relevant parameter: treewidth

2. Some of my contributions (related to treewidth)
   - The number of graphs of bounded treewidth
   - Linear kernels on sparse graphs
   - Fast FPT algorithms parameterized by treewidth

3. Conclusions

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- Conjecture For every connected planar graph H with $|V(H)| \geq 6$, $\mathcal{F}$-Deletion is solvable in time $2^{\Theta(\mathsf{tw} \cdot \log \mathsf{tw})} \cdot n^{\mathcal{O}(1)}$ under the ETH.

Gràcies!

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