An $\mathcal{ALC}$-like Probabilistic Description Logic

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Description logics...

- Description logics offer attractive trade-offs between expressivity and complexity.
  - Now used in ontologies, semantic web.

- Goal: to express...
  - $\forall x : \mathsf{Brazilian}(x) \longrightarrow \mathsf{SouthAmerican}(x)$,
  - $\forall x : P(\mathsf{FootballFan}(x) \mid \mathsf{Brazilian}(x)) \geq 0.85$.

- There are *many* proposals for probabilistic description logics today.

Description logics ($\mathcal{ALC}$)

- Fragment of first-order logic, with Boolean operators, quantifiers, relations, etc.
  - $C \wedge D$,
  - $C \vee D$,
  - $\forall x : \forall y : r(x, y) \rightarrow C(y)$.
- It is a “reasonably tractable” fragment.
- Terminology: set of sentences.
- Intuition: these are assertions about individuals in some domain.

Adding a bit of probability to $\mathcal{ALC}$

- Just allow assessments such as

$$\forall x : P(C(x)|D(x)) = \alpha$$

for *concepts* C and D.

- Also, probabilistic assessments

$$\forall x : \forall y : P(r(x, y)) = \alpha$$

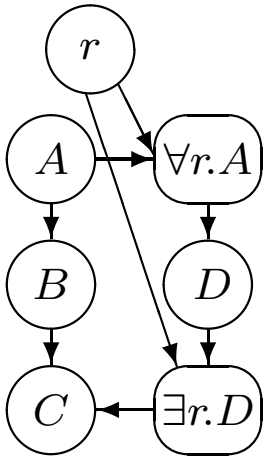
for binary relations $r(x, y)$.

Example I

- Consider a terminology $\mathcal{T}$ with concepts A, B, C, D, where:
  - $\forall x : P(A(x)) = \alpha_1$,
  - $\forall x : B(x) \rightarrow A(x)$,
  - $\forall x : P(B(x)|A(x)) = \alpha_2$,
  - $\forall x : D(x) \Leftrightarrow (\forall y : r(x, y) \rightarrow A(y))$,
  - $\forall x : C(x) \Leftrightarrow B(x) \vee (\exists y : r(x, y) \wedge D(y))$.

Example II

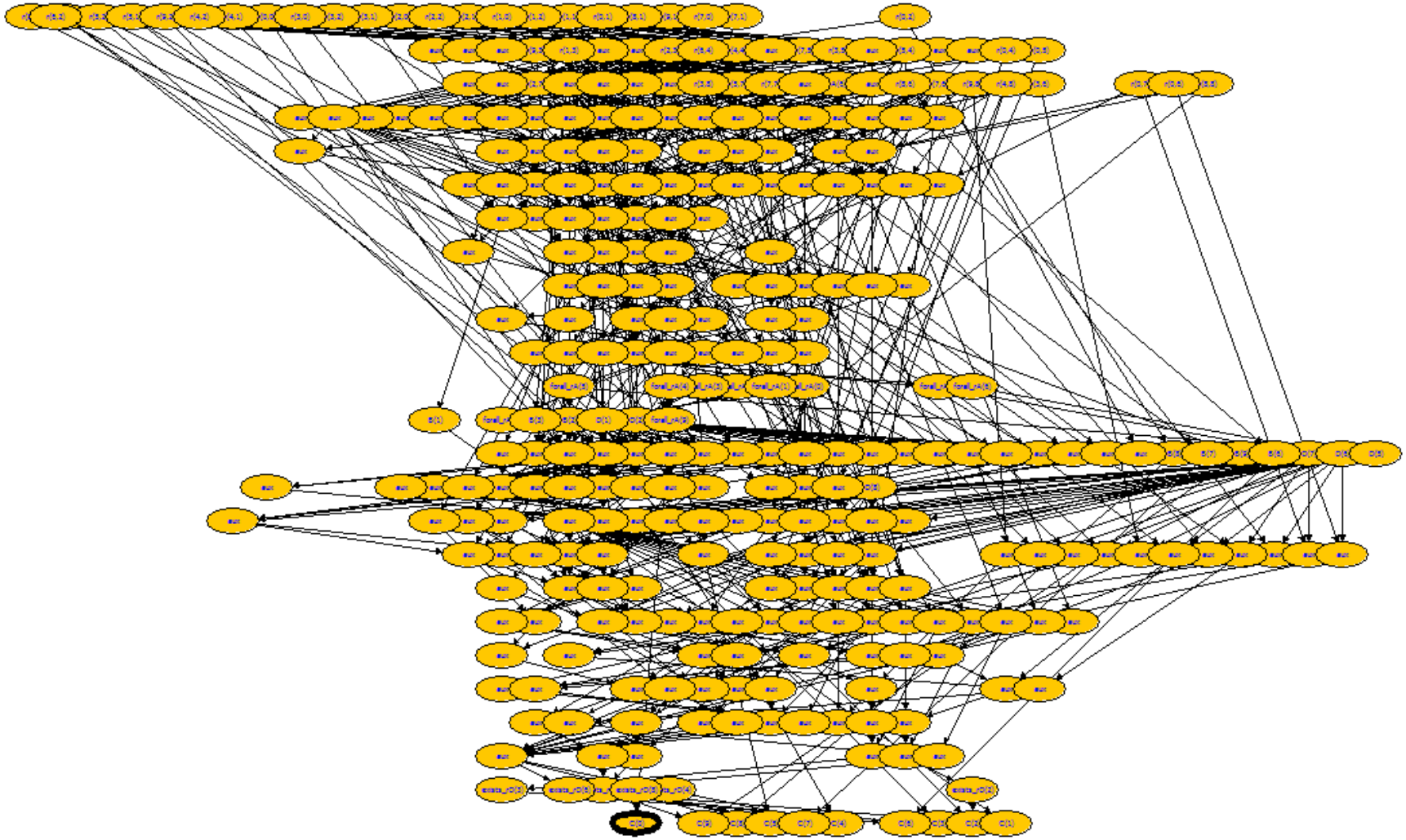
A graphical picture of the assessments...



Usual: acyclicity assumption, Markov condition on groundings...

A “grounded” probabilistic terminology

For a domain with ten elements.



Inference

- After grounding, we have a possibly large credal network.
  - Many relationships are actually deterministic.
- Query: $P(C(a_i)|D(a_j))$.
- How to compute this? L2U is an option.

n	1	5	20	50
L2U: $P(C(a_0))$	$[0.405000$ $0.464500]$	$[0.405000$ $0.405030]$	$[0.405000$ $0.405000]$	$[0.405000$ $0.405000]$

Conclusion

- “Probabilistic logics” (a vague and broad term) has many applications.
- Credal sets are natural (unavoidable?) in that context.
- A possible (killer-)application for our technology.
- Practical problems are within reach given the advances in the last decade (well, with applications and quite a bit of creativity...).
- However, not yet a set of tools widely available.