Exercises: Algorithms for Imprecise Probability and Credal networks

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Build linear program:

- $P(A) \geq \alpha$.
- $B \rightarrow C$.
- $P(B) = \beta$.

Can you give bounds for $P(A \wedge B \wedge C)$?

Coletti and Scozzafava (1999).

- Take H_1, H_2, H_3.
- Assume $H_3 \subset H_1^c \cap H_2$.
- Assessments $P(H_1) = 1/2$, $P(H_2) = 1/5$, $P(H_3) = 1/8$.

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- Take H_1, H_2, H_3.
- Assume $H_3 \subset H_1^c \cap H_2$.
- Assessments $P(H_1) = 1/2$, $P(H_2) = 1/5$, $P(H_3) = 1/8$.
- Also, $P(H_2 | H_1 \cup H_2) \geq 1/2$.

Build linear program.

- Take H_1, H_2, H_3.
- Assume $H_3 \subset H_1^c \cap H_2$.
- Assessments $P(H_1) = 1/2$, $P(H_2) = 1/5$, $P(H_3) = 1/8$.

Build linear program to compute $\underline{P}(H_1 | H_1 \cup H_2)$, applying the Charnes-Cooper transformation.

Consider a variable X with 3 possible values x_1, x_2 and x_3. Suppose the following assessments are given:

$$p(x_1) \leq p(x_2) \leq p(x_3)\,;$$

$$p(x_i) \geq 1/20 \qquad \text{for } i \in \{1, 2, 3\};$$

$$p(x_3 | x_2 \cup x_3) \leq 3/4.$$

Show the credal set determined by these assessments in baricentric coordinates.

Consider assessments:

- $P(A) \geq 1/2$.
- $P(A^c \cap B^c) = 1/2$.
- $P(C|A^c \cap B) = 1/3$.

What is the Krauss-Dubins representation?
What is $P(C|B)$?
What is $P(C^c|A^c \cap B)$?

- If $K(X)$ is an ϵ-contaminated class, what are

$$\underline{E}[f(X)]\,, \quad \overline{E}[f(X)]?$$

  - If P_0 is always nonzero, what is

$$\underline{P}(A|B)\,, \quad \overline{P}(A|B)?$$

- If one gives a measure L such that

$$L(\Omega) < 1,$$

is this an ϵ-contaminated class?
If so, what are P_0 and ϵ?

Credal set $\{P_1, P_2\}$:

$$P_1(s_1) = 1/8, \quad P_1(s_2) = 3/4, \quad P_1(s_3) = 1/8,$$

$$P_2(s_1) = 3/4, \quad P_2(s_2) = 1/8, \quad P_2(s_3) = 1/8,$$

Acts $\{a_1, a_2, a_3\}$:

	s_1	s_2	s_3
a_1	3	3	4
a_2	2.5	3.5	5
a_3	1	5	4.

Which one to select?

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$$P_1(s_1) = 1/8, \quad P_1(s_2) = 3/4, \quad P_1(s_3) = 1/8,$$

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Acts $\{a_1, a_2, a_3\}$:

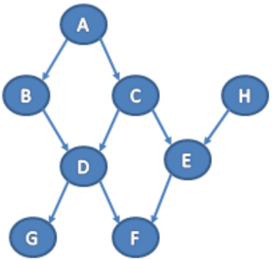
	s_1	s_2	s_3
a_1	3	3	4
a_2	2.5	3.5	5
a_3	1	5	4.

Which one to select?
And if we take convex hull of credal set?

- Show: The Markov condition implies

$$p(X_1, \ldots, X_n) = \prod_i p(X_i | pa(X_i)).$$

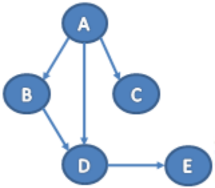
- Evaluate $p(a|e)$ using the Bayesian network just defined. Count the number of multiplications that you need to find the solution.
- Find $\arg\max_{A,C|d} p(A, C|d)$ using the same Bayesian network.
- Using the following Bayesian network, verify which are true: $(A \perp\!\!\!\perp H|F)$, $(G \perp\!\!\!\perp E|C)$, $(G \perp\!\!\!\perp E|A)$, $(B \perp\!\!\!\perp E|A, D)$, $(G \perp\!\!\!\perp H|F)$.



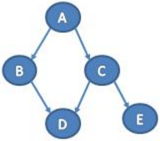
- Give a set of independence relations that can be encoded using a Bayesian network (and show such network) but cannot be encoded using a Markov Random Field.
- Use maximum likelihood to estimate the parameters of the following Bayesian network.
- Repeat using a Dirichlet model with $s = 1$ and uniform $\tau(X|pa(X))$.

$$p(x|pa(X)) = \frac{n_{x,pa(X)} + s \cdot \tau(x|pa(X))}{n_{pa(X)} + s}$$

A	B	C	D	E
a	¬b	¬c	¬d	¬e
a	b	¬c	d	¬e
a	¬b	c	d	¬e



- Evaluate $\overline{p}(a|e)$ using the following credal network.

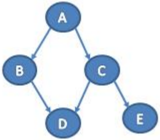


$p(a) \in [0.1, 0.3]$, $p(c|a) = 0.5$, $p(c|\neg a) = 0.8$, $p(e|c) \in [0.6, 0.9]$, $p(e|\neg c) = 0.5$, $p(b|\neg a) \in [0.1, 0.5]$, $p(d|b, c) \in [0.1, 0.5]$, $p(d|\neg b, c) = 0.2$ and other parameters are vacuous.

- Translate the credal network into a Bayesian network using the CCM transformation.
- Find a parameterization that respects the credal network and maximizes the entropy in each local conditional distribution.

- Use Imprecise Dirichlet Model to learn new intervals to the credal network. Use $s = 1$.

A	B	C	D	E
a	¬b	¬c	¬d	¬e
a	b	¬c	d	¬e
a	¬b	c	d	¬e



$p(a) \in [0.1, 0.3], p(c|a) = 0.5, p(c|\neg a) = 0.8, p(e|c) \in [0.6, 0.9], p(e|\neg c) = 0.5, p(b|\neg a) \in [0.1, 0.5], p(d|b, c) \in [0.1, 0.5], p(d|\neg b, c) = 0.2$ and other parameters are vacuous.

- Show how to translate a Bayesian network MAP problem to a credal network belief updating inference.
- Prove that credal MPE in separately specified polytrees can be solved by MPE in polytree Bayesian networks.

- Show that A/R+ provides outer approximations for the credal belief updating problem.
- In polytrees, which reformulation usually produces a simpler optimization program: variable elimination or the bilinear translation idea? Explain your answer.
- Show that no additional constraints is useful while treating binary networks. Provide a useful constraint that could be propagated in a ternary credal network.

Obtain the optimization problem for the following PPL network:

- Three boolean variables A, B, C.
- Logical sentence: $\psi = a \vee c$.
- Probabilistic logic sentence: $p(\phi) \leq 0.3$, where $\phi = \neg a \vee b$.
- Local credal sets $K(A), K(B|a), K(B|\neg a), K(C)$.