A celebrated result of Grötschel, Lovász and Schrijver [2] is that computing clique number $\omega(G)$ and chromatic number $\chi(G)$ of a perfect graph can be done in polynomial time. This result relies on a polyhedral characterization of perfect graphs [1] showing that for such a graph $G$ the stable set polytope $\text{STAB}(G)$ coincides with its linear relaxation $\text{QSTAB}(G)$. However, optimizing over $\text{QSTAB}(G)$ does not work directly [2], but only via a detour involving a geometric representation of graphs. The resulting convex set $\text{TH}(G)$ satisfies $\text{STAB}(G) \subseteq \text{TH}(G) \subseteq \text{QSTAB}(G)$ and has the key property that $\vartheta(G) = \max_{x \in \text{TH}(G)} \{\langle x, \delta(G) \rangle\}$ which also allows to compute $\chi(G) = \omega(G)$ in polynomial time.

We address the question whether this result can be extended to graph classes $\mathcal{G}$ whose members $G$ satisfy the best possible bound on the chromatic number for graph classes containing imperfect graphs, namely $\omega(G) \leq \chi(G) \leq \omega(G) + 1$. For such graphs $G \in \mathcal{G}$ clearly $\omega(G) \leq \vartheta(G) \leq \chi(G) \leq \omega(G) + 1$ holds. Hence $\omega(G) = \lfloor \vartheta(G) \rfloor$ and $\chi(G) = \lceil \vartheta(G) \rceil$ follows for $\vartheta(G) \notin \mathbb{Z}$ and the two parameters can be computed in polytime for all graphs $G \in \mathcal{G}$, provided that we can decide which of the three cases

- $\omega(G) < \lfloor \vartheta(G) \rfloor = \chi(G)$
- $\omega(G) = \lfloor \vartheta(G) \rfloor < \chi(G)$
- $\omega(G) = \lceil \vartheta(G) \rceil = \chi(G)$

occurs if $\vartheta(G) \in \mathbb{Z}$ holds. We apply this method to a superclass of perfect graphs, the circular-perfect graphs defined by a more general coloring concept. The main result is that clique and chromatic number can be computed in polynomial time for all circular-perfect graphs [4] and, using similar techniques, further graph parameters for subclasses of circular-perfect graphs [3, 4].

In contrary, we exhibit that the same approach fails for two prominent graph classes $\mathcal{G}$ with the studied property, namely line graphs and planar graphs, unless $P = NP$.

Références