



# Surgery Simulation

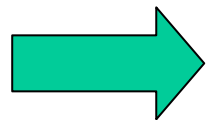
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Hervé Delingette

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DE RECHERCHE  
EN INFORMATIQUE  
ET EN AUTOMATIQUE



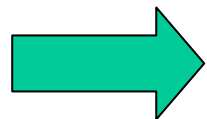
# Motivations of surgery simulation

- Increasing complexity of therapy and especially surgery



Increasing need for training surgeons and residents

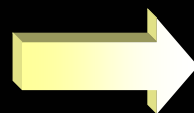
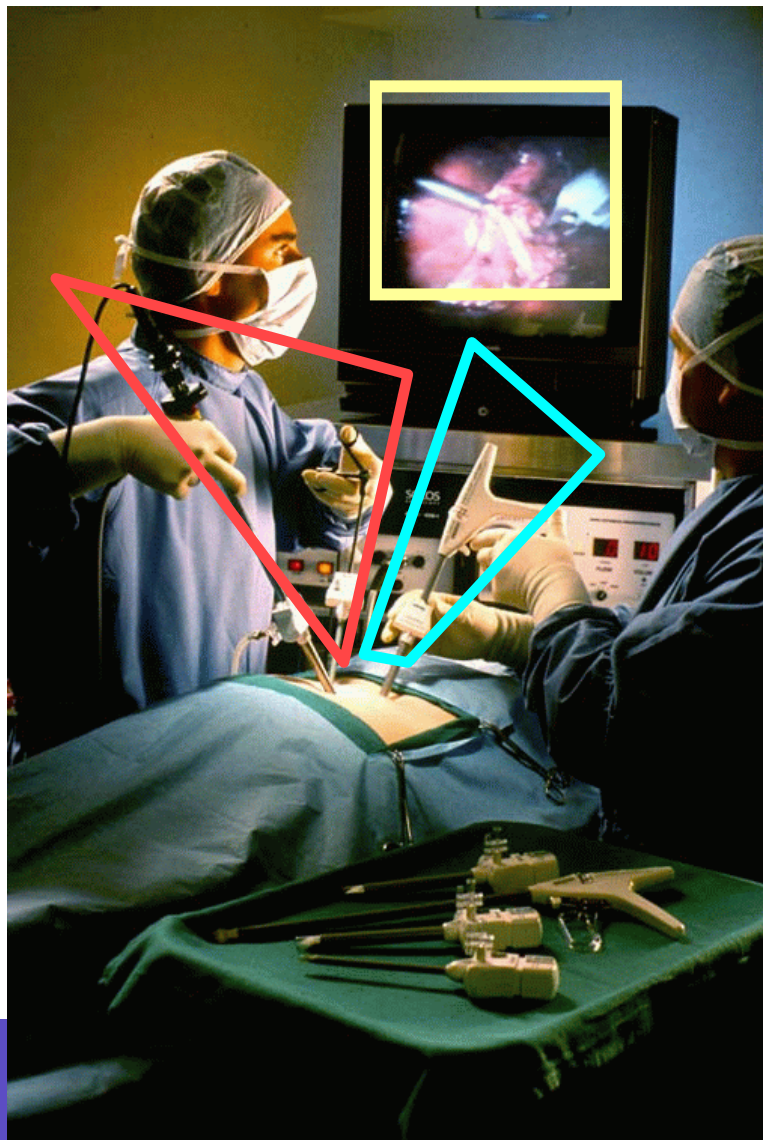
- Medical malpractice has become socially and economically unacceptable



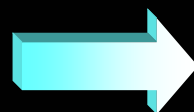
Increasing need for objective evaluation of surgeons  
(see Cordis Nitanol endovascular carotid stent)

- Natural extension of surgery planning

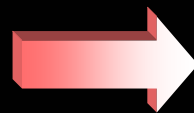
# Need for Training



Hand-eye  
Synchronisation



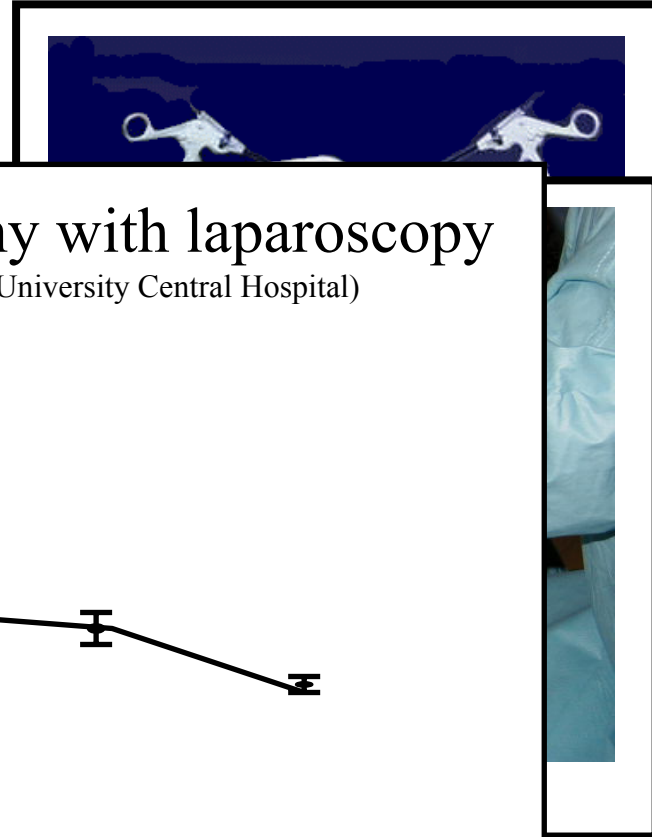
Camera being  
manipulated by an  
assistant



Long instruments  
going through a fixed  
point in the abdomen

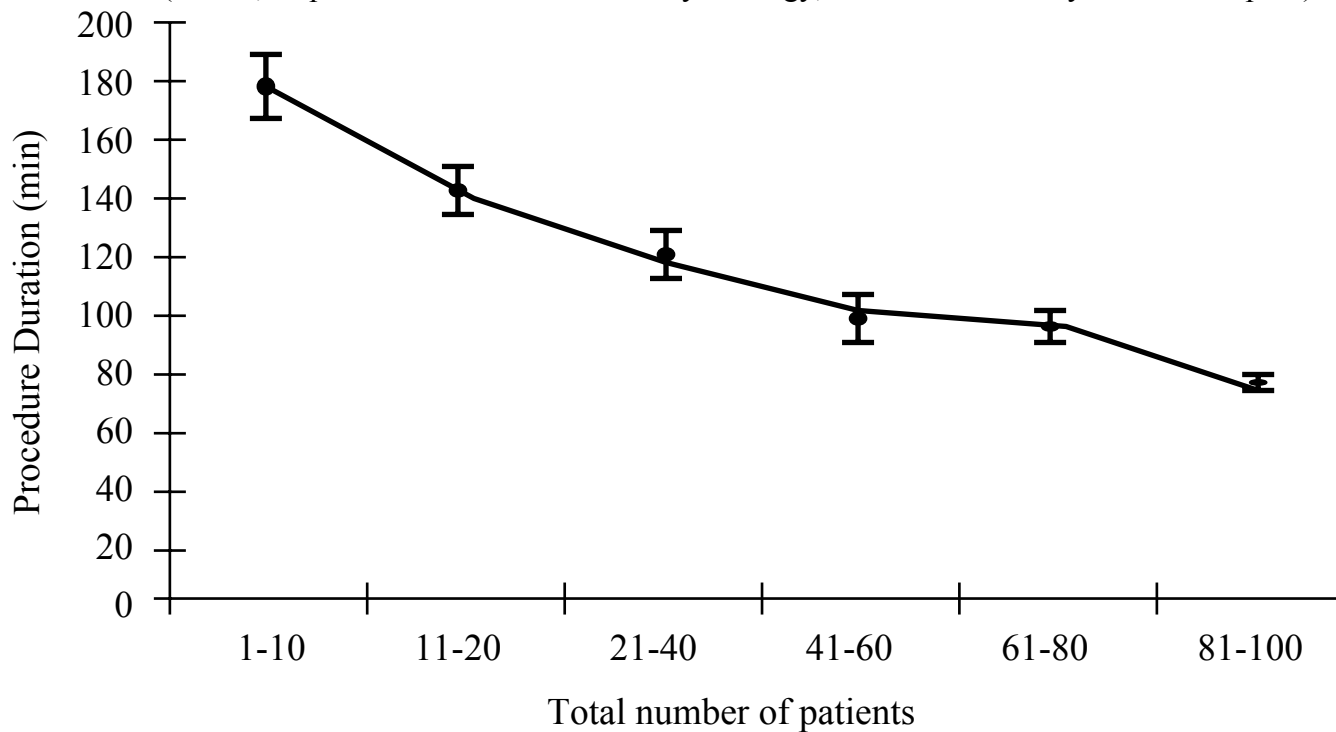
# Current Training Techniques

- Mechanical Simulators



## Average Duration of an hysterectomy with laparoscopy

(source, Department of Obstetrics and Gynecology, Helsinki University Central Hospital)



(source, Ayudamos)

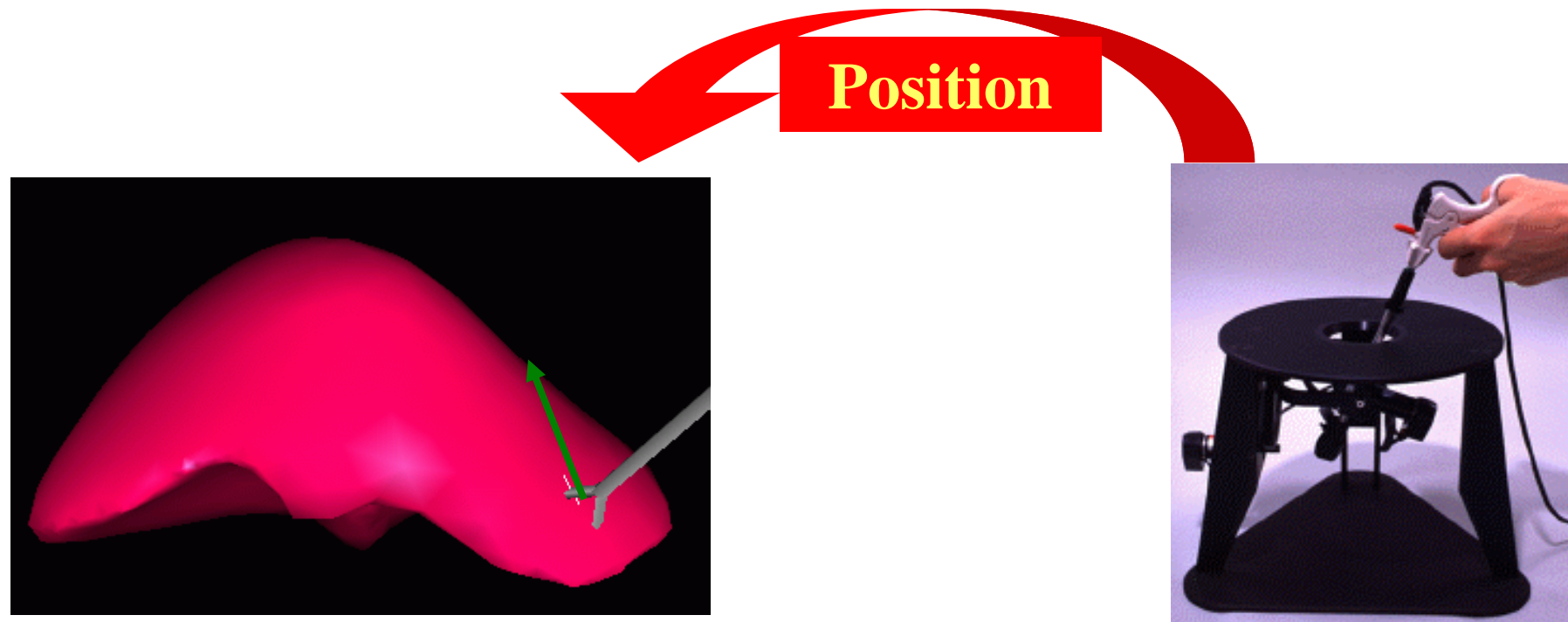
# Training versus Rehearsal

- **Training:** Modelling a *standard* patient for teaching classical or rare situations
- **Rehearsal:** Modelling a *specific* patient to plan and rehearse a delicate intervention, and evaluate consequences beforehand

# Towards Realistic Interactive Simulation

- Surgery Simulation must cope with several difficult technical issues :
  - Soft Tissue Deformation
  - Collision Detection
  - Collision Response
  - Haptics Rendering
- Real-time Constraints :
  - 25Hz for visual rendering
  - 300-1000 Hz for haptic rendering

# Simulator Workflow



- Collision**
- Contact**
- Deformation**
- Force**

# Different Technical Issues

- Mesh Reconstruction from Images
- Soft Tissue Modeling
- Tissue Cutting
- Collision Detection
- Contact Modeling
- Surface Rendering
- Haptic Feedback

With real-  
time  
constraints

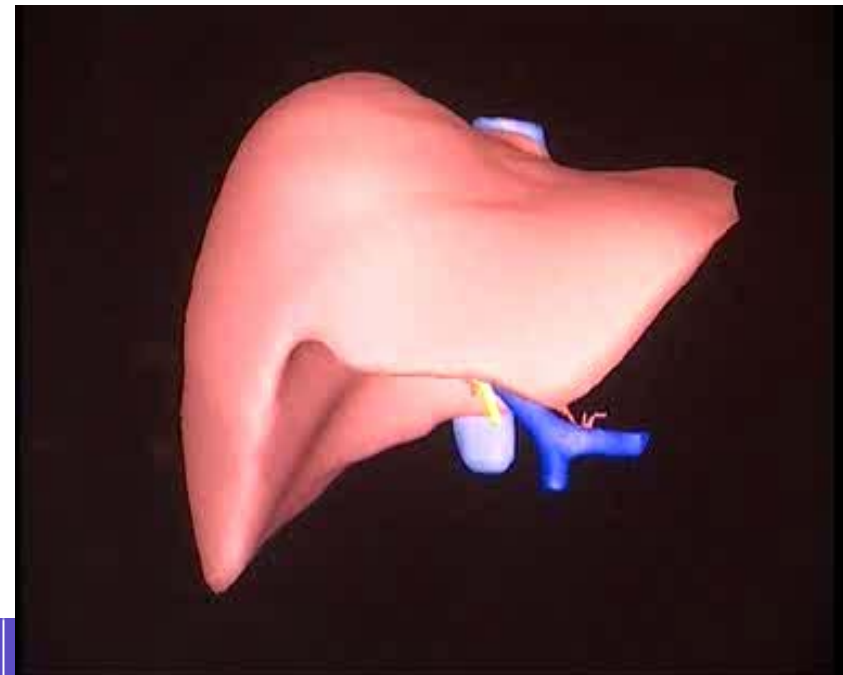
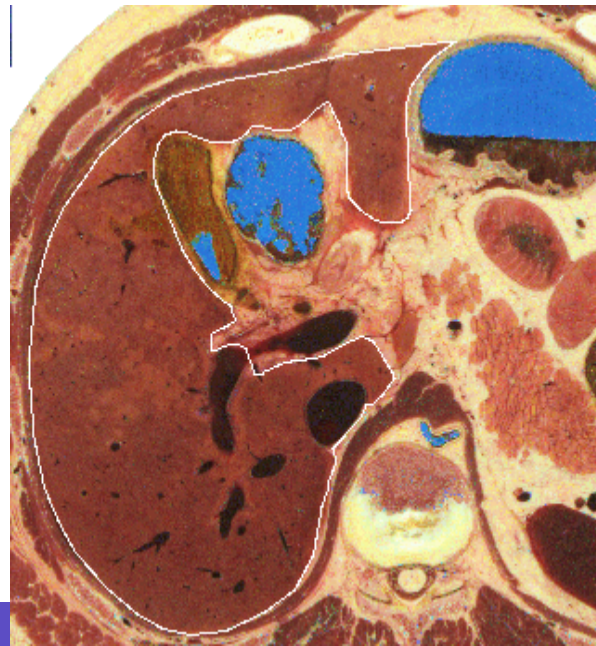


# Different Technical Issues

- Mesh Reconstruction from Images
- Soft Tissue Modeling
- Tissue Cutting
- Collision Detection
- Contact Modeling
- Surface Rendering
- Haptic Feedback

# Liver Reconstruction

**Deformation from a reference model  
reconstructed from the  
« *Visible Human Project* »**



# Different Technical Issues

- Mesh Reconstruction from Images
- Soft Tissue Modeling
- Tissue Cutting
- Collision Detection
- Contact Modeling
- Surface Rendering
- Haptic Feedback

# Soft Tissue Characterization

- Biomechanical behavior of biological tissue is very complex
- Most biological tissue is composed of several components :
  - Fluids : water or blood
  - Fibrous materials : muscle fiber, neuronal fibers, ...
  - Membranes : interstitial tissue, Glisson capsule
  - Parenchyma : liver or brain

# Estimating material parameters

- Complex for biological tissue :
  - Heterogeneous and anisotropic materials
  - Tissue behavior changes between in-vivo and in-vitro
  - Ethics clearance for performing experimental studies
  - Effect of preconditioning
  - Potential large variability across population

# Soft Tissue Characterization

- Different possible methods
  - In vitro rheology
  - In vivo rheology
  - Elastometry
  - Solving Inverse problems

# Soft Tissue Characterization

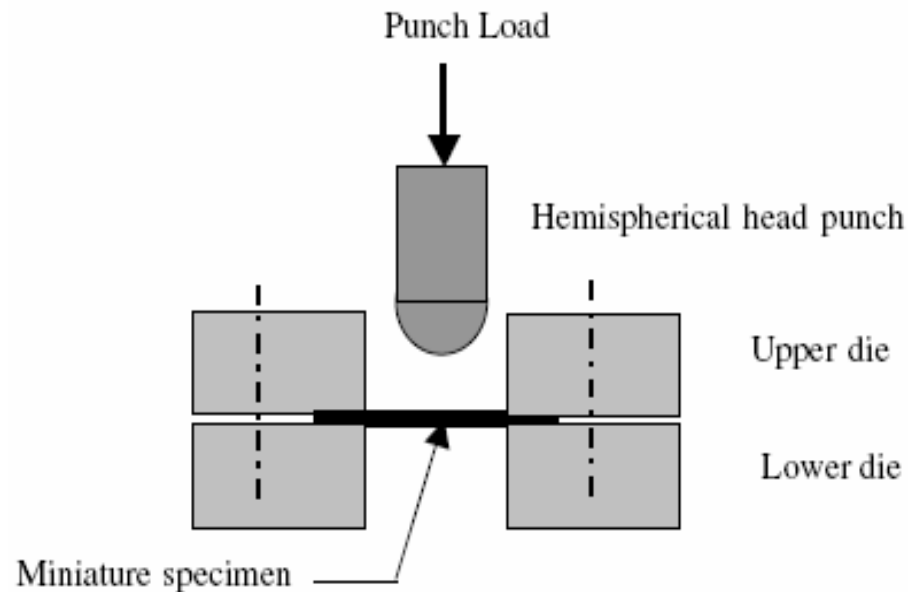
- In vitro rheology



- can be performed in a laboratory.  
Technique is mature



- Not realistic for soft tissue (perfusion, ...)



# Soft Tissue Characterization

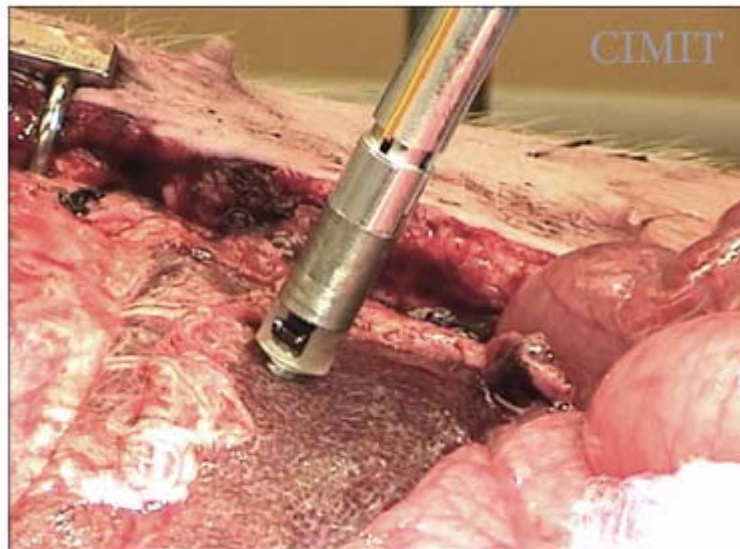
- In vivo rheology



- can provide stress/strain relationships at several locations



- Influence of boundary conditions not well understood



Source : Cimit, Boston USA



# Soft Tissue Characterization

- Elastometry (MR, Ultrasound)



- measure property inside any organ non invasively



- validation ? Only for linear elastic materials



Fibroscan



Source Echosens, Paris

# Soft Tissue Characterization

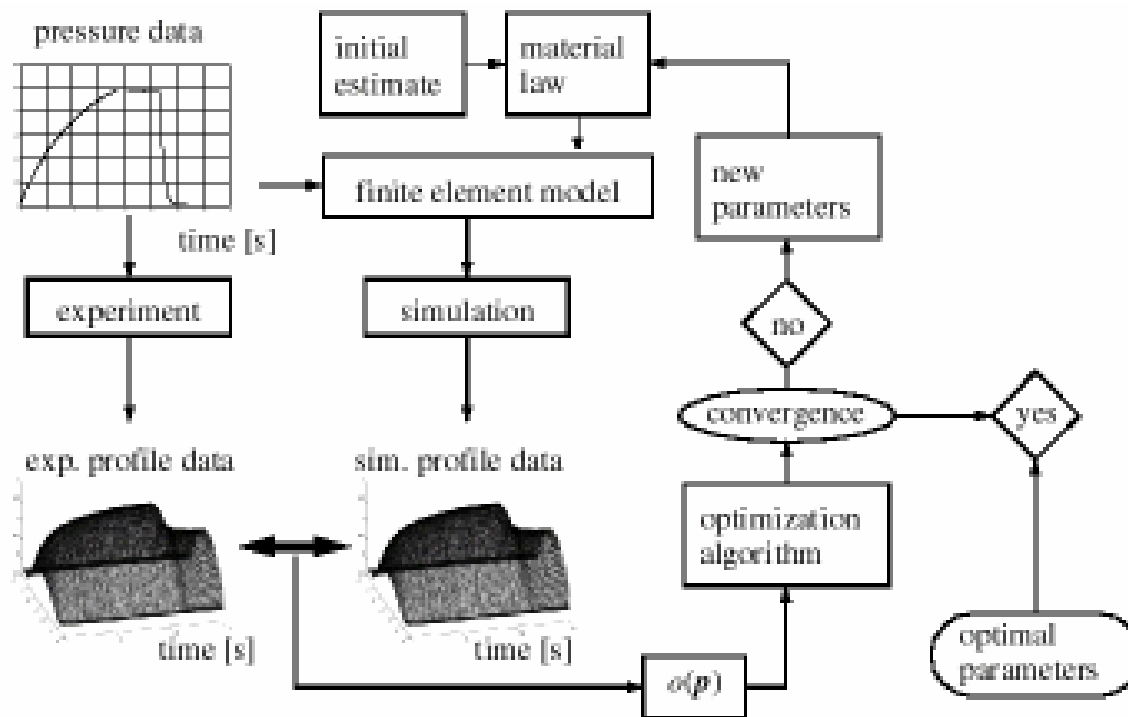
- Inverse Problems



- well-suited for surgery simulation (computational approach)

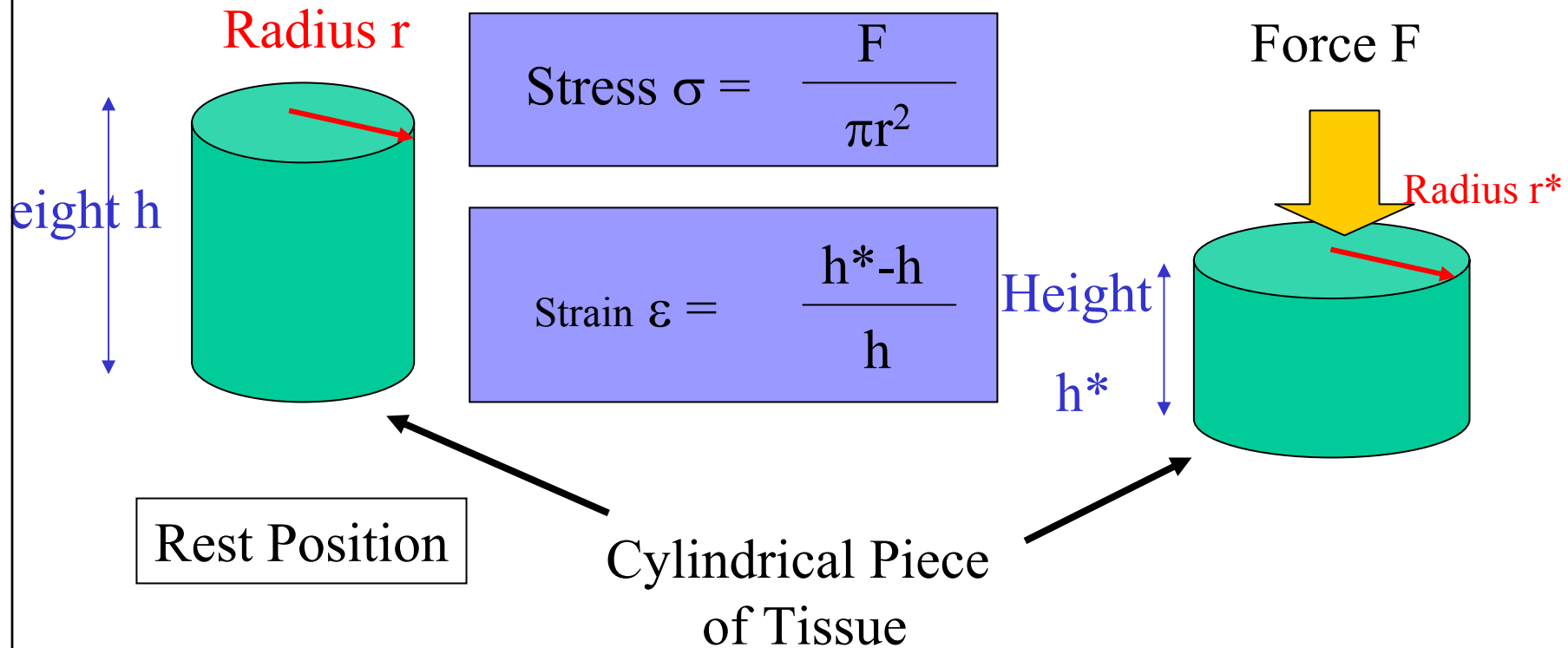


- require the geometry before and after deformation



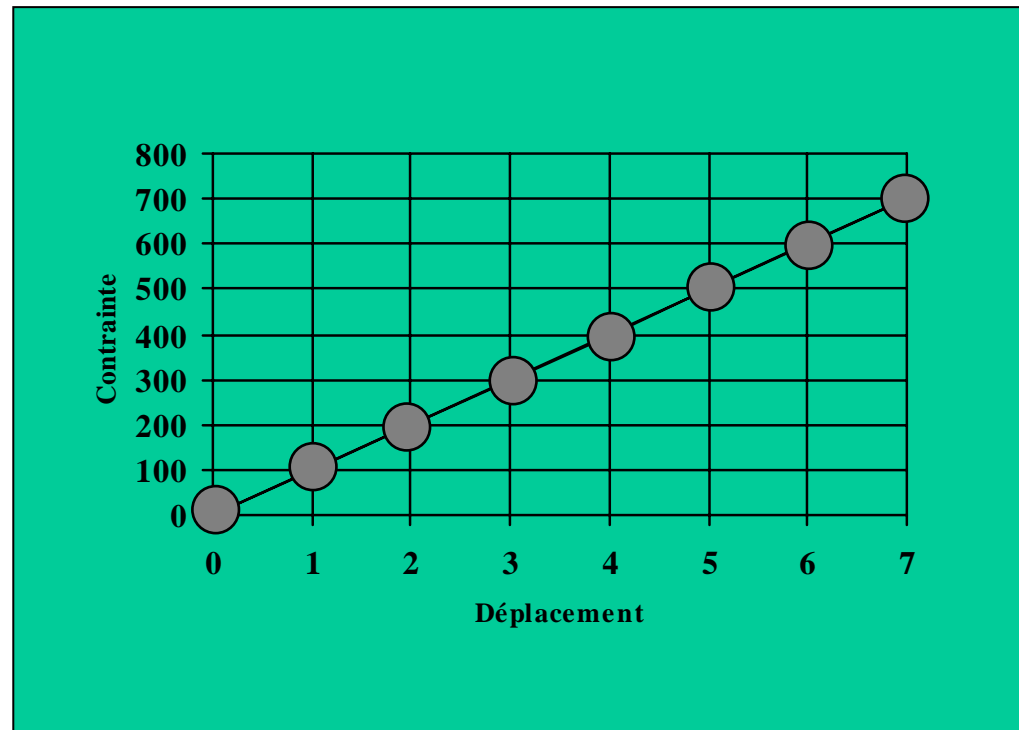
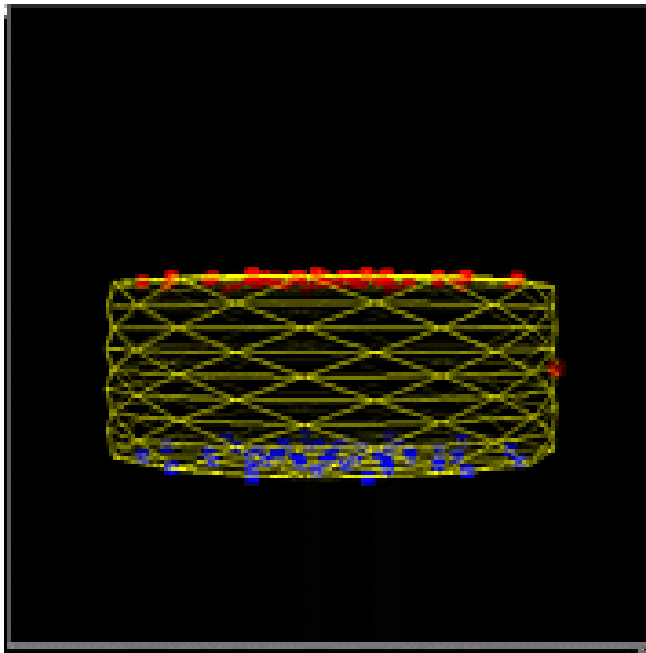
# Soft Tissue Characterization

- To characterize a tissue, its stress-strain relationship is studied



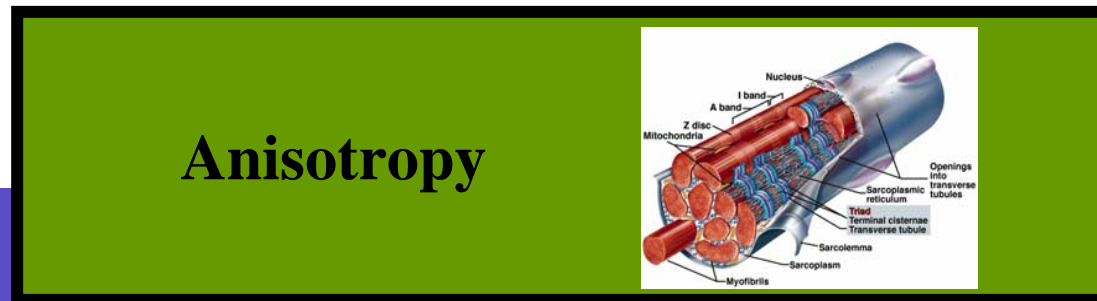
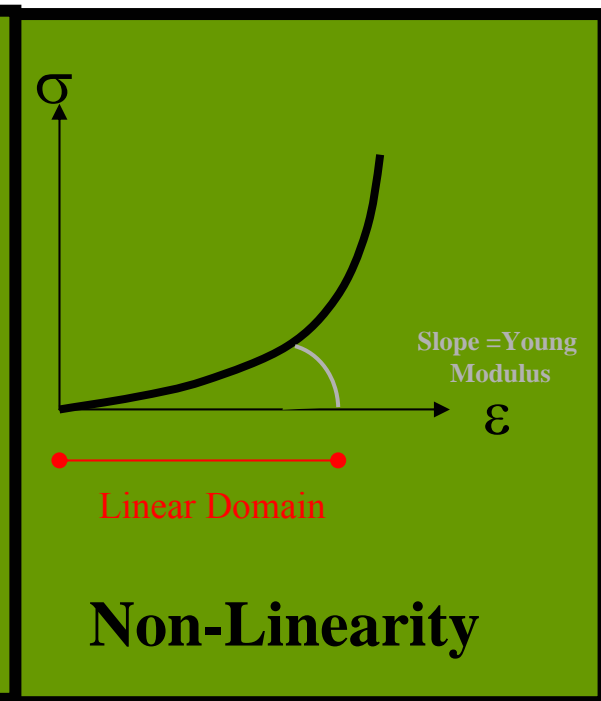
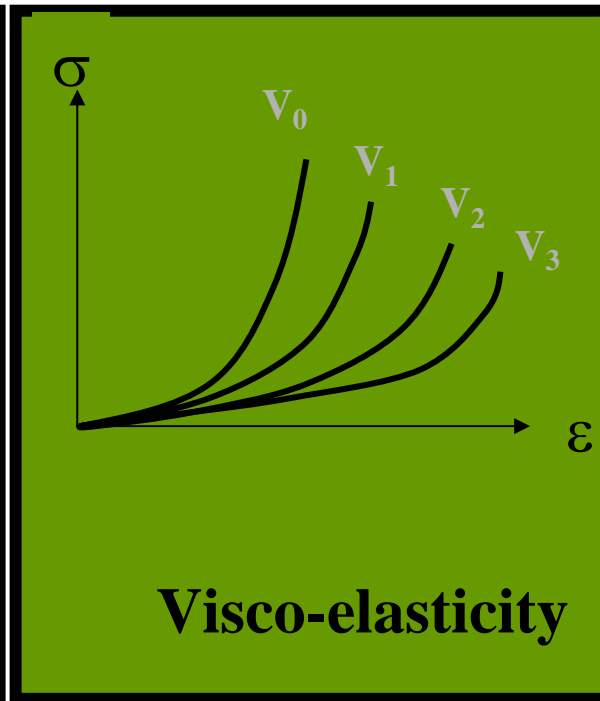
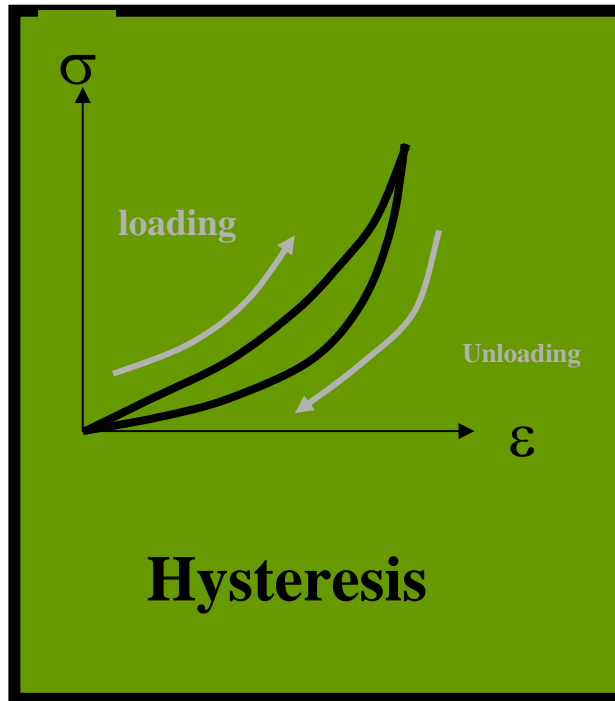
# Linear Elastic Material

- Simplest Material behaviour
- Only valid for small deformations (less than 5%)

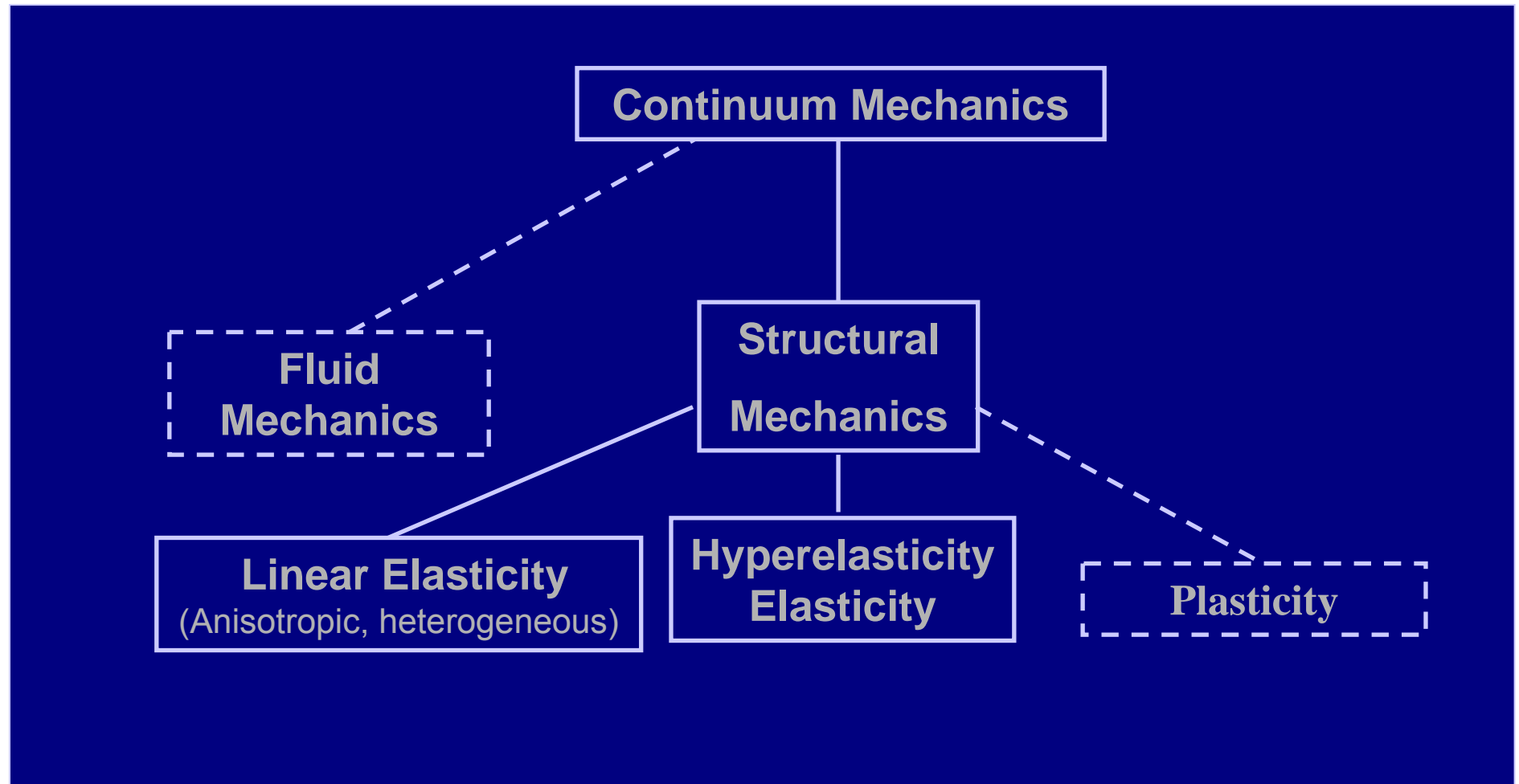


# Biological Tissue

- Far more complex phenomena arises



# Continuum Mechanics



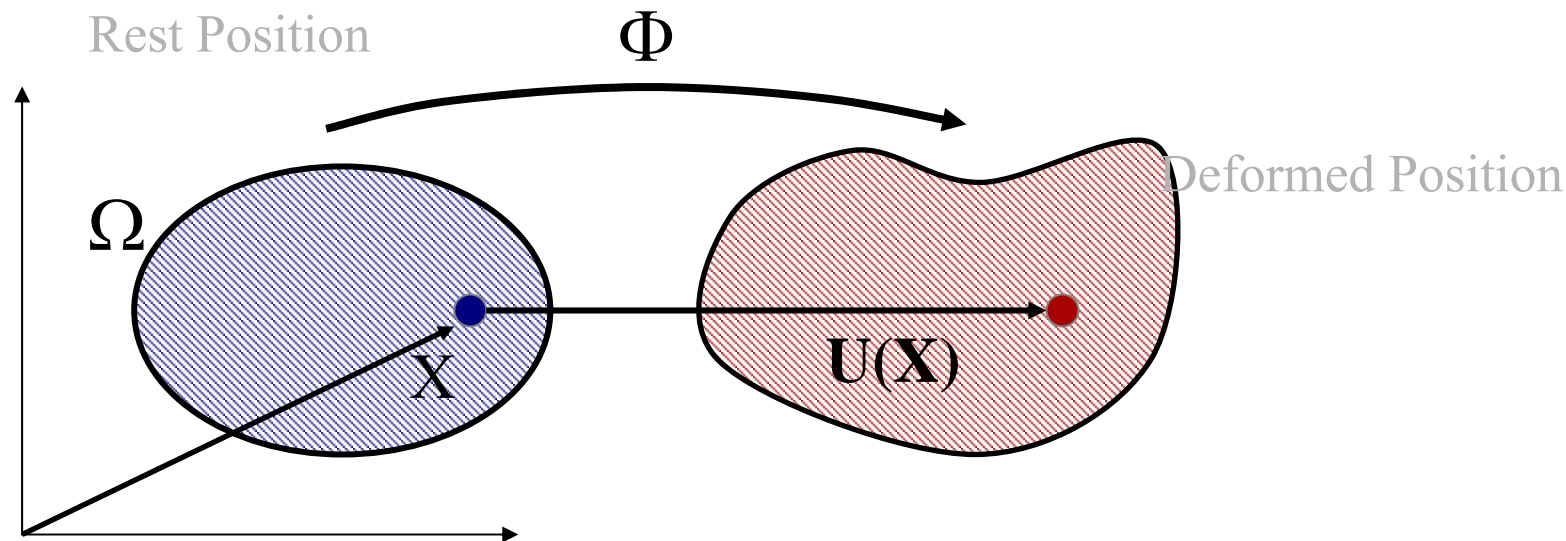
# Basics of Continuum Mechanics

- Deformation Function

$$X \in \Omega \mapsto \phi(X) \in \mathcal{R}^3$$

- Displacement Function

$$U(X) = \phi(X) - X$$

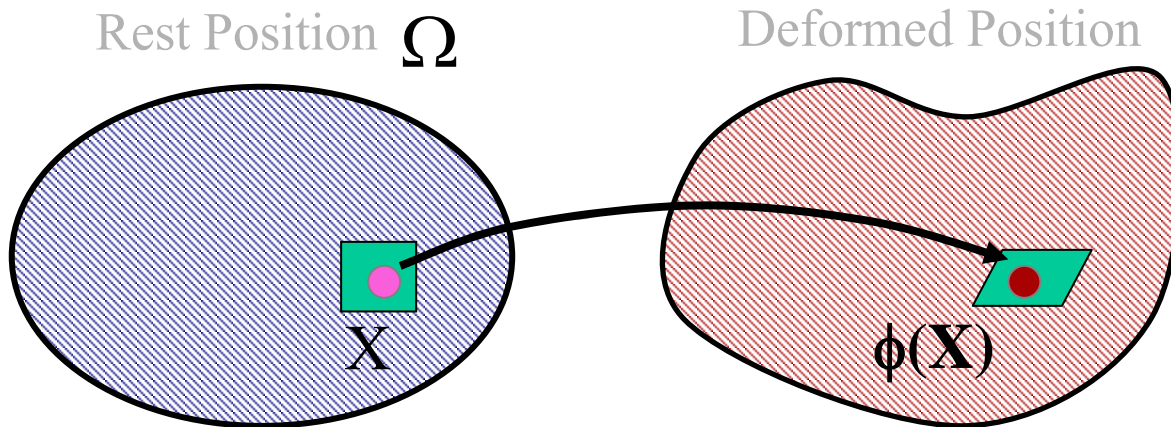


# Basics of Continuum Mechanics

- The local deformation is captured by the deformation gradient :

$$F = \frac{\partial \phi}{\partial X}$$

$$F_{ij} = \frac{\partial \phi_i}{\partial X_j} = \begin{bmatrix} \frac{\partial \phi_1}{\partial X_1} & \frac{\partial \phi_1}{\partial X_2} & \frac{\partial \phi_1}{\partial X_3} \\ \frac{\partial \phi_2}{\partial X_1} & \frac{\partial \phi_2}{\partial X_2} & \frac{\partial \phi_2}{\partial X_3} \\ \frac{\partial \phi_3}{\partial X_1} & \frac{\partial \phi_3}{\partial X_2} & \frac{\partial \phi_3}{\partial X_3} \end{bmatrix}$$

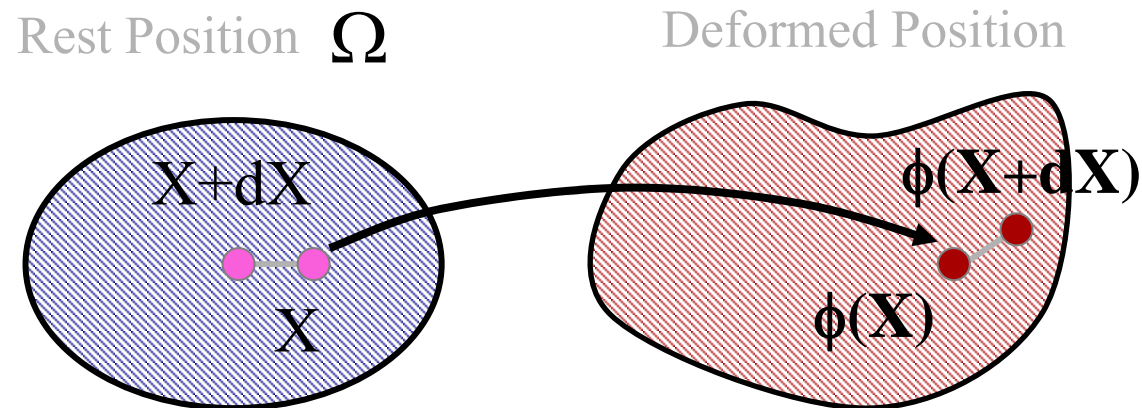


$F(X)$  is the local affine transformation that maps the neighborhood of  $X$  into the neighborhood of  $\phi(X)$



# Basics of Continuum Mechanics

- Distance between point may not be preserved



- Distance between deformed points

$$(ds)^2 = \|\phi(X + dX) - \phi(X)\|^2 \approx dX^T (\nabla \phi^T \nabla \phi) dX$$

- Right Cauchy-Green Deformation tensor

$$C = \nabla \phi^T \nabla \phi$$

Measures the change of metric in the deformed body

# Basics of Continuum Mechanics

- Example : Rigid Body motion entails no deformation

$$\phi(X) = RX + T$$

$$F(X) = \nabla \phi(X) = R$$

$$C = R^T R = Id$$

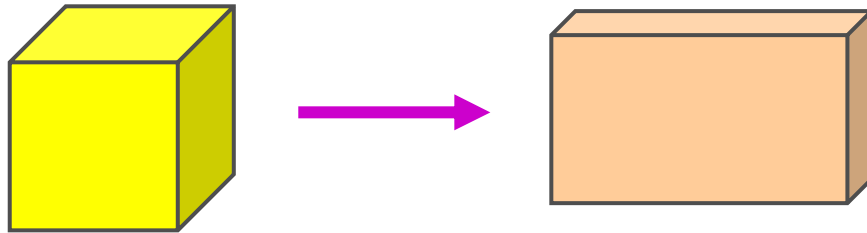
- Strain tensor captures the amount of deformation

- It is defined as the “distance between C and the Identity matrix”

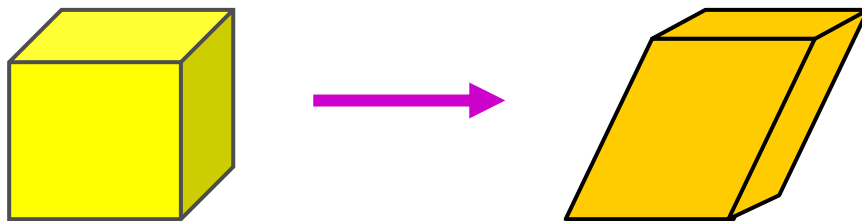
$$E = \frac{1}{2} (\nabla \phi^T \nabla \phi - Id) = \frac{1}{2} (C - Id)$$

# Strain Tensor

- Diagonal Terms :  $\epsilon_i$ 
  - Capture the length variation along the 3 axis



- Off-Diagonal Terms :  $\gamma_i$ 
  - Capture the shear effect along the 3 axis



$$E = \begin{bmatrix} \epsilon_x & \frac{1}{2} \gamma_{xy} & \frac{1}{2} \gamma_{xz} \\ \frac{1}{2} \gamma_{xy} & \epsilon_y & \frac{1}{2} \gamma_{yz} \\ \frac{1}{2} \gamma_{xz} & \frac{1}{2} \gamma_{yz} & \epsilon_z \end{bmatrix}$$

# Linearized Strain Tensor

- Use displacement rather than deformation

$$\nabla \phi(X) = Id + \nabla U(X)$$

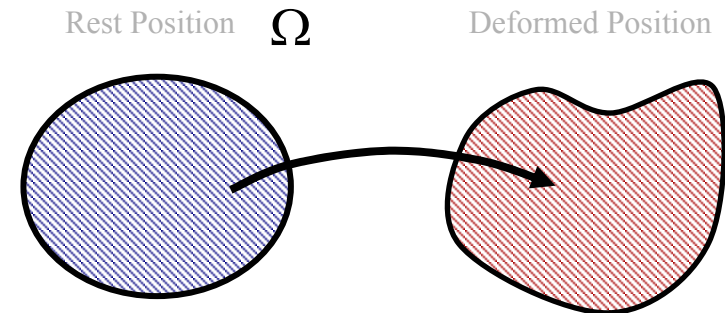
$$E = \frac{1}{2} (\nabla U + \nabla U^T + \nabla U^T \nabla U)$$

- Assume small displacements

$$E_{Lin} = \frac{1}{2} (\nabla U + \nabla U^T)$$

# Hyperelastic Energy

- The energy required to deform a body is a function of the invariants of strain tensor  $E$  :
  - Trace  $E = I_1$
  - Trace  $E^*E = I_2$
  - Determinant of  $E = I_3$



$$W(\phi) = \int_{\Omega} w(I_1, I_2, I_3) dX$$

Total Elastic Energy

# Linear Elasticity

- Isotropic Energy

$(\lambda, \mu)$  : Lamé coefficients

$$w(X) = \frac{\lambda}{2} (\text{tr } E_{Lin})^2 + \mu \text{tr } E_{Lin}^2$$

Hooke's Law

$w(X)$  : density of elastic energy

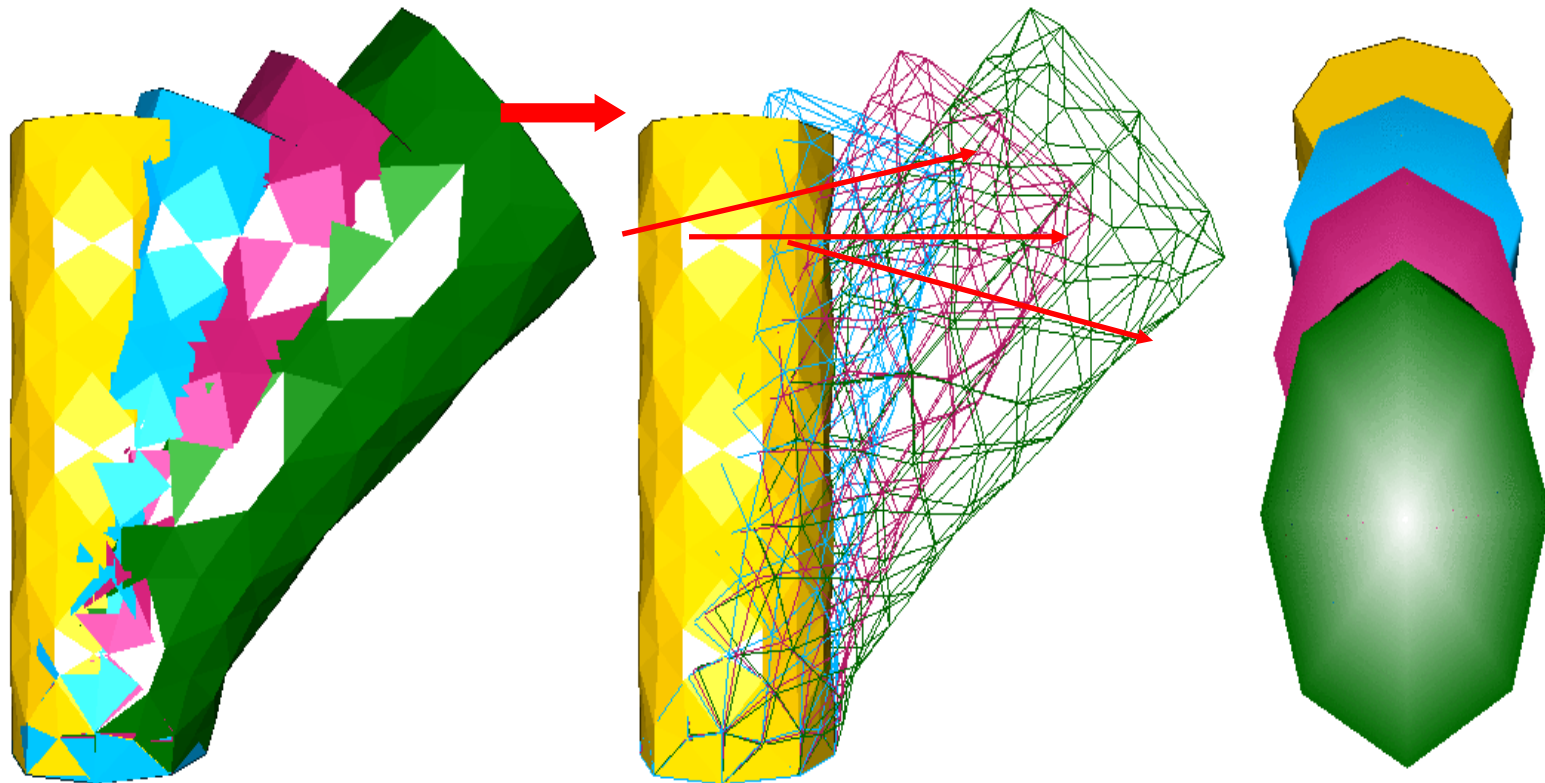
- Advantage :
  - Quadratic function of displacement

$$w = \frac{\lambda}{2} (\text{div } U)^2 + \mu \|\nabla U\|^2 - \frac{\mu}{2} \|\text{rot } U\|^2$$

- Drawback :
  - Not invariant with respect to global rotation
- Extension for anisotropic materials

# Shortcomings of linear elasticity

- Non valid for « large rotations and displacements »



# St-Venant Kirchhoff Elasticity

- Isotropic Energy

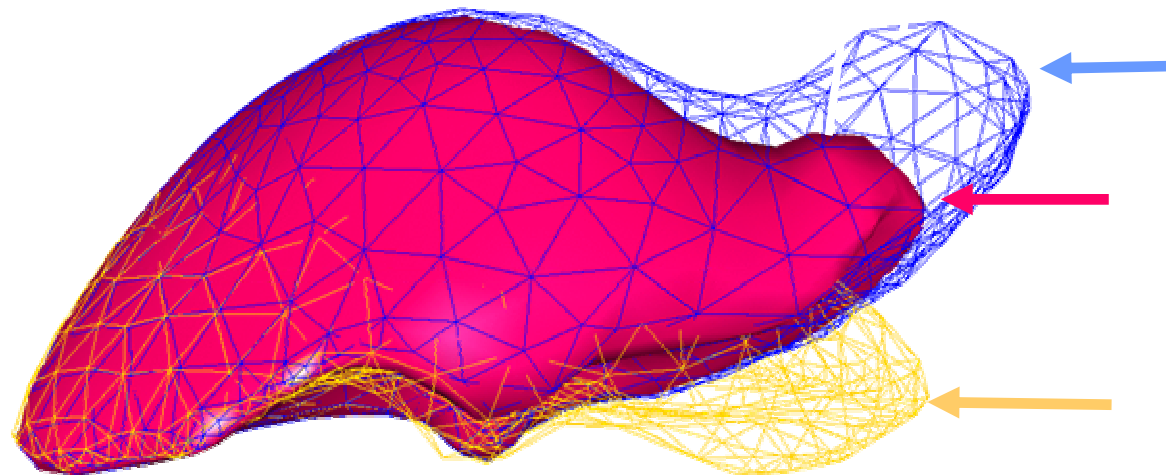
$$w(X) = \frac{\lambda}{2} (\text{tr } E)^2 + \mu \text{tr } E^2$$

$(\lambda, \mu)$  : Lamé coefficients

- Advantage :
  - Generalize linear elasticity
  - Invariant to global rotations
- Drawback :
  - Poor behavior in compression
  - Quartic function of displacement
- Extension for anisotropic materials



# St Venant Kirchhoff vs Linear Elasticity



Linear  
St Venant  
Kirchoff

Rest Position

Linear



St Venant  
Kirchoff

# Other Hyperelastic Material

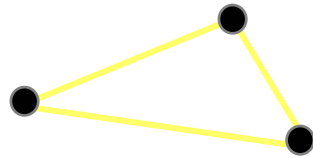
- Neo-Hookean Model  $w(X) = \frac{\mu}{2} \text{tr}E + f(I_3)$
- Fung Isotropic Model  $w(X) = \frac{\mu}{2} e^{\text{tr}E} + f(I_3)$
- Fung Anisotropic Model  $w(X) = \frac{\mu}{2} e^{\text{tr}E} + \frac{k_1}{k_2} (e^{k_2(I_4-1)} - 1) + f(I_3)$
- Veronda-Westman  $w(X) = c_1 (e^{\gamma \text{tr}E}) + c_2 \text{tr}E^2 + f(I_3)$
- Mooney-Rivlin :  $w(X) = c_{10} \text{tr}E + c_{01} \text{tr}E^2 + f(I_3)$

# Discretisation techniques

- Four main approaches :
  - Volumetric Mesh Based
  - Surface Mesh Based
  - Meshless
  - Particles

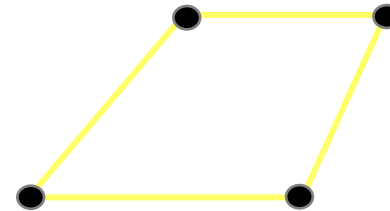
# Different types of meshes

- Surface Elements :



**Triangle**

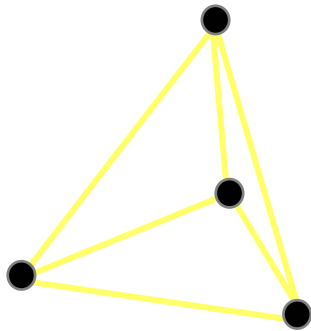
3, 12 nodes and more



**Quad**

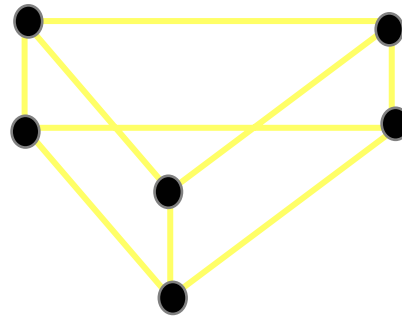
4, 8 nodes and more

- Volume Elements



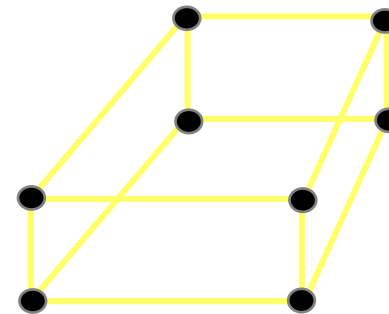
**Tetrahedron**

4, 10 nodes



**Prismatic**

6, 15 nodes and more



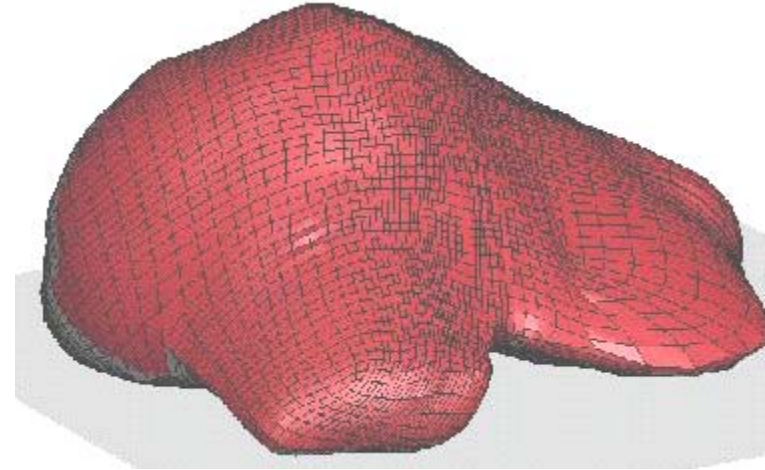
**Hexahedron**

8, 20 nodes and more

# Structured vs Unstructured meshes

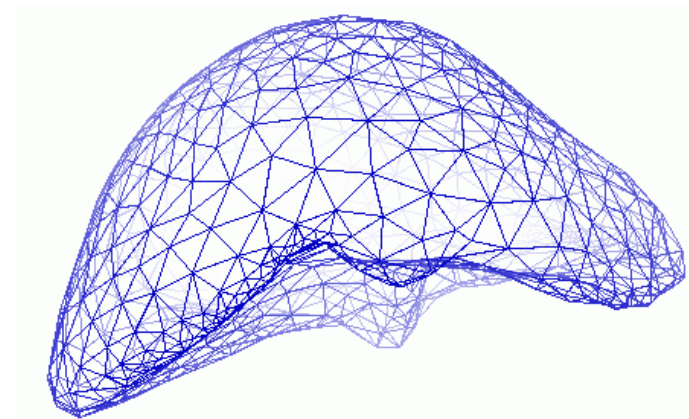
- Example 1 : Liver meshed with hexahedra

3 months work  
(courtesy of ESI)



- Example 2: Liver meshed with tetrahedra

Automatically  
generated (10s)

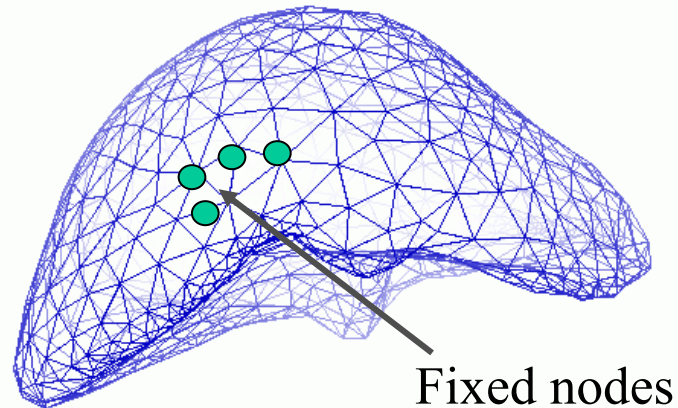
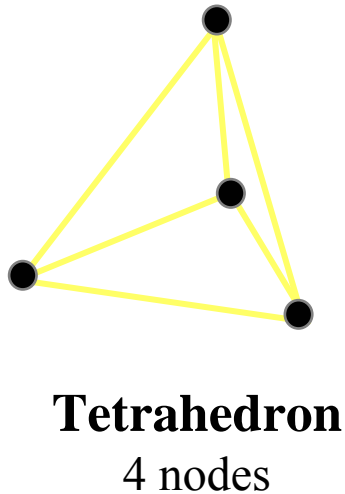


# Volumetric Mesh Discretization

- Classical Approaches :
  - Finite Element Method (weak form)
  - Rayleigh Ritz Method (variational form)
  - Finite Volume Method (conservation eq.)
  - Finite Differences Method (strong form)
- FEM, RRM, FVM are equivalent when using linear elements

# Rayleigh-Ritz Method

- Step1 : Choose
  - Finite Element (e.g. linear tetrahedron)
  - Mesh discretizing the domain of computation
  - Hyperelastic Material with its parameters
  - Boundary Conditions



$$w(X) = \frac{\lambda}{2} (tr E)^2 + \mu tr E^2$$

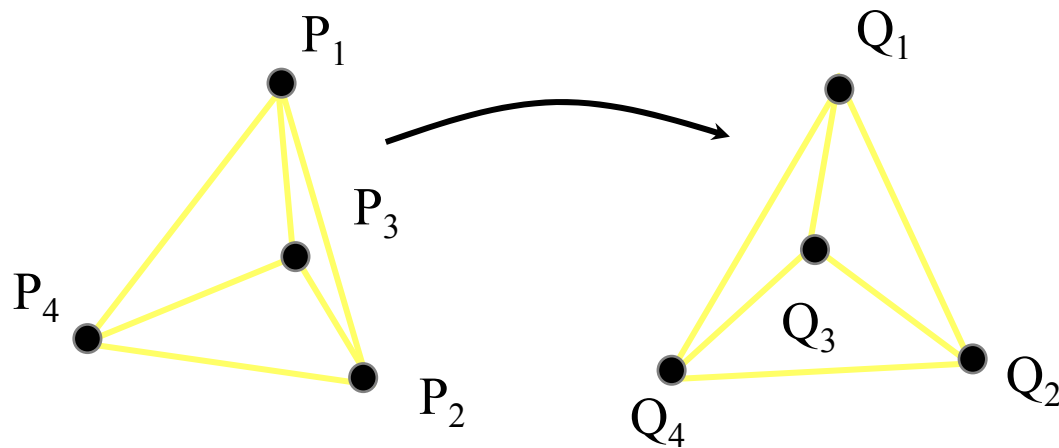
Young Modulus

Poisson Coefficient

# Rayleigh-Ritz Method

- Step2

- Write the elastic energy required to deform a single element



$$u(P_i) = Q_i - P_i = U_i$$

$$u(X) = \sum_{i=1}^4 \lambda_i(X) u(P_i)$$

$$\nabla \lambda_i(X) = -\frac{M_i}{6V(T)}$$

$$trE = -\sum_i \frac{M_i \cdot U_i}{6V(T)}$$

$$W_{T_i} = \sum_{jk} U_j^t [\mathbf{K}_{jk}^{T_i}] U_k$$

$$[\mathbf{K}_{jk}^{T_i}] = \frac{1}{36 \cdot V(T_i)} (\lambda_i \mathbf{M}_k \mathbf{M}_j^T + \mu_i \mathbf{M}_j \mathbf{M}_k^T + \mu_i (\mathbf{M}_j \cdot \mathbf{M}_k) [\mathbf{Id}_{3 \times 3}])$$



# Rayleigh-Ritz Method

- Step3
  - Sum to get the total elastic energy

$$W(U) = \int_{\Omega_h} w(I_1, I_2, I_3) dX = \sum_{T_i} W_{T_i} = U^T K U$$

- Write the conservation of energy

$$W(U) = \underbrace{F^T U}_{\text{Internal Energy}} + \underbrace{\int_{\Omega} \rho(X) (X \cdot g) dX}_{\text{Gravity Potential Energy}}$$

Nodal Forces

# Rayleigh-Ritz Method

- Step3
  - Write first variation of the energy :

## Linear Elasticity

$$KU = R$$

Static case

$$M\ddot{U} + C\dot{U} + KU = R(t)$$

Dynamic case

## HyperElasticity=NonLinear Elasticity

$$K(U) = R$$

Static case

$$M\ddot{U} + C\dot{U} + K(U) = R(t)$$

Dynamic case

# Surface-Based Methods

- Only consider the mesh surface under some hypothesis :
  - Linear Elastic Material (sometimes homogeneous)
  - Only interact with organ surface
- Pros :
  - No need to produce volumetric meshes
  - Much faster than volumetric computation
- Cons :
  - Only linear material
  - No cutting

# Evolution

- Dynamic evolution

- Discrete models = lumped mass particles submitted to forces
- Newtonian evolution (1<sup>st</sup> order differential system):

$$\begin{cases} \delta P = V \cdot dt \\ \delta V = M^{-1} F(P, V) \cdot dt \end{cases}$$

- Explicit schemes:

- Euler: 
$$\begin{cases} \delta P = V_t \cdot dt \\ \delta V = M^{-1} F(P_t, V_t) \cdot dt \end{cases}$$

- Runge-Kutta: several evaluations to better extrapolate the new state [press92]  
→ Unstable for large time-step !!

- Semi-Implicit schemes:

- Euler: 
$$\begin{cases} \delta P = V_{t+dt} \cdot dt \\ \delta V = M^{-1} F(P_t, V_t) \cdot dt \end{cases} \rightarrow \begin{cases} P_{t+dt} = 2P_t - P_{t-dt} + M^{-1} F(P_t, V_t) \cdot dt^2 \\ V_{t+dt} = (P_{t+dt} - P_t) dt^{-1} \end{cases}$$

# Evolution

– Implicit schemes [terzopoulos87], [baraff98], [desbrun99], [volino01], [hauth01]

- First-order expansion of the force:

$$F(\mathbf{P}_{t+dt}, \mathbf{V}_{t+dt}) \approx F(\mathbf{P}_t, \mathbf{V}_t) + \partial F / \partial \mathbf{P} \delta \mathbf{P} + \partial F / \partial \mathbf{V} \delta \mathbf{V}$$

- Euler implicit

$$\rightarrow \left\{ \begin{array}{l} \delta \mathbf{P} = \mathbf{V}_{t+dt} \cdot dt \\ \delta \mathbf{V} = \mathbf{H}^{-1} \mathbf{Y} \end{array} \right. \quad \text{with} \quad \begin{array}{l} \mathbf{H} = \mathbf{I} - \mathbf{M}^{-1} \partial F / \partial \mathbf{V} dt - \mathbf{M}^{-1} \partial F / \partial \mathbf{P} dt^2 \\ \mathbf{Y} = \mathbf{M}^{-1} F(\mathbf{P}_t, \mathbf{V}_t) + \mathbf{M}^{-1} \partial F / \partial \mathbf{P} \mathbf{V}_t dt^2 \end{array}$$

- Backward differential formulas (BDF) : Use of previous states

→ Unconditionally stable for any time-step

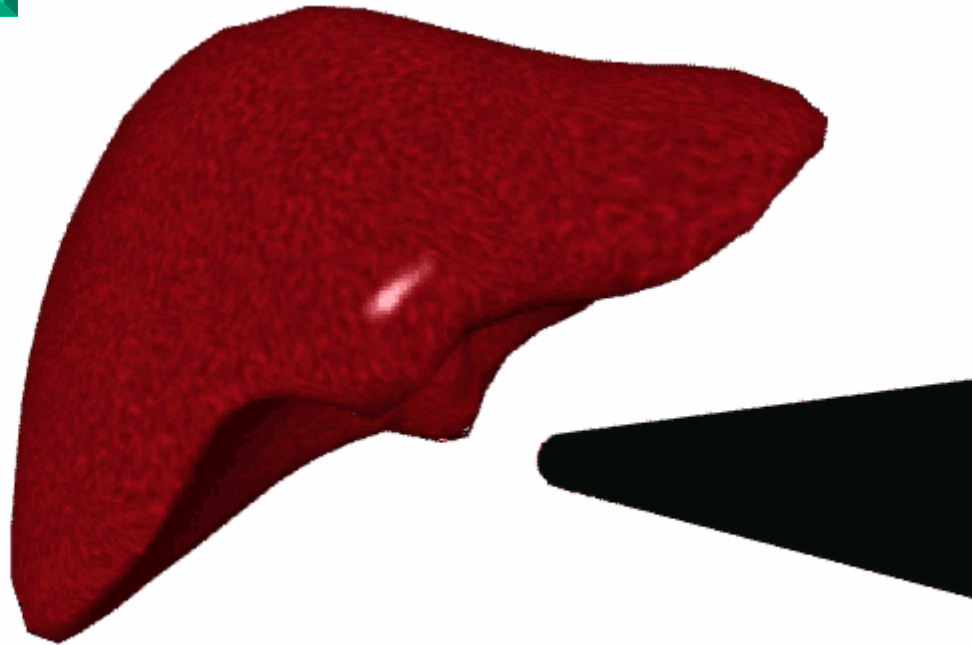
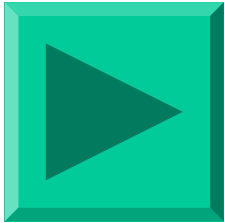
... But requires the inversion of a large sparse system

- Choleski decomposition + relaxation
- Conjugate gradient
- Speed and accuracy can be improve through preconditioning (alteration of  $\mathbf{H}$ )

# Example of Soft Tissue Models

	Pre-computed Elastic Model	Tensor-Mass and Relaxation-based Model	Non-Linear Tensor-Mass Model
Computational Efficiency	+++	+	-
Cutting Simulation	-	++	++
Large Displacements	-	-	+

# Precomputed linear elastic model



9517  
Tetrahedra

AISIM 1999  
Epidaure IMAGIS Sinus

# Different Technical Issues

- Mesh Reconstruction from Images
- Soft Tissue Modeling
- Tissue Cutting
- Collision Detection
- Contact Modeling
- Surface Rendering
- Haptic Feedback



# Different algorithms for cutting tetrahedral meshes

- Split of tetrahedra

*[Bielser, 2000] [Mohr, 2000]*

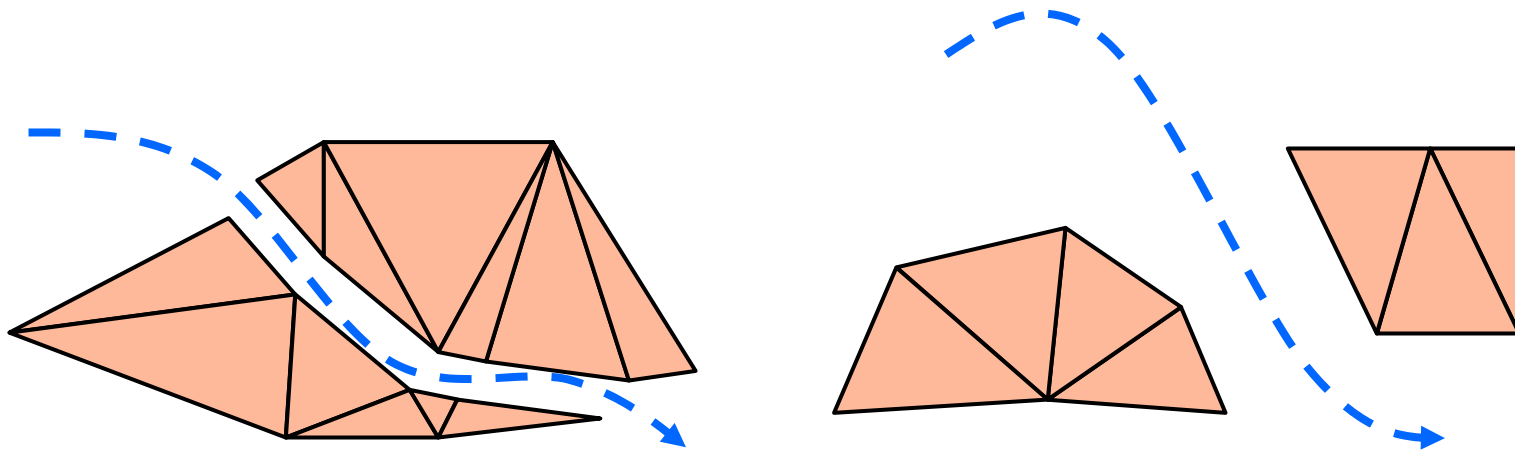
*[Nienhuys, 2001]*

- + Accurate, realistic
- - Decrease of Mesh Quality

- Removing Tetrahedra

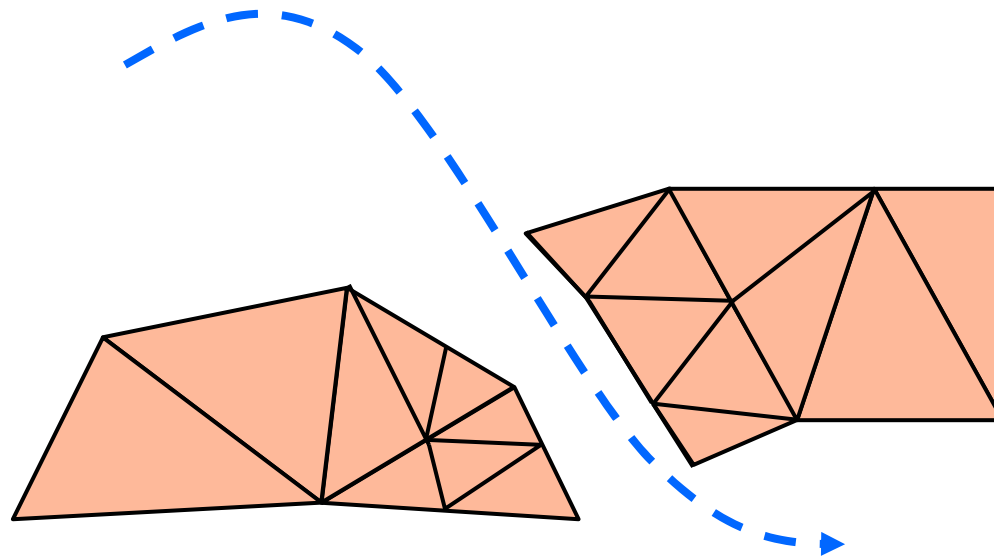
*[Forest, 2002]*

- + Keeps a good mesh quality
- - Gross cut

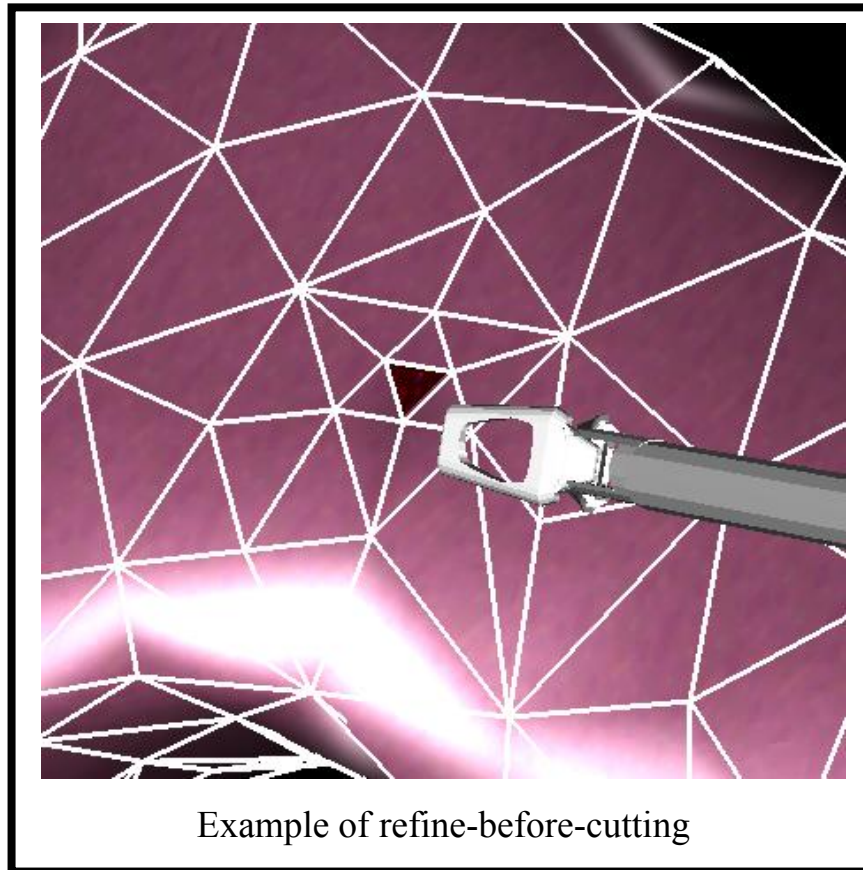


# Proposed Technique

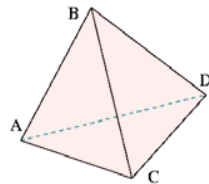
- Remove Tetrahedra
- Refine Mesh before removing material



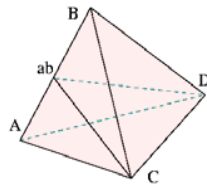
# Dynamic Refinement



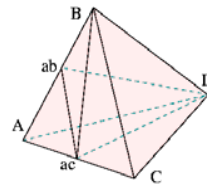
# Refinement by Edge Split



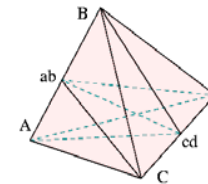
ABCD



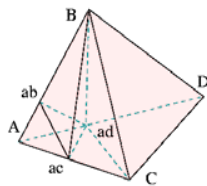
AabCD, abBCD



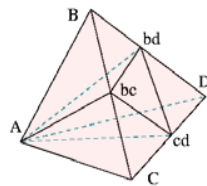
en supposant  $B < C$   
AabacD, abBacD, acBCD



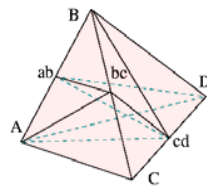
AabCcd, AacdD, abBCcd, abBcdD



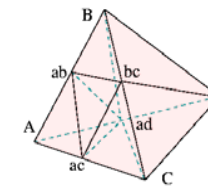
en supposant  $B < C < D$   
Aabacad, abBacad, acBCad, adBDC



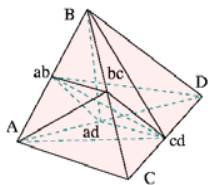
ABbcdbd, AbcCcd,  
Abccdbd, AbdcdD



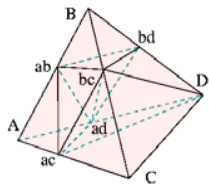
AabcdD  
si  $A < C$  Aabbccd, AbcCcd  
sinon abbcCcd, AabCcd  
si  $B < D$  abBbccd, abBedD  
sinon abbccdD, abBbcD



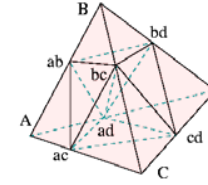
Aabacad, abbcacad  
si  $C < D$  abbcacD, acbcCD  
sinon acbcCad, adbcCD  
si  $B < C$  abBbcad, adBbcD  
sinon adabbcD, abBbcD



si  $A < C$  Aabedad, Aabbccd, AbcCcd  
sinon abbcCcd, adabCcd, AabCad  
si  $B < D$  abBedad, abBbccd, cdBbcD  
sinon abBbcD, abbccdD, adabcdD



en supposant  $C < D$   
Aabacad, abBbcdbd, acabbcad, adabbcdbd  
acbcCad, adbcCbd, abbdcdD

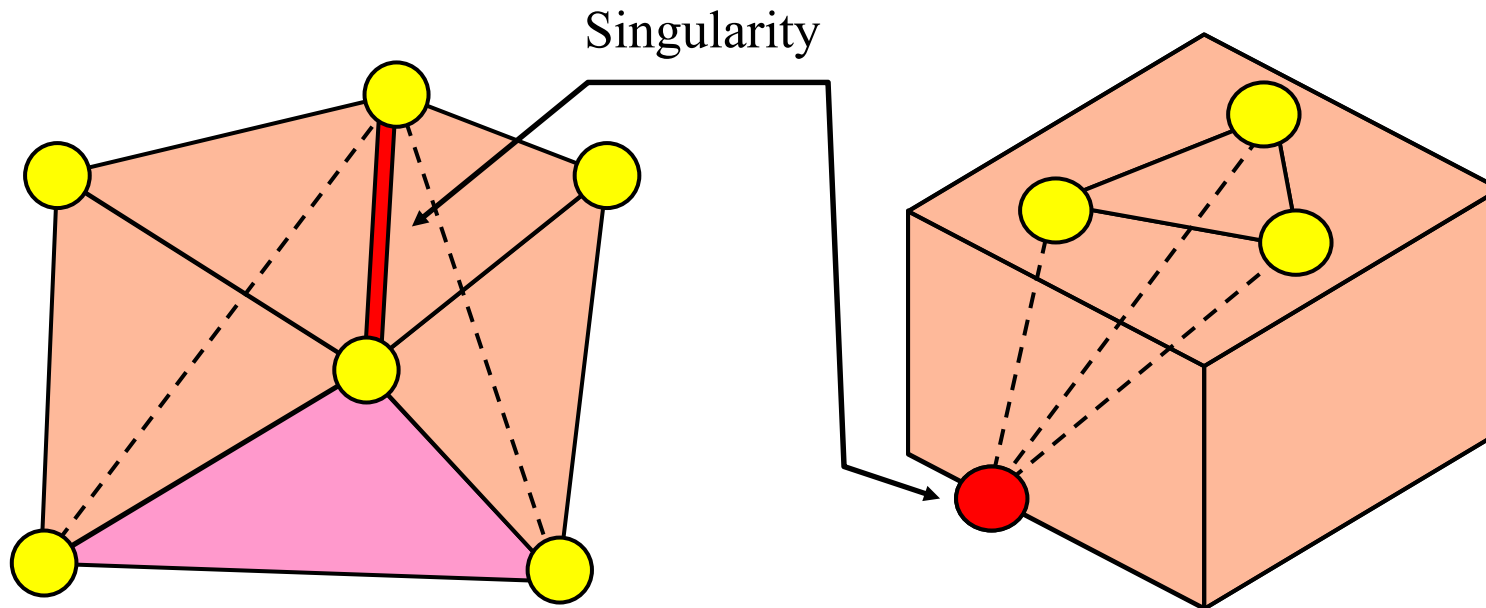


Aabacad, abBbcdbd, acbcCcd, abbdcdD  
abacadbc, cdacbcad, abbdbcad, abdbbccd

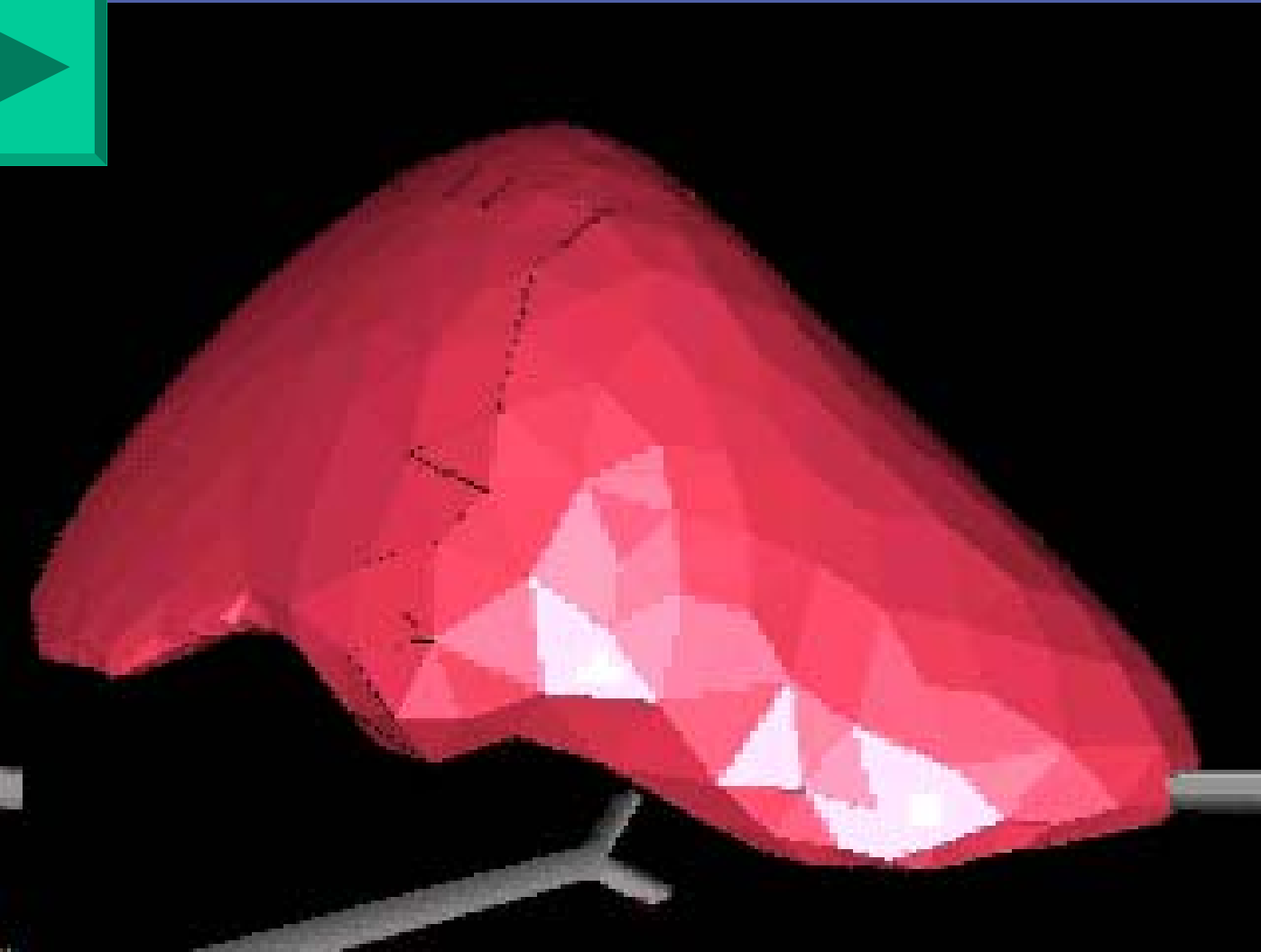
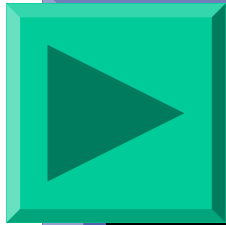
Decomposition of tetrahedra by edge split

# Topological Singularities

- Removing a tetrahedron may create a singularity (zero thickness at edge and vertices) (see [forest])



# Non-linear Tensor-Mass Models



# Different Technical Issues

- Mesh Reconstruction from Images
- Soft Tissue Modeling
- Tissue Cutting
- Collision Detection
- Contact Modeling
- Surface Rendering
- Haptic Feedback

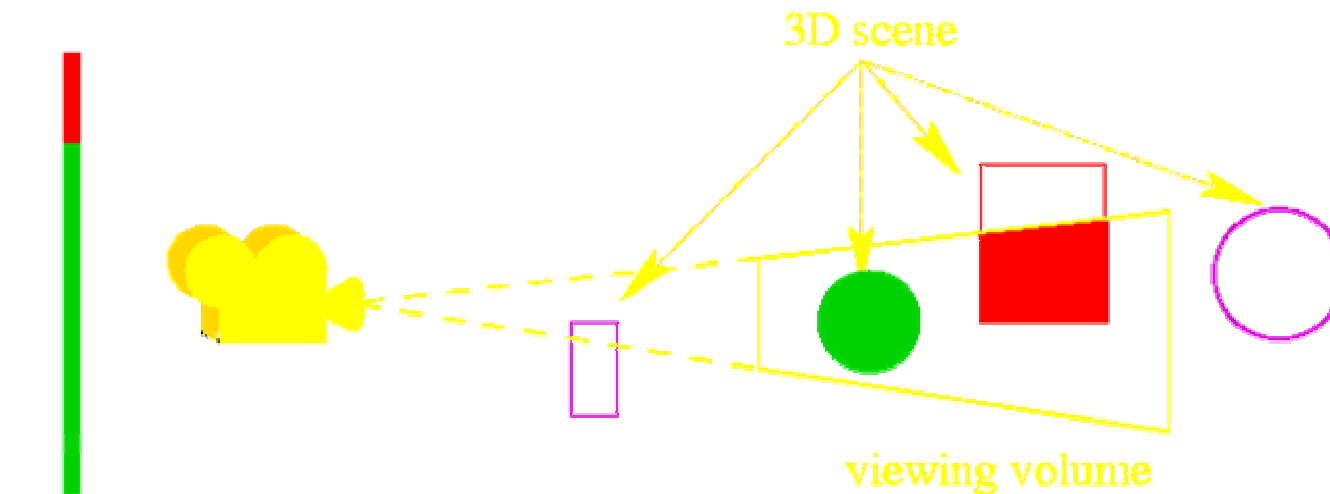
# Previous Work

- A lot of research on Collision Detection
- Hierarchy of oriented bounding boxes:  
Gottshalk & al. - *Obb-tree: A hierarchical structure for rapid interference detection* - SIGGRAPH'96
- public domain package *RAPID*
- Very efficient,  
but needs pre-computation



# The Rendering Process

- Camera = viewing volume + projection



- Two steps: geometry & rasterization

# Collision Detection and Rendering analogy

a tool collides the organ



a part of the organ is inside the tool



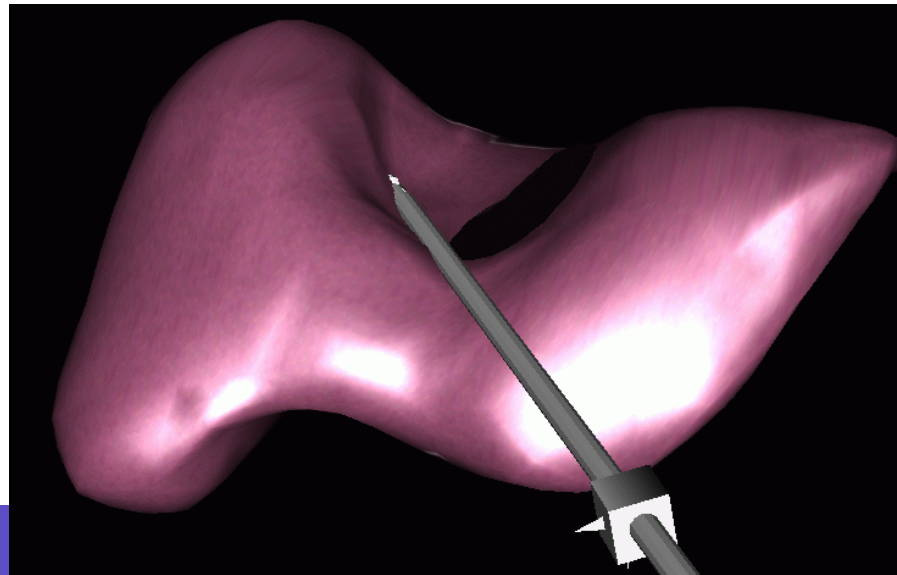
if we define a camera with a viewing volume that matches the tool geometry, the organ will be in the picture.

# Different Technical Issues

- Mesh Reconstruction from Images
- Soft Tissue Modeling
- Tissue Cutting
- Collision Detection
- Contact Modeling
- Surface Rendering
- Haptic Feedback

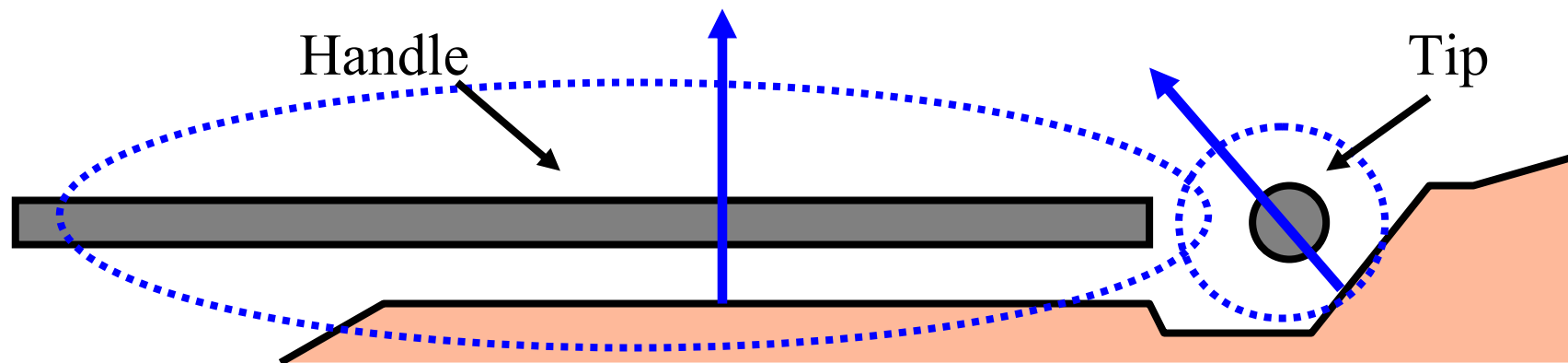
# Tool-Soft Tissue Interaction

- Prevent penetration of tool inside the soft tissue
  - Detect intersections
  - Push explicitly mesh vertices outside the tool



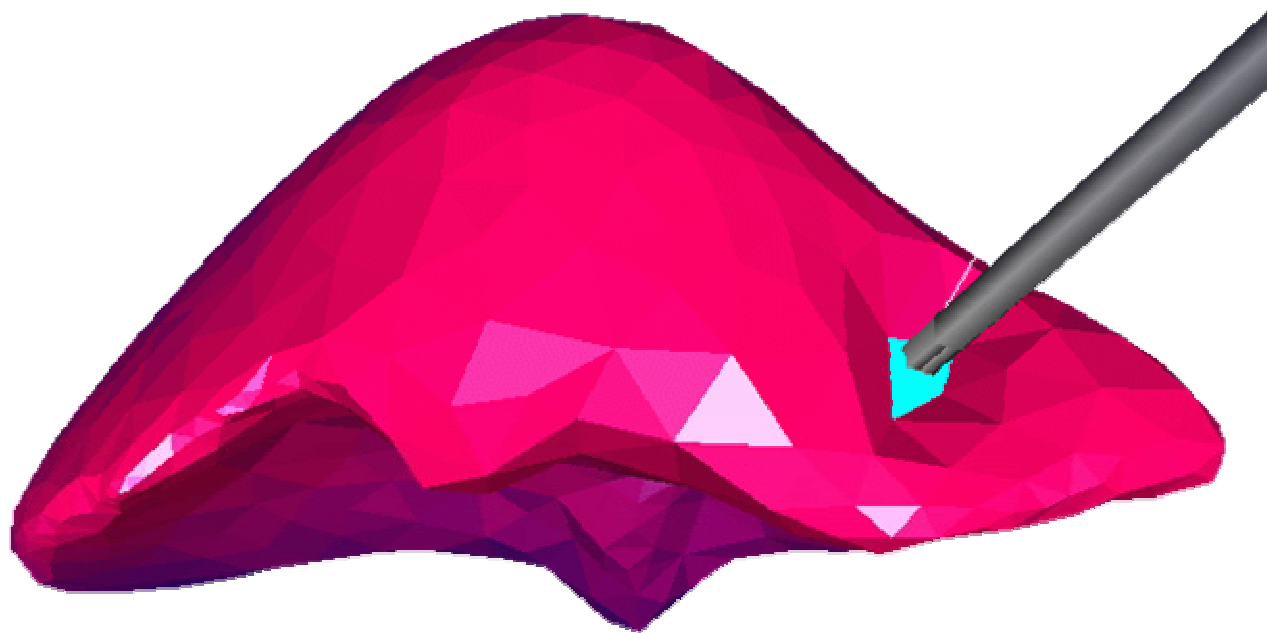
# First Approach [*Picinbono, 2001*]

- 2 different tools : tip and handle
- Compute average normal in the neighborhood of the contact
- Projection of vertices in this plane



# Collision Processing

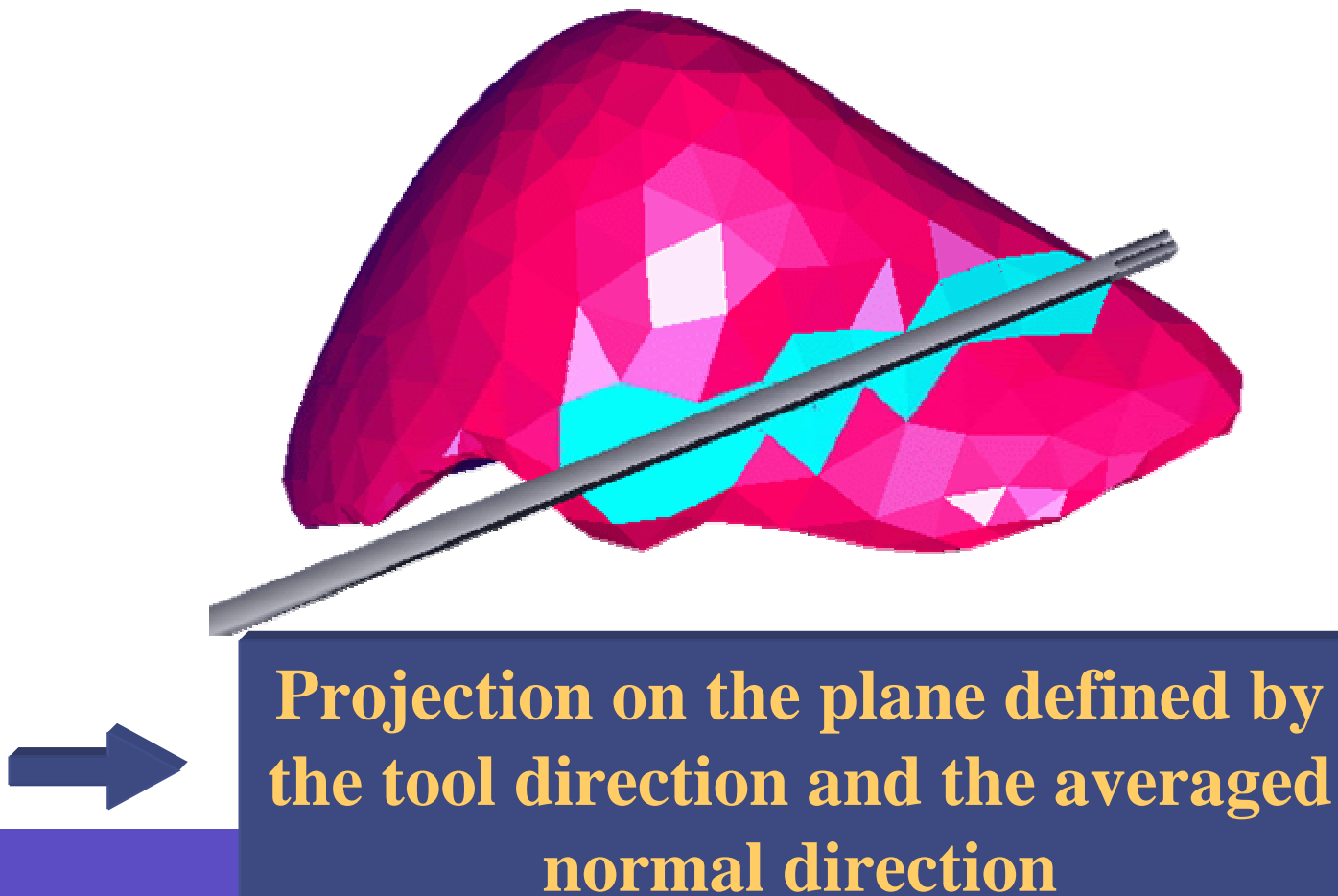
- Contact with the tip of the instrument



**Projection on the plane defined by the tip of the instrument and the average normal of intersected triangles**

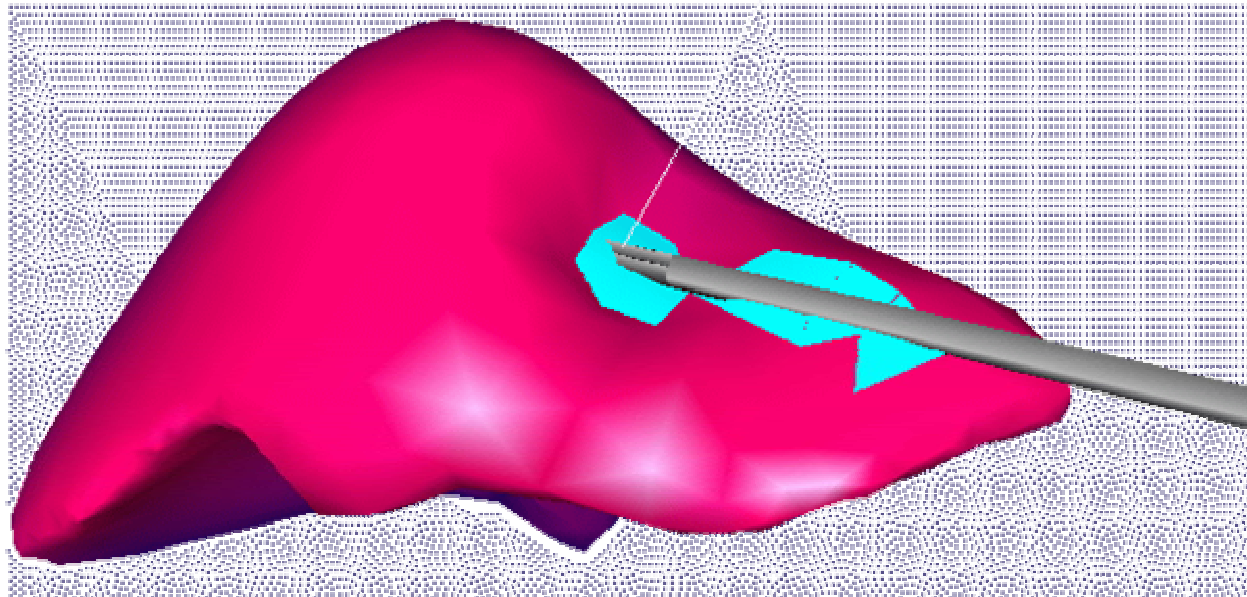
# Collision Processing

- Contact with the handle of the tool



# Collision Processing

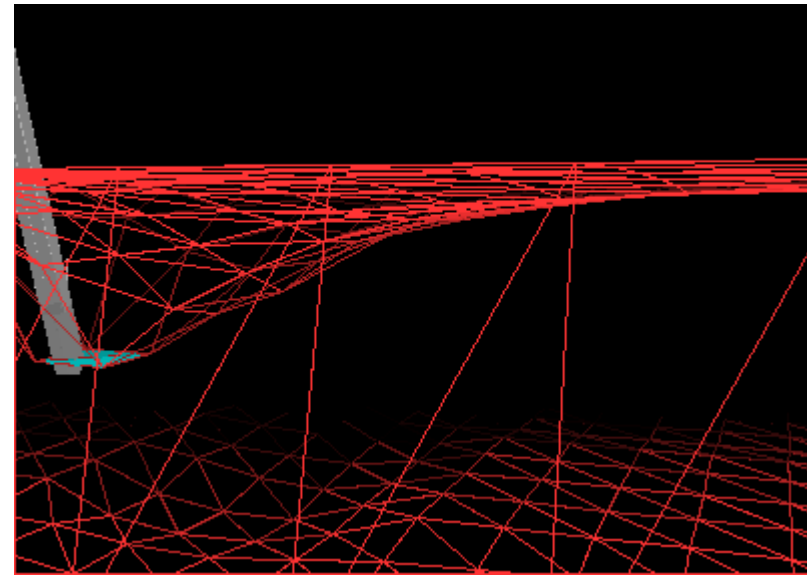
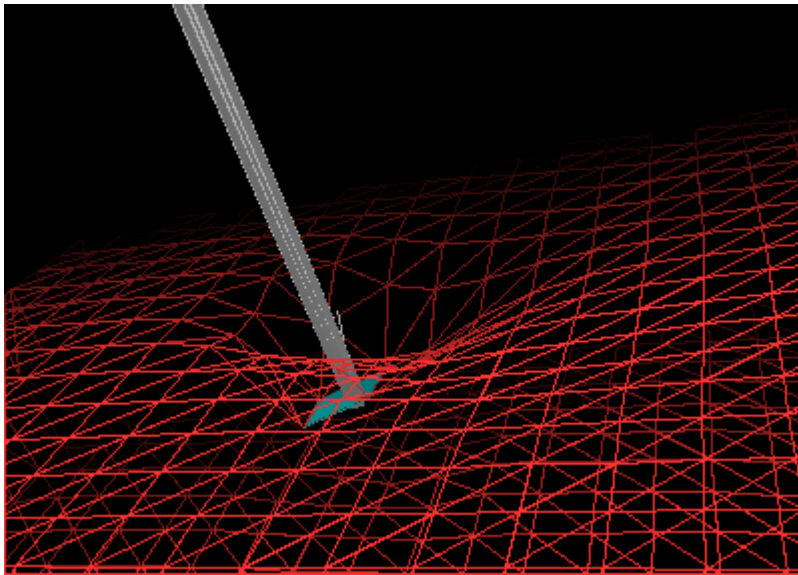
- Perform 2 detections simultaneously





# Possible interactions

- Slip on the surface

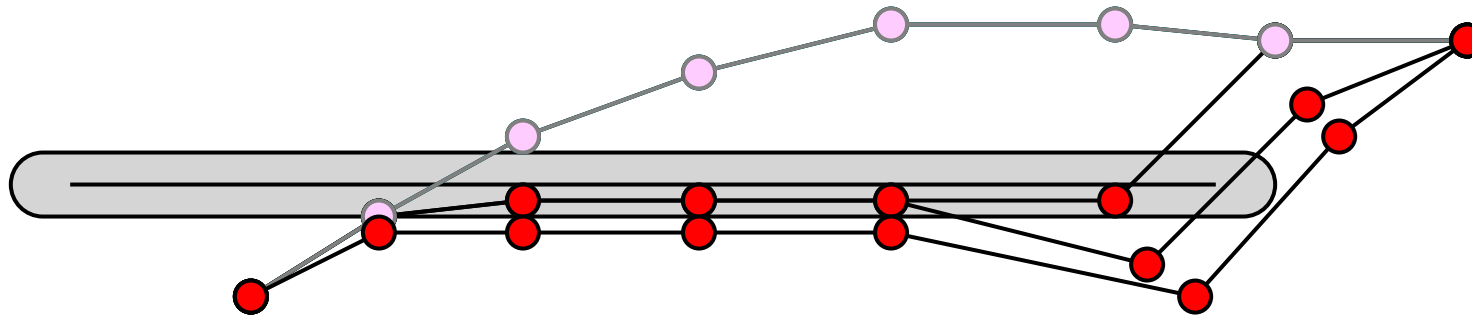


# Limitations of this approach

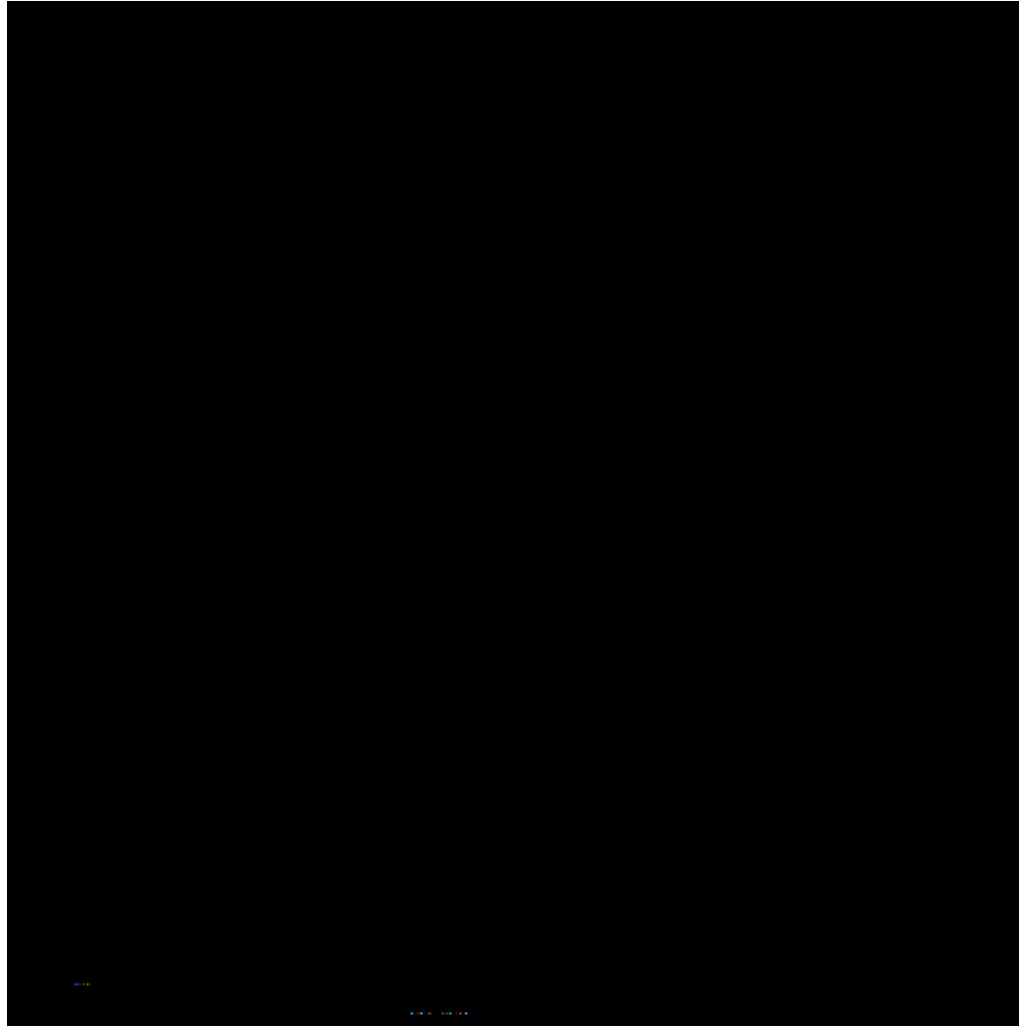
- Same normal vector for all triangles in the same neighborhood
- Leads to instabilities when handling a complex geometry

# New approach

- Three steps
  - Prevent vertices to collide with the tool axis
  - Move vertices near the tip of the tool
  - Move vertices outside the volume of the tool



# Example

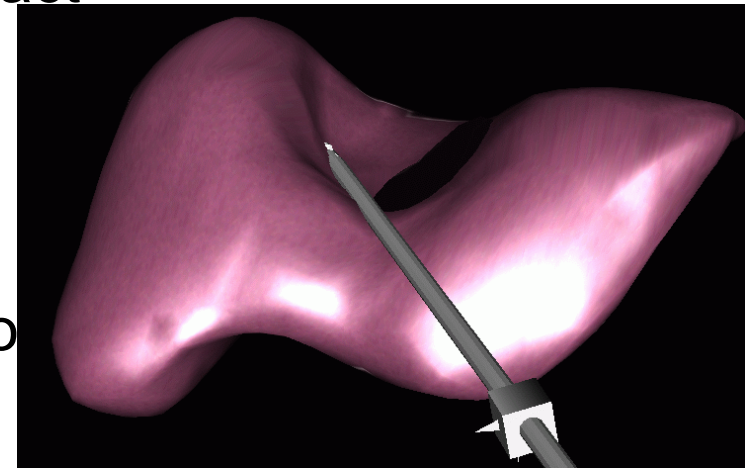


# Different Technical Issues

- Mesh Reconstruction from Images
- Soft Tissue Modeling
- Tissue Cutting
- Collision Detection
- Contact Modeling
- Surface Rendering
- Haptic Feedback

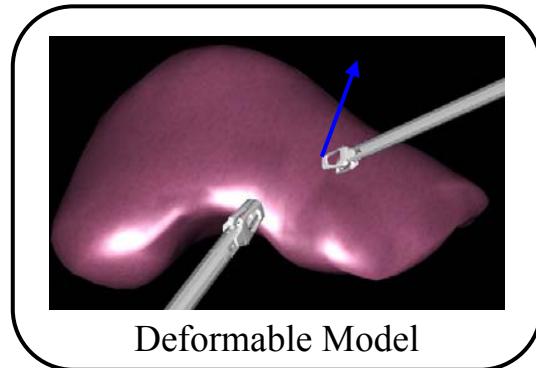
# Haptic Feedback

- Principle
  - Give a realistic sense of contact with the soft tissue
- Motivation
  - Increase realism
  - Naturally limit the amplitude of hand motion
- Pitfalls
  - Frequency update of haptics  $> 500$  Hz
  - Frequency update of deformable models  $\approx 30$  Hz



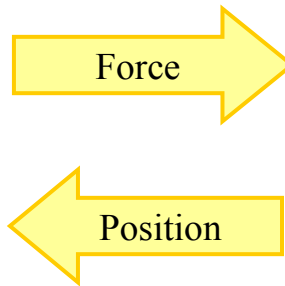
Mouvement non contraint par le retour d'effort

# First approach [Picinbono, 2001]



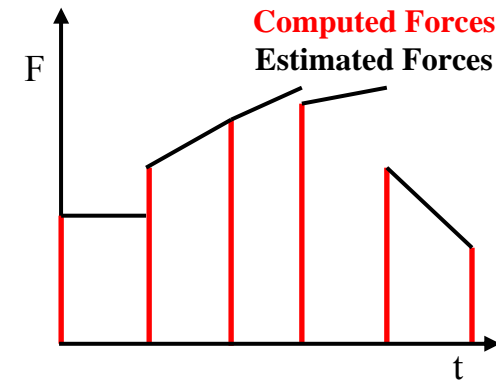
Deformable Model

Frequency Update 20 Hz  
Force Computation



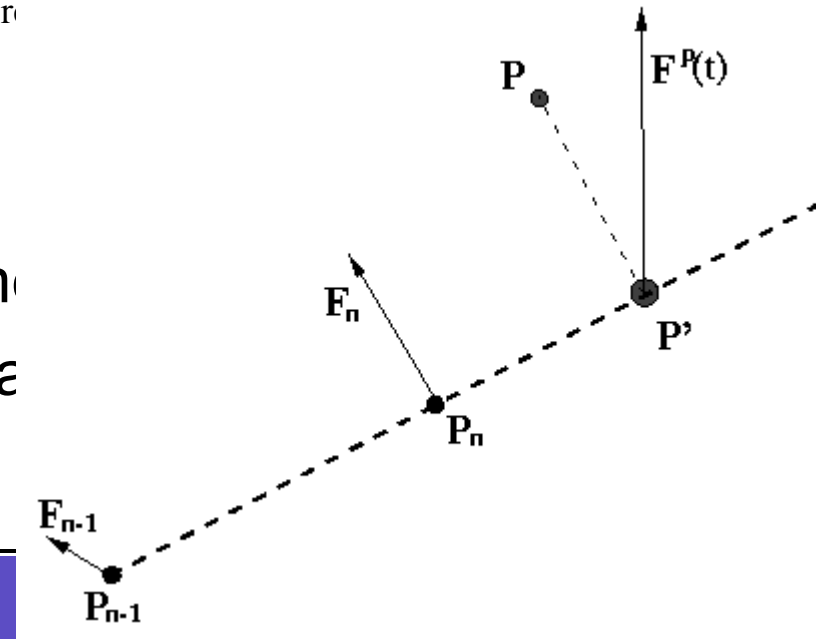
Force Feedback

Frequency Update 200 Hz  
Force Feedback



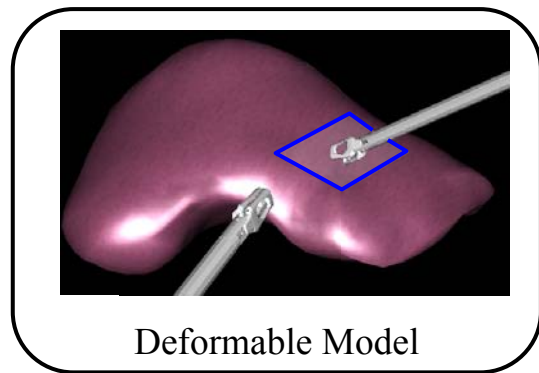
- Unstable if complex geom.
- Difficult extrapolation for large

$$F^P(t) = F_n + \frac{\|P' - P_n\|}{\|P_n - P_{n-1}\|} (F_n - F_{n-1})$$

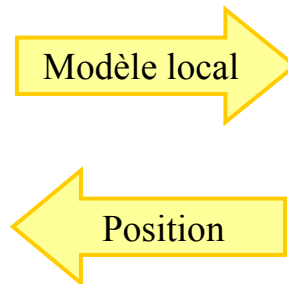


# Local Model

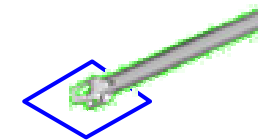
*[Mendoza, 2001] [Balaniuk, 1999] [Mark, 1996]*



Update Frequency 20 Hz  
Computation of a local model



Update Frequency 300 Hz Force Computation from a local model

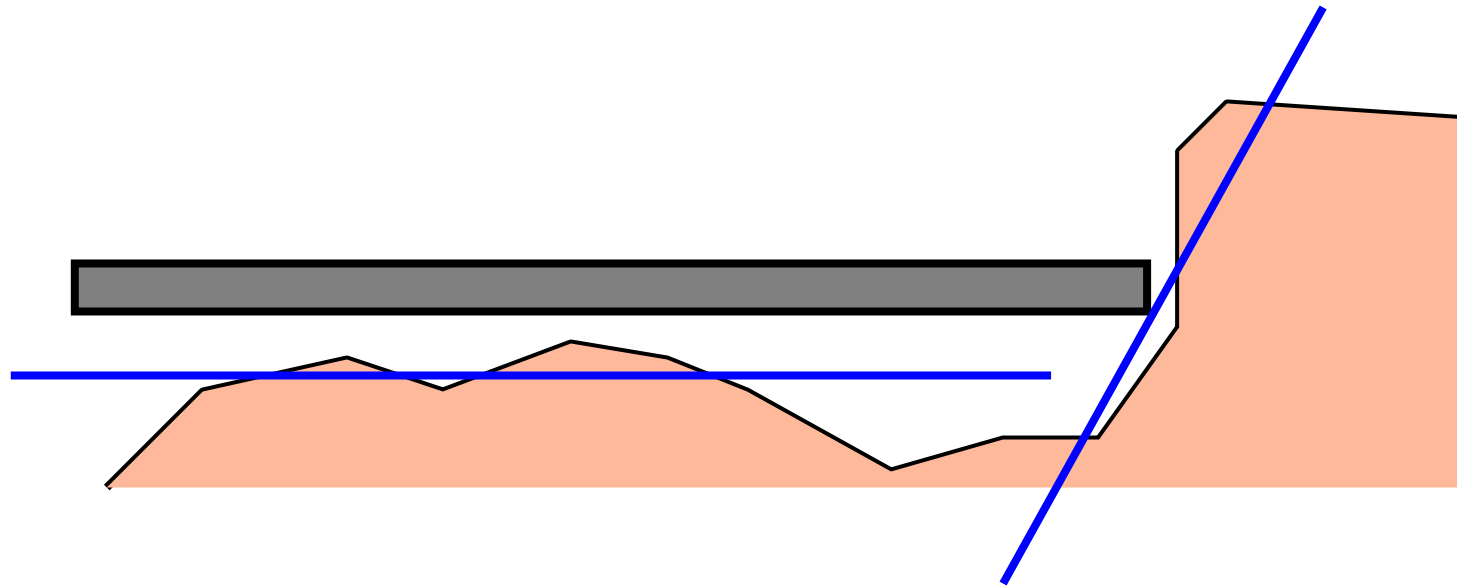


- Smooth Transition from one local model to the next



# Computing the local model

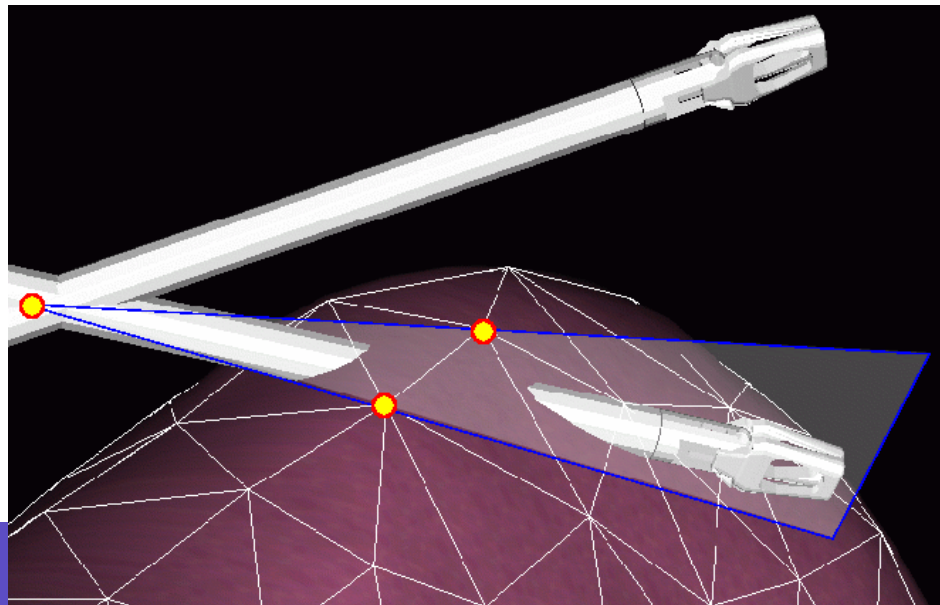
- Described as a set of planes
- One model for the tip
- One model for the handle



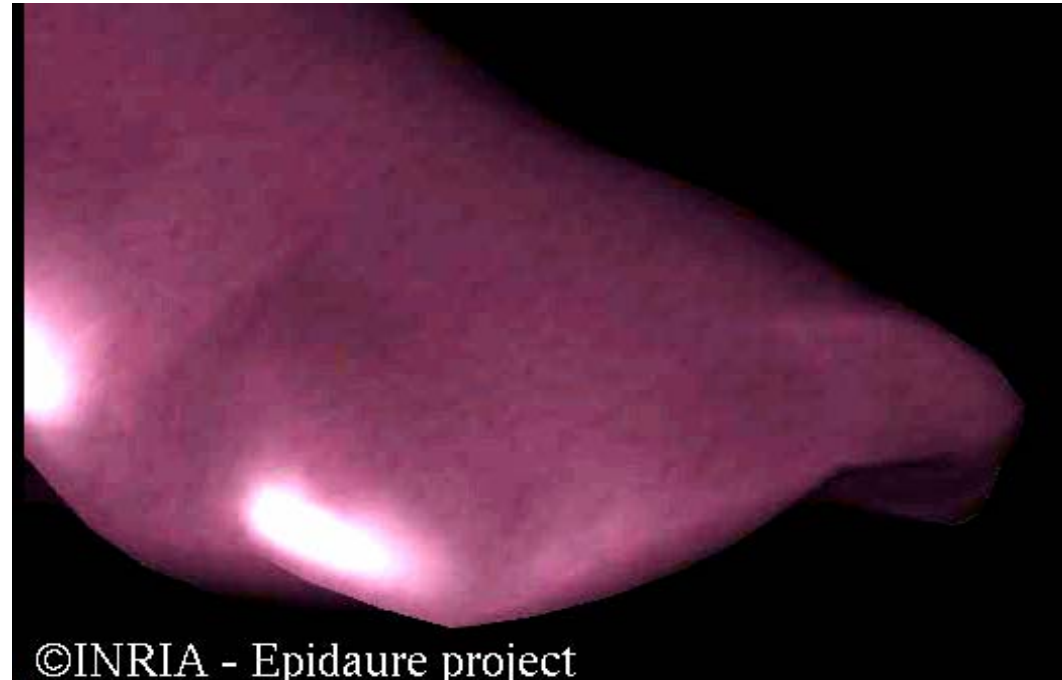
# Force Computation

- Proportional to the penetration of the tool tip in the planes described by the local model

$$F = k.(EndP - O_P).\vec{n}_P$$

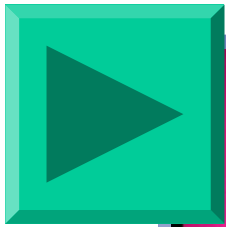


# Tensor-Mass Models (low resolution)

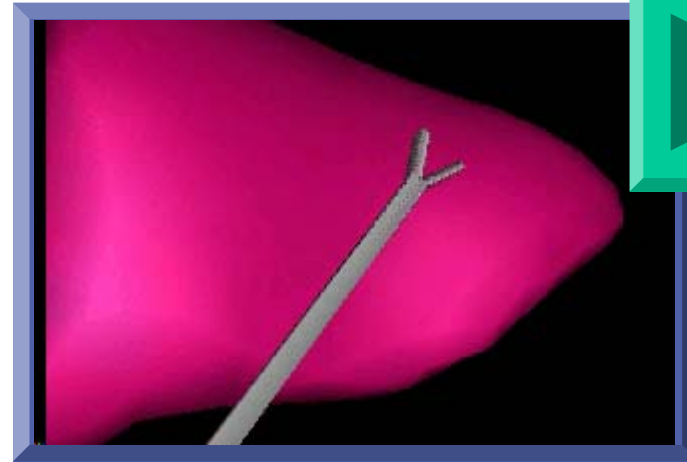
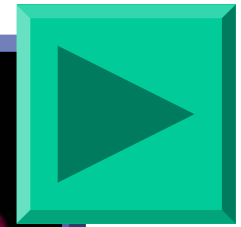


$N = 1394$  (6342 Tétraèdres)

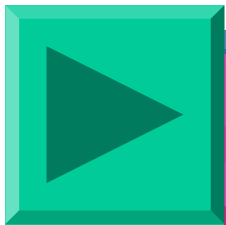
# Simulation of surgical gestures



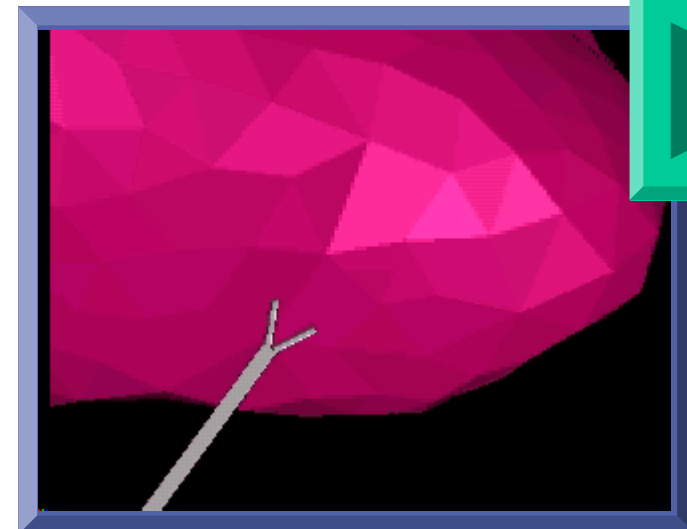
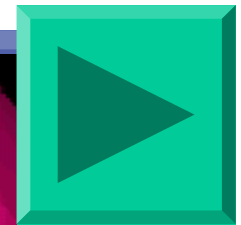
Gliding



Gripping

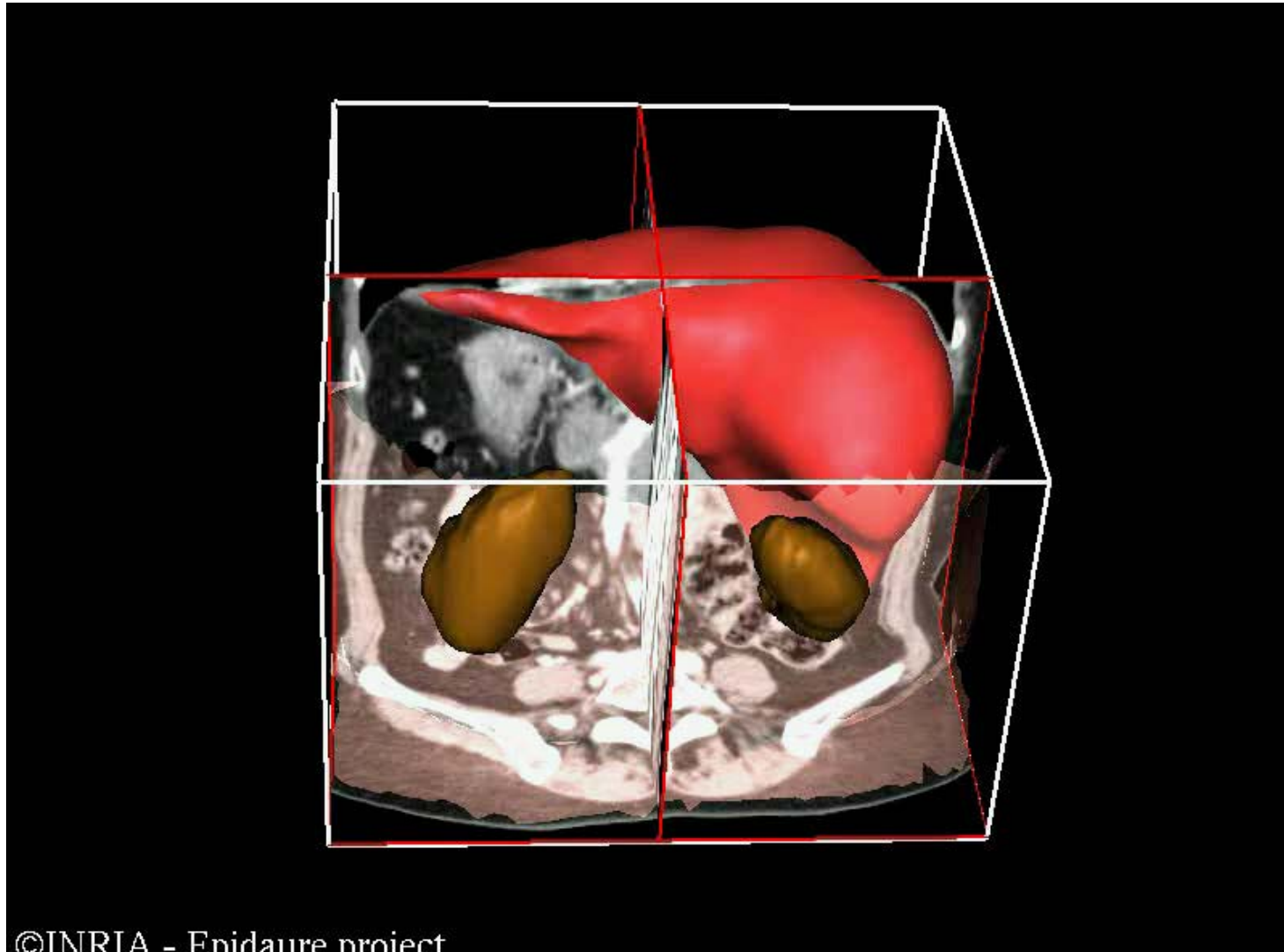
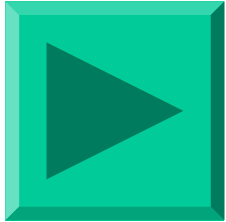


Cutting (pliers)



Cutting (US)

# Hepatic Surgery Simulation



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