1 Force Sensing

1.1 Load Cell

A **"Load Cell"** is a structure which supports the load and deflects a known amount in response to applied forces and torques. The deflections are measured to characterize the applied forces and torques.

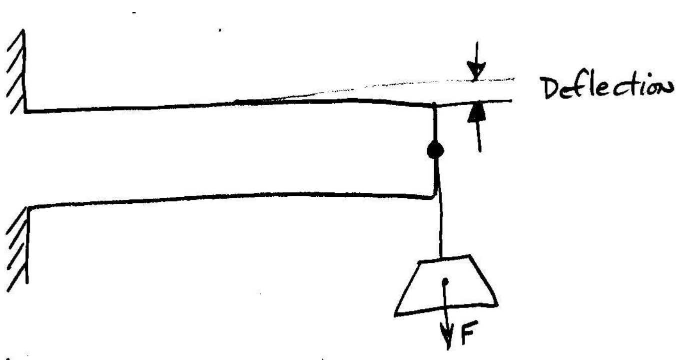

Figure 1: Cantilever Beam. When load F is applied to the end of the beam, the beam deflects a known amount.

Example: Cantilever Beam

1.2 Stress and Strain

1.2.1 Stress

- Stress (σ): Force per unit area in one dimension.

$$\sigma = \frac{F}{A}$$

  Units: Pascals = $\frac{N}{M^2}$, psi.

- Stress can be refered to as "Compressive" (tending to compress the material)and "Tensile" (tending to elongate it).

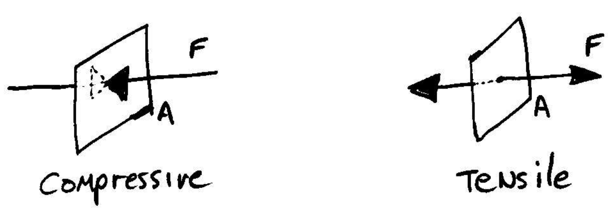

Figure 2: Illustrations of Compressive and Tensile stress through a patch of area A.

- Shear stress refers to forces *in* the plane. Shear stress can cause material to tear.

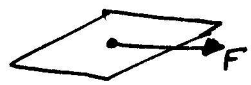

Figure 3: Shear stress. Shear stress is due to forces in the plane.

- 3-Dimensions: Full analysis of stress in 3-dimensions is complex an requires notion of *Stress Tensor* — a 3×3 matrix giving the complete state of stress at a given point in the material.

1.2.2 Strain

- Strain, (ϵ), is relative deformation.
- 1 Dimension: Elongation Ratio

$$\epsilon = \frac{\Delta L}{L}$$

- if $\frac{\Delta L}{L} = 10^{-5}$ we say "10 micro-strain".
- Strains of typical metal structures are very small (i.e. micro-strain).
- Strains in biological tissues can be much larger ($\epsilon \approx 1$).
- Advanced: Several Types of Strain

1.2.3 Hooke's Law (Elastic Deformation)

$$\sigma = \mathrm{E}\epsilon$$

where E is Young's Modulus.

- Biological materials rarely obey Hooke's Law.
- Metals obey Hooke's Law very well for stresses below the *elastic limit.* If stress is greater, material permanently deforms.

1.3 Cantilever Beam

We want to derive $\epsilon_{surface}$ as a function of the applied load, the dimensions of the beam, and the beam material's stiffness (Young's modulus). First, we assume that the beam is at static equilibrium around the centerline along the wall. For this we equate the moment developed in the structure around the center line with the moment applied by the external load.

$$Fl = \int_{-\frac{T}{2}}^{\frac{T}{2}} r\sigma(r)tdr$$

where r, t, F, l are defined in the figure, and $\sigma(r)$ is the internal stress where the beam joins the wall. Let us assume that $\sigma(r)$ varies linearly through the beam so that

$$\sigma(r) = \alpha r$$

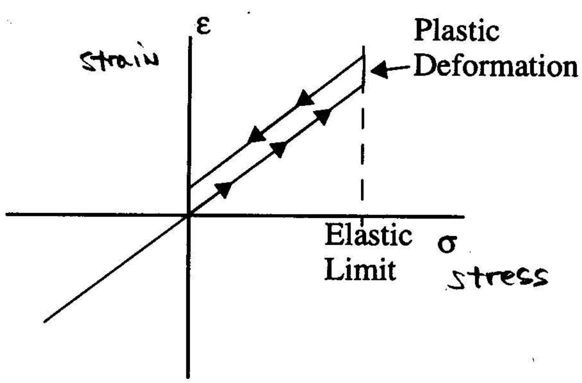

Figure 4: Hookean behavior with Elastic Limit. If stress increases beyond Elastic Limit, material is permanently deformed.

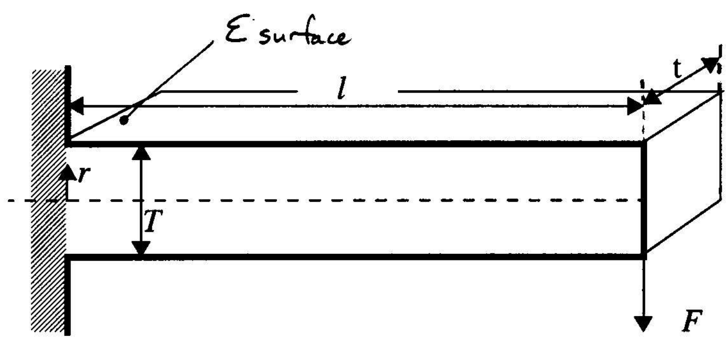

Figure 5: Rectangular cantilever beam is a simple load cell structure for basic analysis.

where α is some positive constant.

Then we have

$$Fl = \alpha t \int_{-\frac{T}{2}}^{\frac{T}{2}} r^2 dr = \frac{\alpha t T^3}{12}$$

Solving for α,

$$\alpha = \frac{12l}{tT^3}F$$

$$\sigma(r) = \frac{12lr}{tT^3}F$$

Finally, we can evaluate $\sigma(r)$ at the surface and use Young's modulus to convert stress to strain:

$$\sigma(\frac{T}{2}) = \frac{12l\frac{T}{2}}{tT^3}F$$

$$= \frac{6l}{tT^2}F$$

$$\epsilon_{surface} = \frac{6l}{\mathrm{E}tT^2}F$$

1.4 Strain Gages

(see `http://www.measurementsgroup.com/guide/index.htm`)

A strain gage is a device which transduces strain on a surface. There are two types in common use, resistive metal foil, and semiconductor. Both transduce strain by taking advantage of the variation of electrical resistance with strain.

Metal Foil Strain Gages Metal foil gages are bonded to the surface and their resistance changes in proportion to the surface strain.

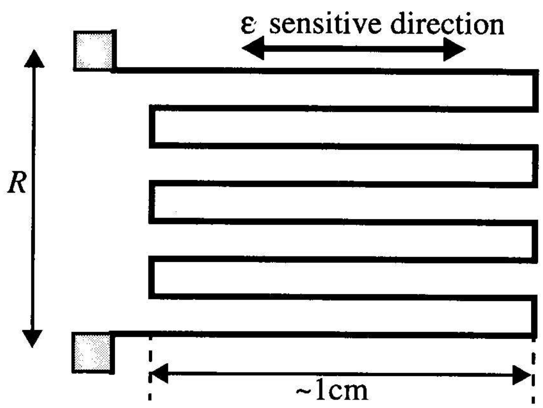

Figure 6: Diagram of a metal foil strain gage. Conductor in the shape shown is attached to surface. As surface deforms, conductive pattern changes length which changes resistance between contact pads (left).

In simplified form,

$$R = R_0(1 + \epsilon g)$$

where $g = \frac{\Delta R}{R_0 \epsilon}$ is called the "gage factor". Occasionally, the gage factor is not normalized by R_0 in which case it has the units of resistance, Ohms (Ω), sometimes called "Ohms per strain". For metal foil gages, $g \approx 3$.

Semiconductor strain gages These are small chips of silicon with a single region of doping to create a resistor. The chip is bonded to the load cell and its resistance varies with strain. With a semiconductor strain gage, the designer can expect signficantly higher gage factor of around $g \approx 150$.

Properties of Strain Gages

1. Very linear. 1% is achievable due to Hooke's law. Elastic limit must be avoided with a safety margin.

2. Wide dynamic range. Example: Force/Torque sensor designed by JPL for Space Shuttle RMS robot arm design range: 1N to 4×10^5N. 112db.

3. Temperature Sensitivity. Both types are highly sensitive to temperature.

4. Interface between gage and structure requires care due to different coefficients of thermal expansion between gage and structure.

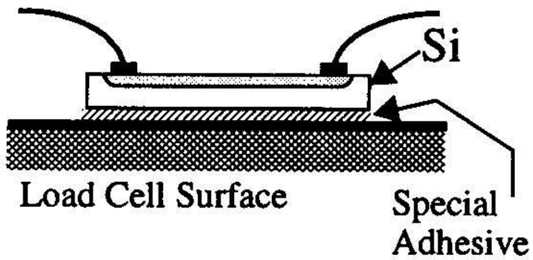

Figure 7: Diagram of a semiconductor strain gage.

5. Practicalities. Foil gages are robust and cheap. Semiconductor gages are delicate and expensive. Foil gages are fixed with an adhesive which usually requires curing in an oven. Semiconductor gages require an elaborate process for attachment which must be done by a specialist.

1.5 Temperature Sensitivity

Careful thermal analysis is important for accurate force sensing with strain gages. Two effects account for the temperature sensitivity of metal foil strain gages:

- Thermal Coefficient of Resistance of gage material.
- Difference in thermal coefficient of expansion of gage and load cell structure.

The first causes resistance to change only as a function of temperature and not of strain. The second causes strains in the gage as a function of temperature only.

The temperature dependence of the gage resistance can be described by:

$$r = \frac{\frac{\Delta R}{R_0}}{\Delta T} = \beta_g + g\left(\frac{1+K_t}{1+\nu_0 K_t}\right)(\alpha_s - \alpha_g)$$

Where

β_g = thermal coefficient of resistance of the gage material.

K_t = transverse sensitivity of the gage: the gage factor for strains orthogonal to the sensitive direction (extremely small for most gages and will be neglected).

ν_0 = Poisson's ratio (typically 0.3)

α_s, α_g = coefficients of thermal expansion of substrate and gage respectively.

The first step to reduce temperature sensitivity is to make sure the thermal expansion coefficients of the gage (α_g) and substrate (α_s) are as close as possible.

We can then relate resistance to strain and temperature through

$$R = R_0(1 + g\epsilon)(1 + r\Delta T)$$

The second step to compensating for the remaining temperature sensitivity is to make a differential measurement of two gages which have the same temperature but opposite strains. In the case of a cantilever beam, if we assume that the beam is at a uniform temperature, this can be arranged by putting gages on either side of the beam. By symmetry, $\epsilon_1 = -\epsilon_2$.

Dual Gage beam By symmetry, $\epsilon_1 = -\epsilon_2$.

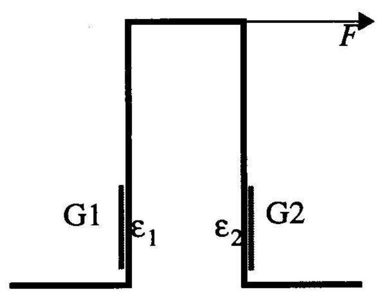

Figure 8: Cantilever beam load cell with gages on either side. Strains ϵ_1 and ϵ_2 will be equal and opposite. If temperature is same on both sides of beam, we can correct for temperature sensitivity.

1.6 Whetstone Bridge

Subtraction of the two strain signals is usually accomplished right in the load cell itself using a Whetstone Bridge Circuit.

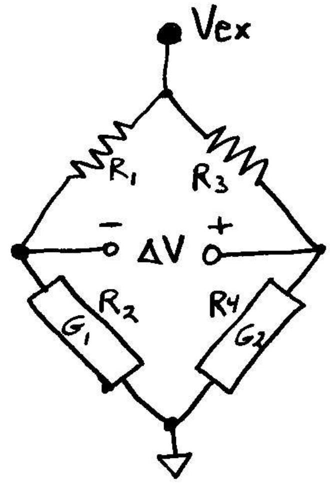

Figure 9: Whetstone Bridge circuit used to subtract resistance changes of the two strain gages.

Analysis of the bridge circuit yields:

$$\frac{\Delta V}{V_{ex}} = \frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} = \frac{R_4 R_1 - R_2 R_3}{(R_1 + R_2)(R_3 + R_4)}$$

In this example, we place the two strain gages into the bottom legs of the bridge, (R_2, R_4). The other two legs of the bridge are non-strain-sensitive resistors (but still temperature sensitive).

We define $\hat{R} = R_0(1 + r\Delta T)$.

$$R_1, R_3 = \hat{R}$$

$$R_2 = \hat{R} + (1 + r\Delta T)gR_0\epsilon_1$$
$$R_4 = \hat{R} + (1 + r\Delta T)gR_0\epsilon_2$$

From the load cell design, we have $\epsilon_2 = -\epsilon_1$.
Computing the output voltage we get:

$$\frac{\Delta V}{V_{ex}} = \frac{\hat{R}(1 + r\Delta T)2gR_0\epsilon_2}{4\hat{R}^2 - (1 + r\Delta T)^2 g^2 R_0^2 \epsilon_1^2}$$

Expanding $\hat{R}$ an canceling terms:

$$\frac{\Delta V}{V_{ex}} = \frac{2g\epsilon_2}{4 - g^2\epsilon_1^2}$$

Ignoring the high order terms:

$$\frac{\Delta V}{V_{ex}} = \frac{g\epsilon_2}{2}$$

Temperature variation is eliminated! Non-linear term contributes insignificant errors because ϵ is very small.

1.7 Multi-Axis Sensing

In robotics and surgery it is rarely of interest to measure a single force. In general we want to measure the 6 components of force and torque,

F_X
F_Y
F_Z
τ_X
τ_Y
τ_Z

We want to characterize these force/torque components at some interface between two objects. Examples of these interfaces: robot wrist — robot hand, surgical tool handle — surgical tool body, etc. So that the sensor does not distort the forces and torques present, it should be as rigid as possible. Therefore the two objects connected by the sensor should normally be rigidly connected.

Design of load cells for this measurement takes substantial effort and is beyond the scope of this course. However it is useful to consider two example designs.

1.7.1 "Maltese Cross" Sensor

Variations of this design have been produced in a number of laboratories and are available commercially (Fig. 10). In this design, the spokes of a wheel are instrumented with gages and calibrated to measure forces and torques applied to the hub relative to the rim.

Four gages are applied to each beam for a total of 8 differential measurements.

A 6x8 calibration matrix C is computed to relate the strain measurements to forces/torques

$$\begin{bmatrix} F_x \\ F_Y \\ F_Z \\ \tau_X \\ \tau_Y \\ \tau_Z \end{bmatrix} = \begin{bmatrix} & & \\ & C & \\ & & \end{bmatrix}_{6\times 8} \begin{bmatrix} \\ \epsilon \\ \\ \end{bmatrix}_{8\times 1}$$

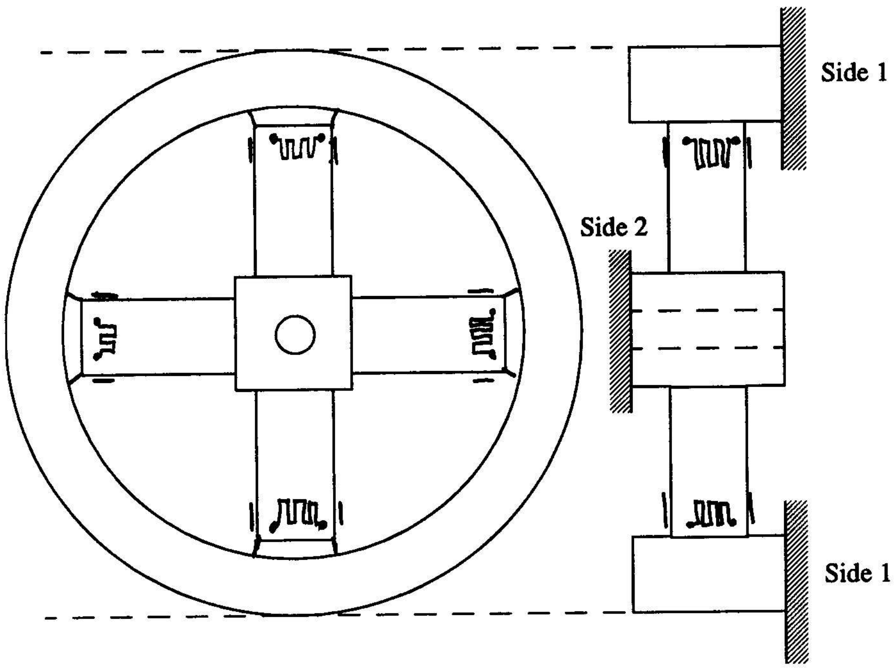

Figure 10: "Maltese Cross" force-torque sensor. Forces and torques applied to the surface labeled 'side 1' cause strains in the spokes which are picked up by 8 attached gages.

In practice, manufacturing tolerances are not adequate to compute C from the load cell design with sufficient accuracy so it must be obtained from measurements of strain outputs as a function of applied forces and torques for each device using regression analysis.

1.7.2 Parallel-Plate Structure

(See T. Yoshikawa and T. Miyazaki, "A six-axis force sensor with three dimensional cross shape structure," Proc. Intl. Conference on Robotics and Automation, pp. 249-254, 1989.)

An alternative to the bending beam load cell is the parallel plate structure (also known as a flexure). In this case, two thin beams are arranged in parallel (Figure 11).

Analysis At the point of maximum strain concentration, labeled ϵ_{max}, the strain is

$$\epsilon_{max} = \frac{3lF}{\mathrm{E}tT^2}$$

where t is the width of the parallel plate structure, and E is Young's Modulus for the material.

1.8 Overload Protection

We saw that above a certain critical strain, the "elastic limit," a material will permanently (plastically) deform. This has two important consequences. First, if

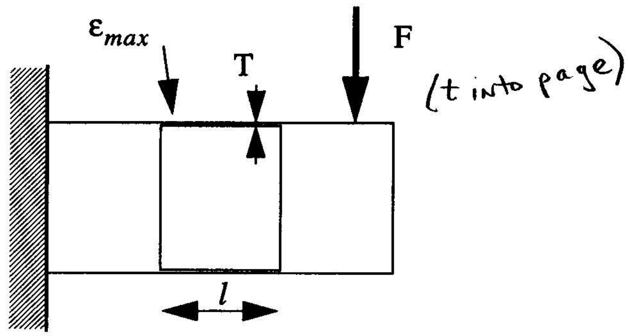

Figure 11: Parallel plate flexure load cell for a single axis.

the material deforms plastically, the zero reading of the load cell will have to be recalibrated. Second, the load cell will be damaged and will eventually break or suffer reduced life.

In most realistic force sensing applications, it is difficult or impossible to predict the maximum forces likely to be experienced by the sensor. This is because collisions between the sensor and the environment often occur, either deliberately or as a result of errors. The transient forces which occur in these collisions (especially with hard objects) are hard to control and may exceed the design force limit for the sensing beam.

To prevent damage from these types of events, overload protection is sometimes designed into force/torque sensors. An overload protection device must allow safe deflection of the load cell in all active directions without disturbance forces, but must provide greatly increased stiffness and strength for deflections above the safe operating point (which may be a factor of two or more *below* the elastic limit.)

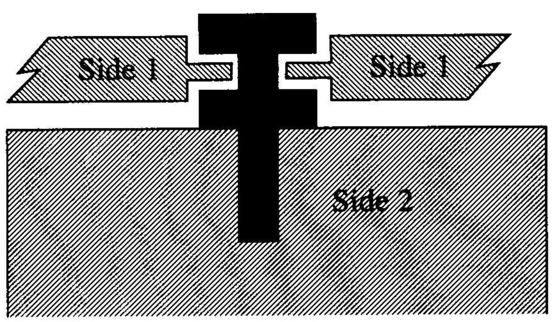

Figure 12: Overload protection scheme for a force/torque sensor. a high-strength structural element connects the two sides when deflection exceeds the size of its gap to protect the sensing structure from excessive strain.

In the example illustrated in Figure 12, a pin of strong material (i.e. steel in an aluminum load cell design) is affixed betwene the two sides of the load cell so that there is a calibrated gap between the two sides. When the strain reaches the maximum safe operating point, the two sides touch and the stiffness of the overall loadcell is greatly increased. Once the two sides come into contact through the pin, the amount of force required to reach the elastic limit is greatly increased.

Any further deflection is thus reduced or eliminated.