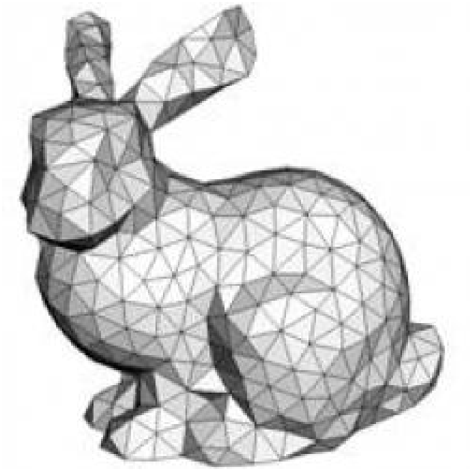
Orient to encode

Benjamin Lévêque

CNRS, G-SCOP, Grenoble

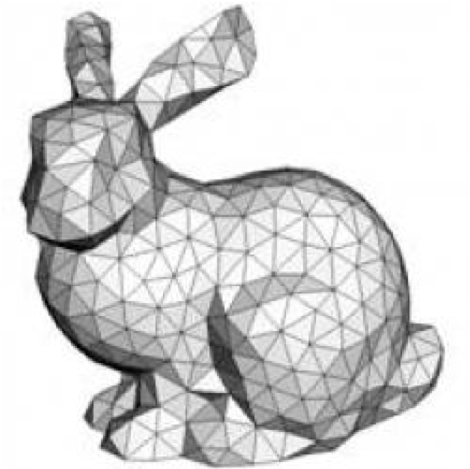
Why encoding ?

Why encoding ?



Why encoding ?

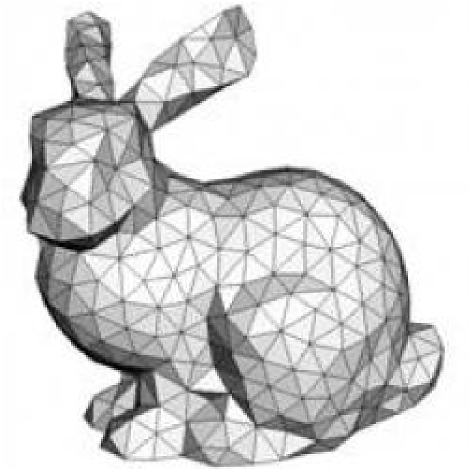
Compactness
number of bits



Why encoding ?

Compactness
number of bits

Efficiency
complexity of the coding and decoding procedure



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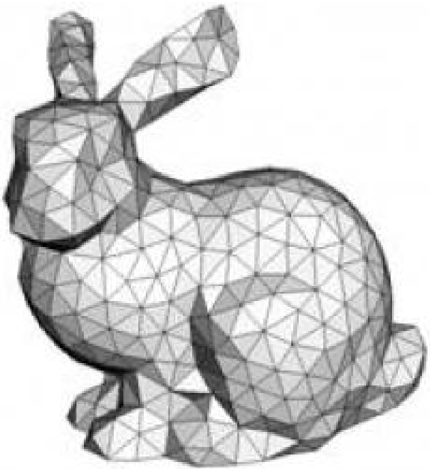
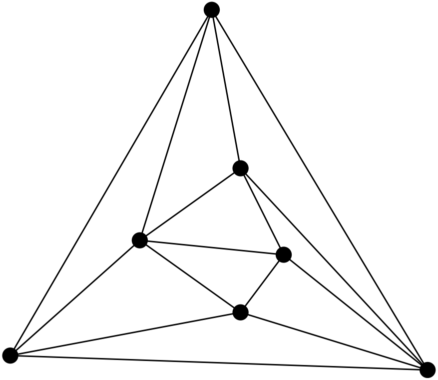


Effective approach, encode separetely :

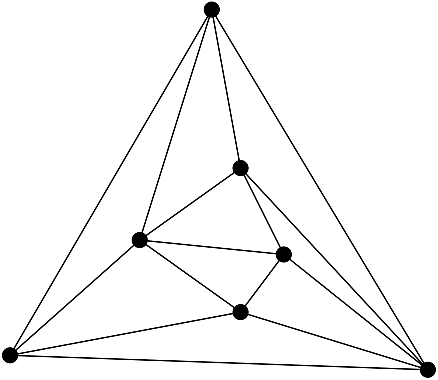
- Vertex coordinates
- Combinatorial structure (i.e. the planar triangulation here)

Planar triangulations

Planar triangulations

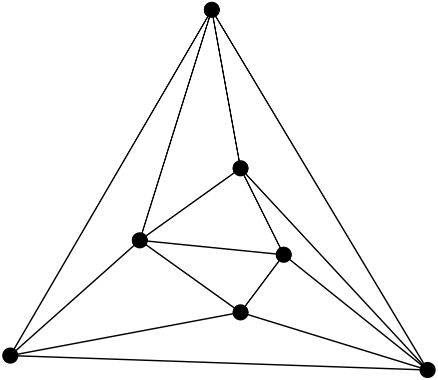


Planar triangulations



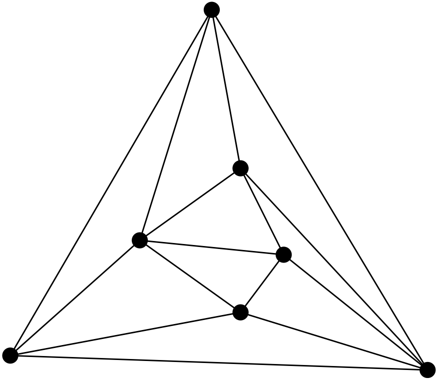
Euler : $n - m + f = 2$

Planar triangulations



Euler : $n - m + f = 2$
Triangulation : $3f = 2m$

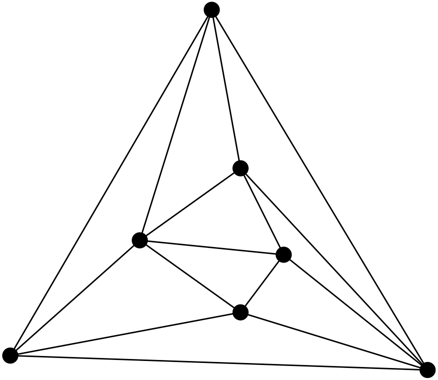
Planar triangulations



Euler : $n - m + f = 2$ $\qquad$ $m \;=\; 3n - 6$

Triangulation : $3f = 2m$ $\quad \Longrightarrow$

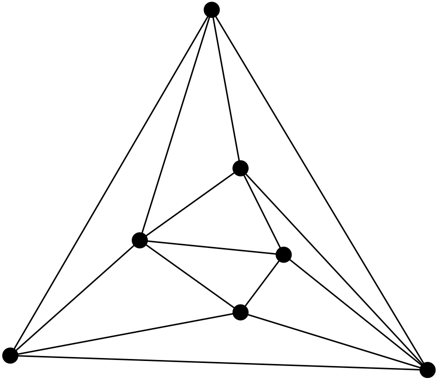
Planar triangulations



Euler : $n - m + f = 2$
Triangulation : $3f = 2m$ $\implies$

$$\begin{aligned} m &= 3n-6 \\ (m-3) &= 3(n-3) \end{aligned}$$

Planar triangulations



Euler : $n - m + f = 2$
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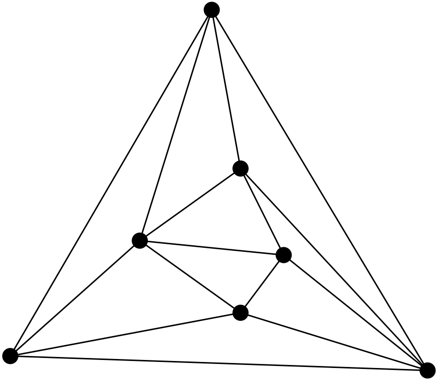

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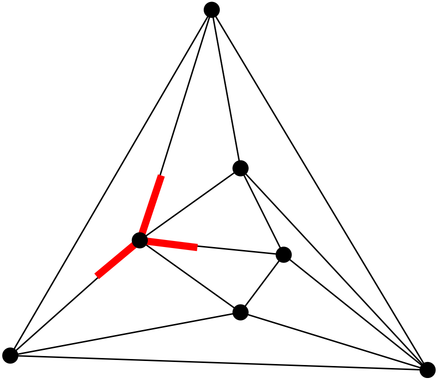
$\rightsquigarrow$ Associate to each internal vertex three incident edges

Planar triangulations



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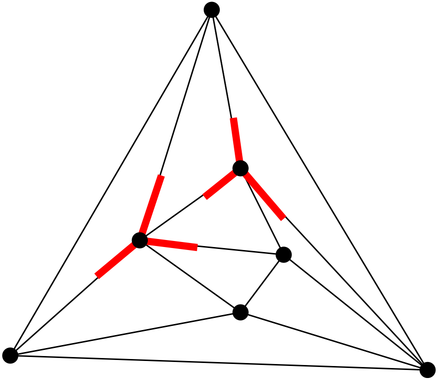

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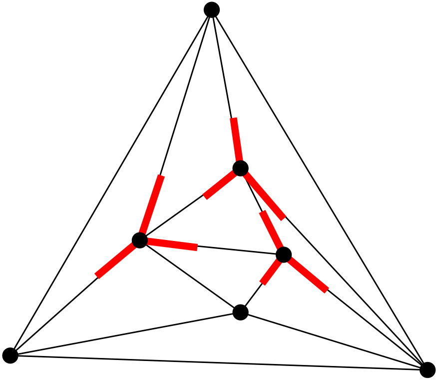

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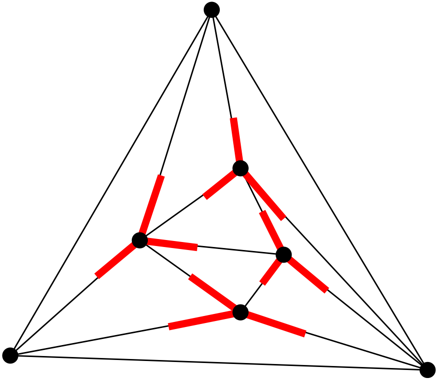

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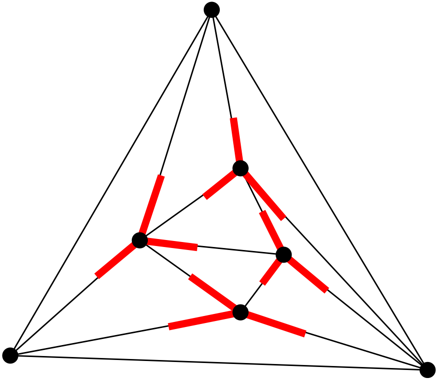

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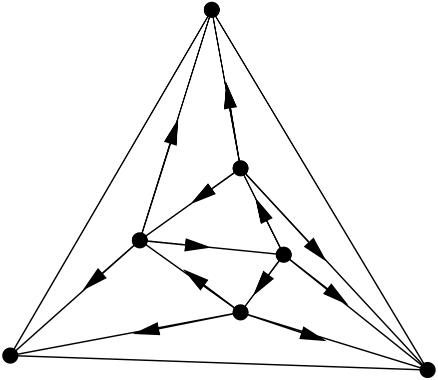
⇝ Associate to each internal vertex three incident edges and orient

Planar triangulations



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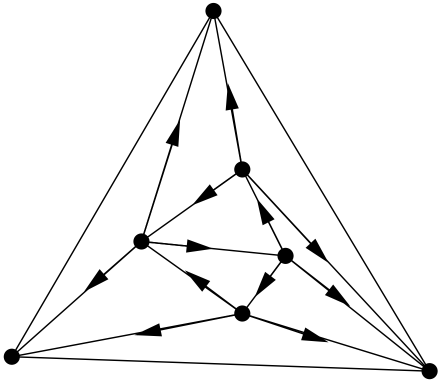
3-orientations of planar triangulations



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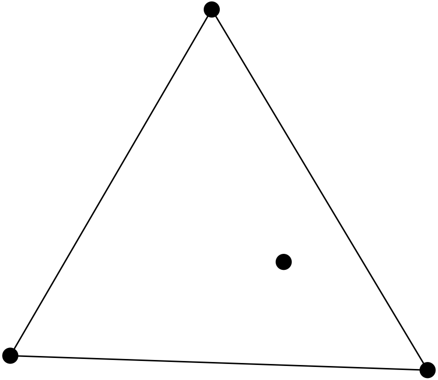

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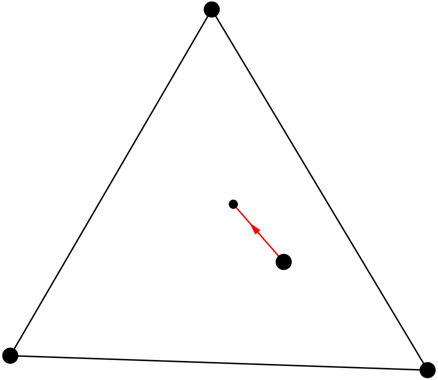

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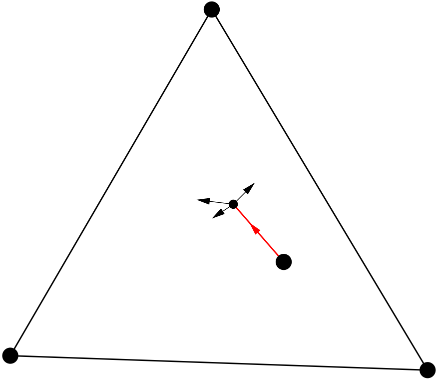

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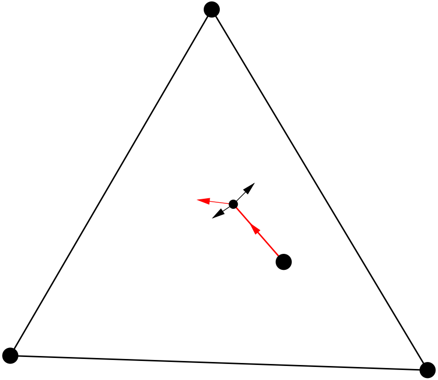
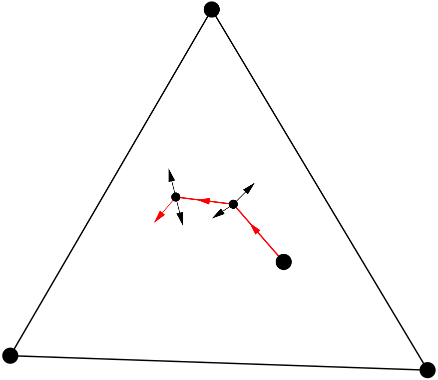


"Middle paths"

3-orientations of planar triangulations



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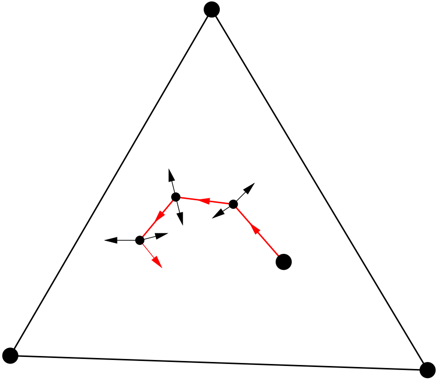

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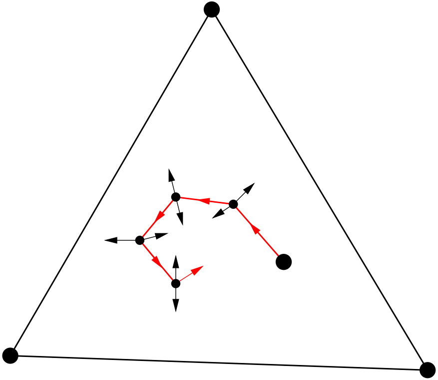

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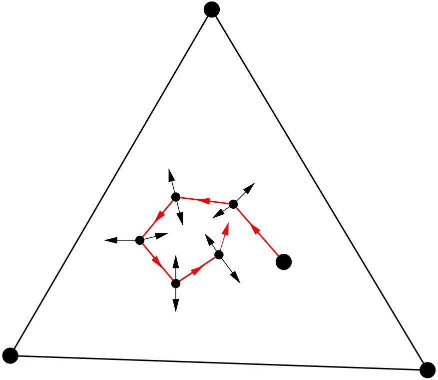

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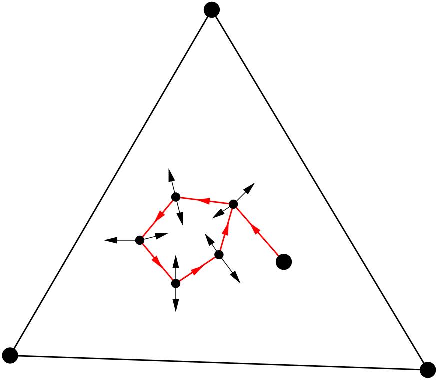

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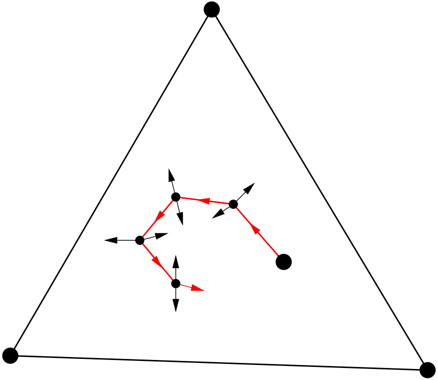

"Middle paths" do not cycle

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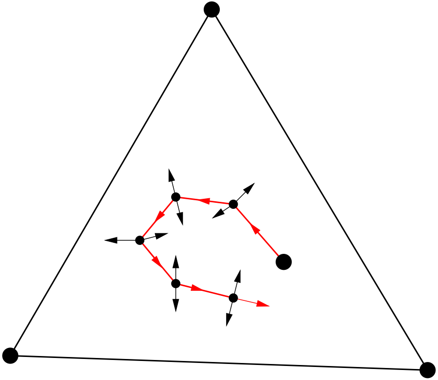

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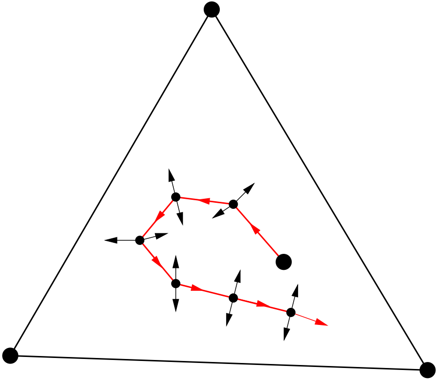

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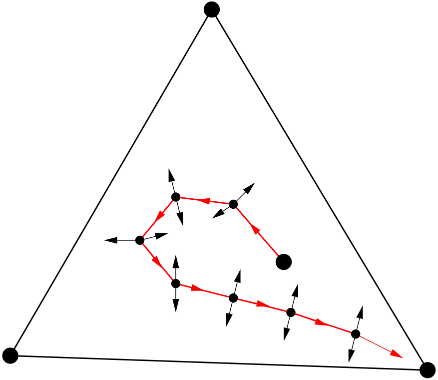

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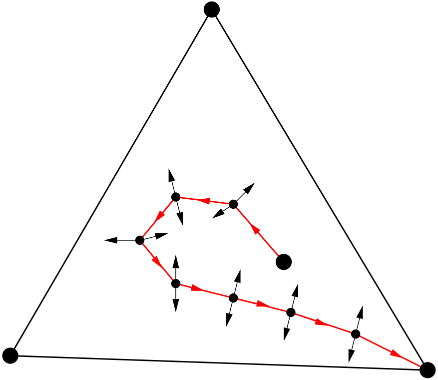

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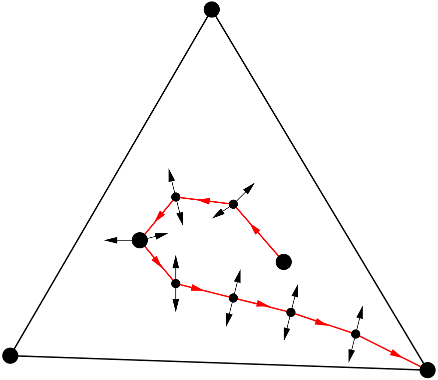

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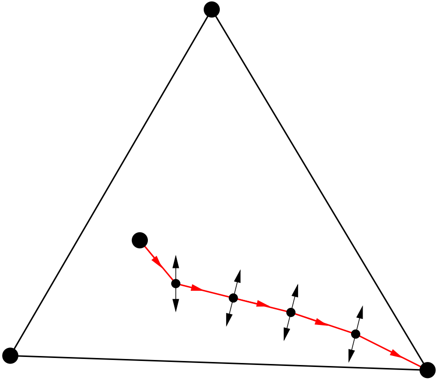

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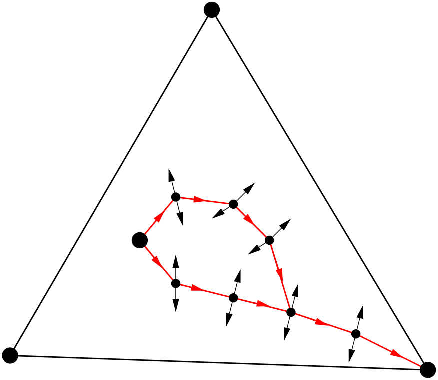

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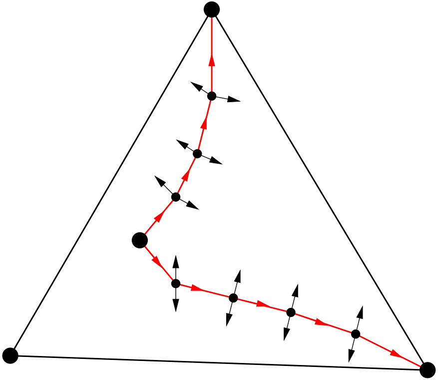

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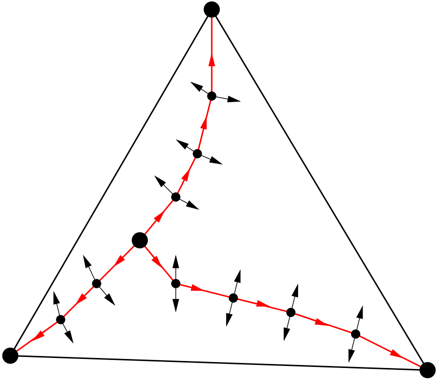

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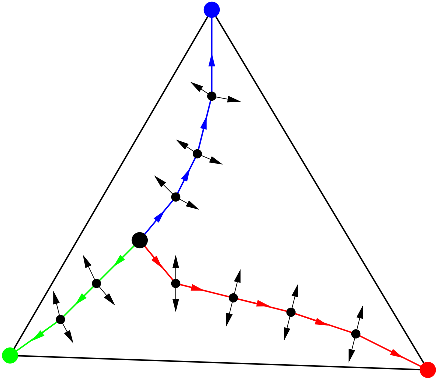

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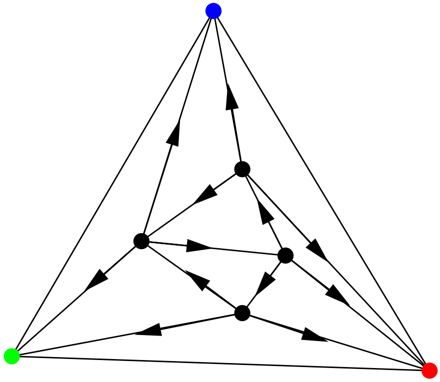

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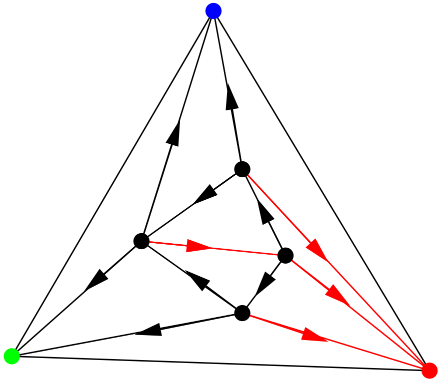

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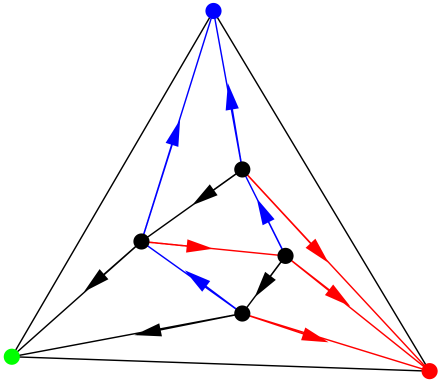

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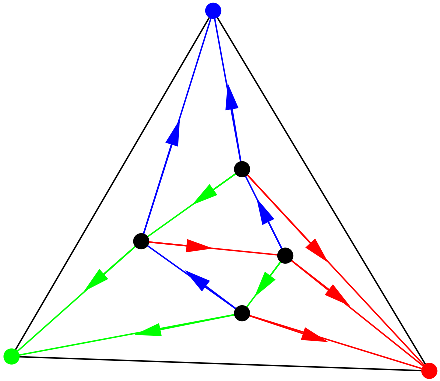

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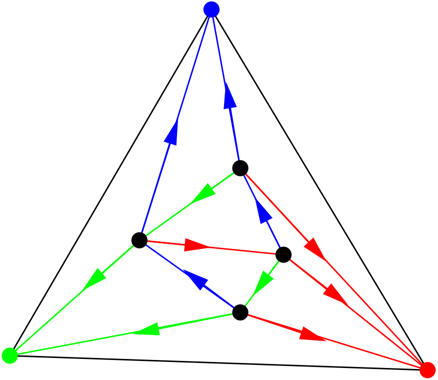
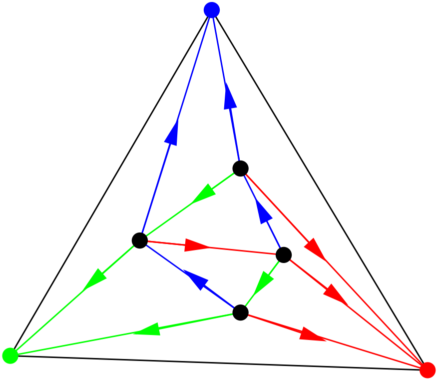
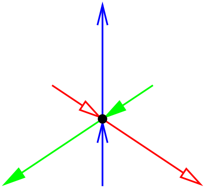


$\rightsquigarrow$ Schnyder woods

3-orientations of planar triangulations



⇝ Schnyder woods



Applications of Schnyder woods

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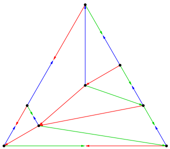
- Dushnik-Miller dimension

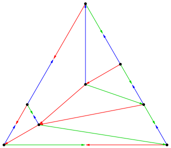
Applications of Schnyder woods

- Dushnik-Miller dimension
- Graph drawing



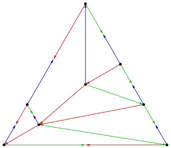
Applications of Schnyder woods

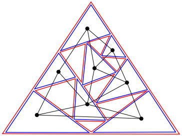
- Dushnik-Miller dimension
- Graph drawing
- Spanners



Applications of Schnyder woods

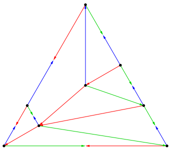
- Dushnik-Miller dimension
- Graph drawing
- Spanners
- Contact systems of polygons

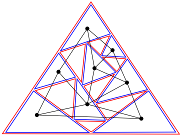


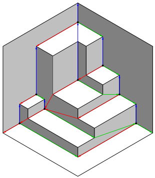


Applications of Schnyder woods

- Dushnik-Miller dimension
- Graph drawing
- Spanners
- Contact systems of polygons
- Orthogonal surfaces

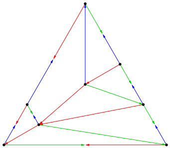


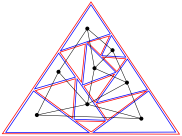


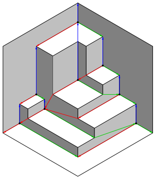


Applications of Schnyder woods

- Dushnik-Miller dimension
- Graph drawing
- Spanners
- Contact systems of polygons
- Orthogonal surfaces
- Encoding

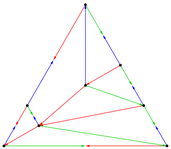


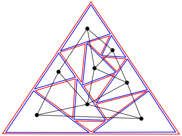


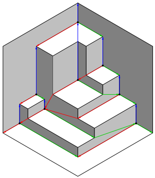


Applications of Schnyder woods

- Dushnik-Miller dimension
- Graph drawing
- Spanners
- Contact systems of polygons
- Orthogonal surfaces
- Encoding
- Counting

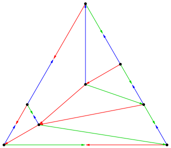


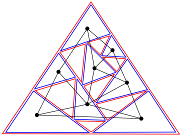


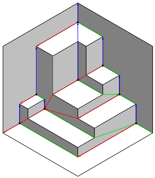


Applications of Schnyder woods

- Dushnik-Miller dimension
- Graph drawing
- Spanners
- Contact systems of polygons
- Orthogonal surfaces
- Encoding
- Counting
- Sampling

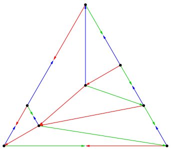


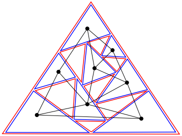


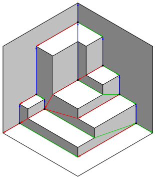


Applications of Schnyder woods

- Dushnik-Miller dimension
- Graph drawing
- Spanners
- Contact systems of polygons
- Orthogonal surfaces
- Encoding
- Counting
- Sampling
- 3-connected planar maps

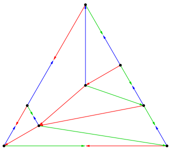


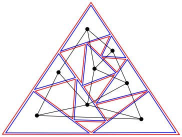


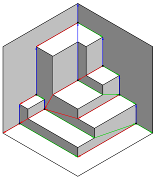


Applications of Schnyder woods

- Dushnik-Miller dimension
- Graph drawing
- Spanners
- Contact systems of polygons
- Orthogonal surfaces
- Encoding
- Counting
- Sampling
- 3-connected planar maps
- Duality







Applications of Schnyder woods

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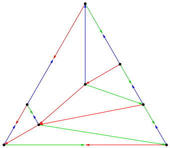
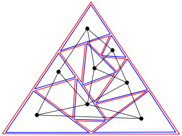
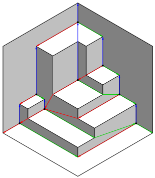
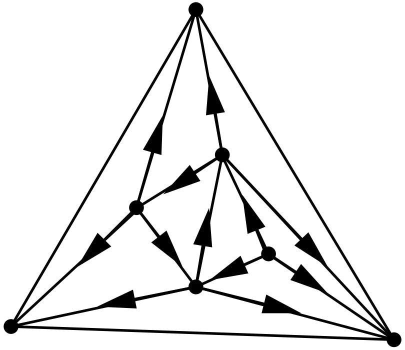
- ...







Encoding - Poulalhon, Schaeffer (2003)



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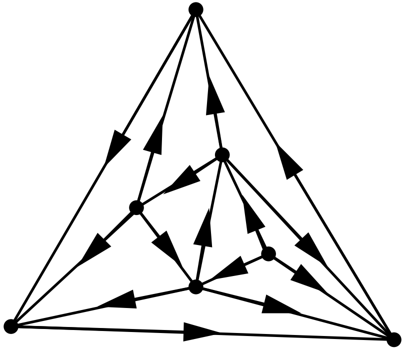

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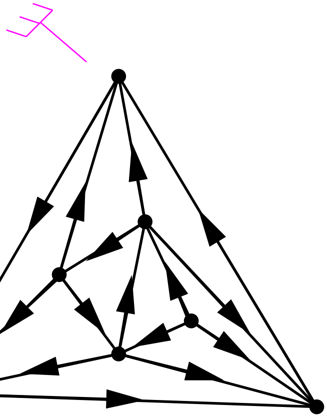

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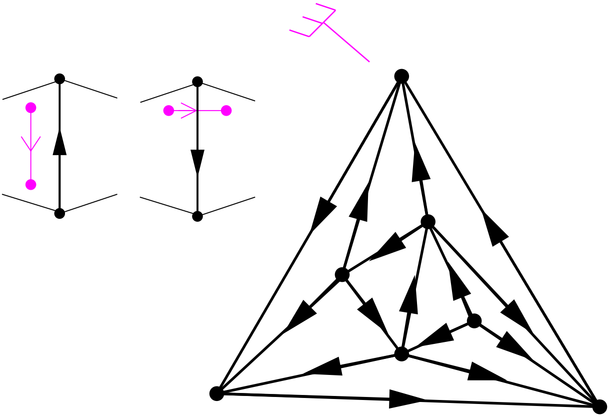

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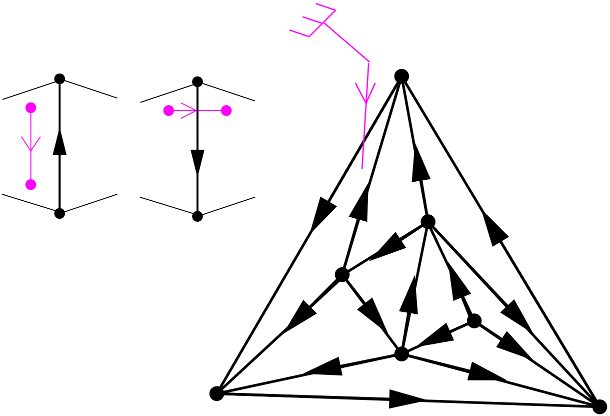

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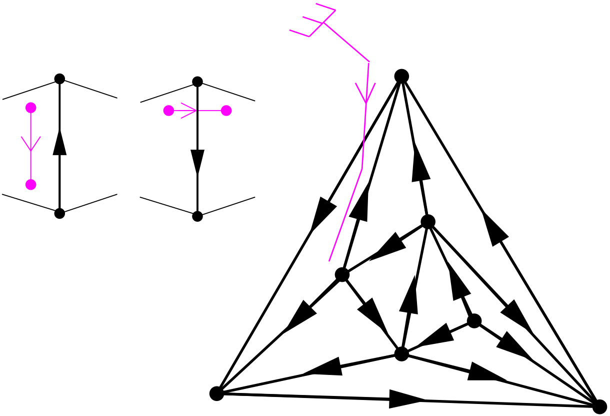

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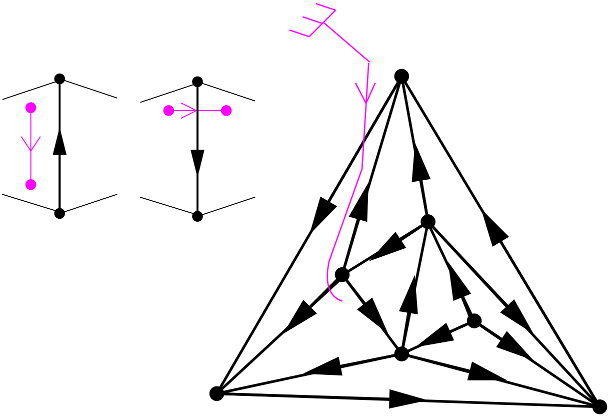

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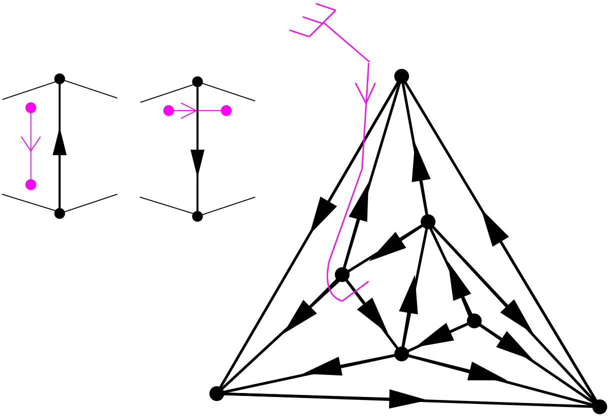

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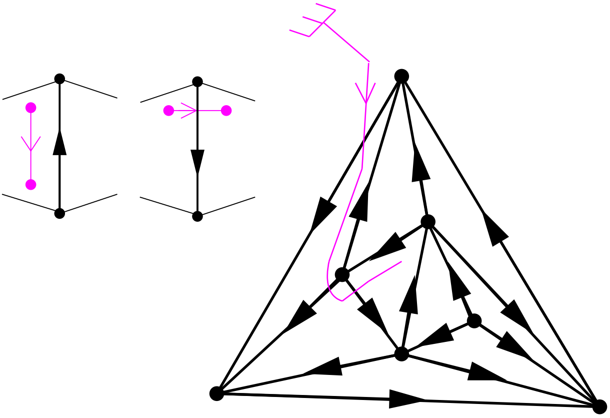

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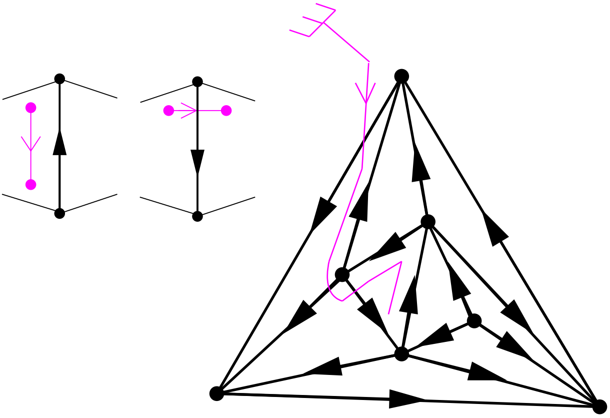

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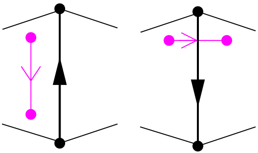

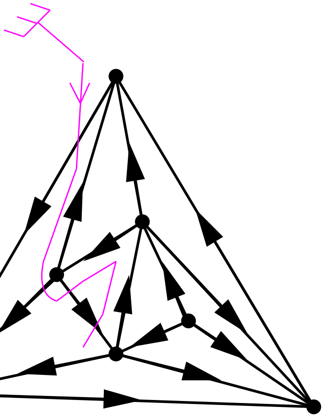

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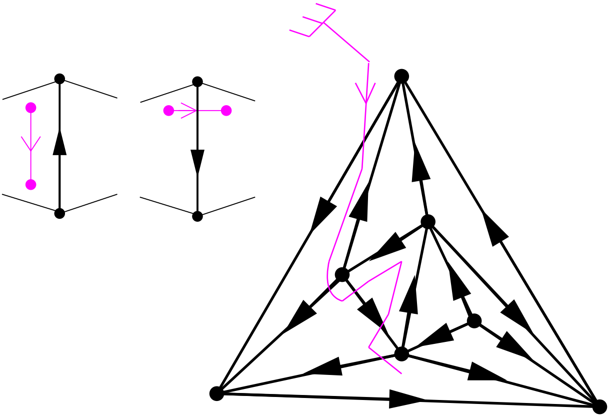

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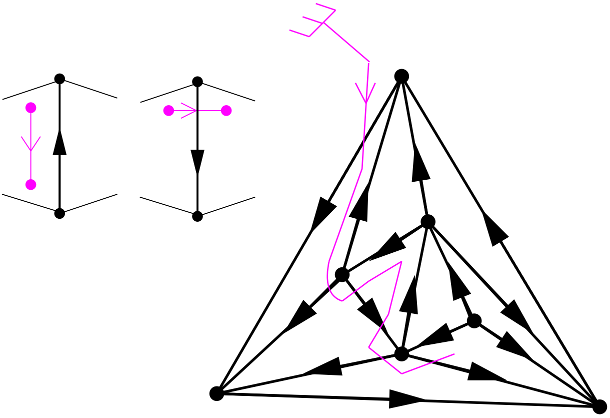

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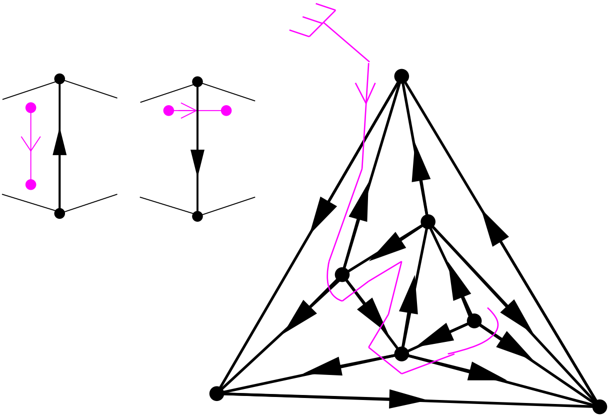

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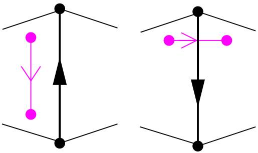

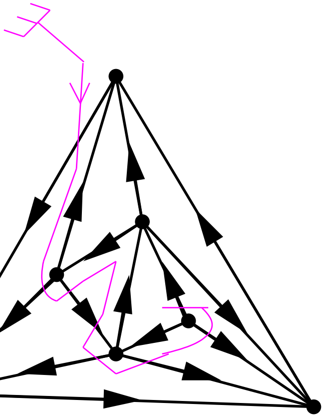

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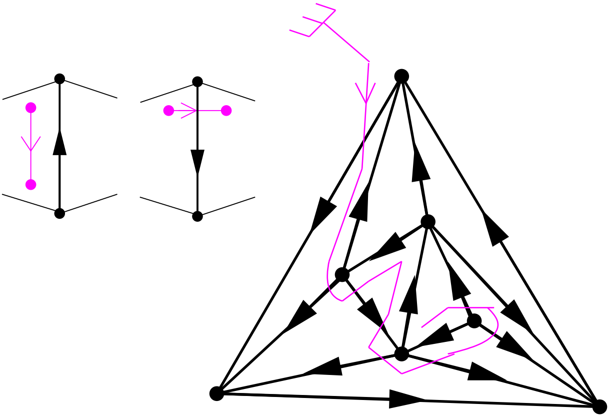

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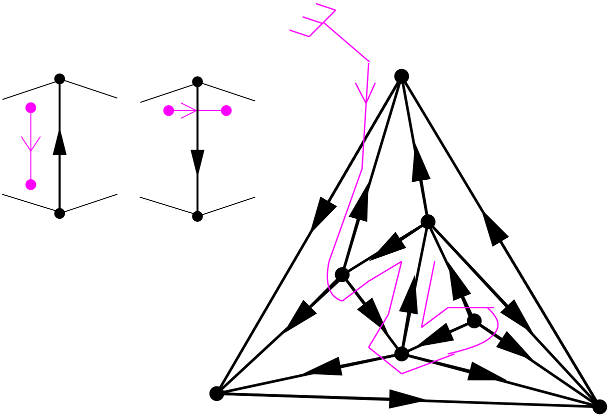

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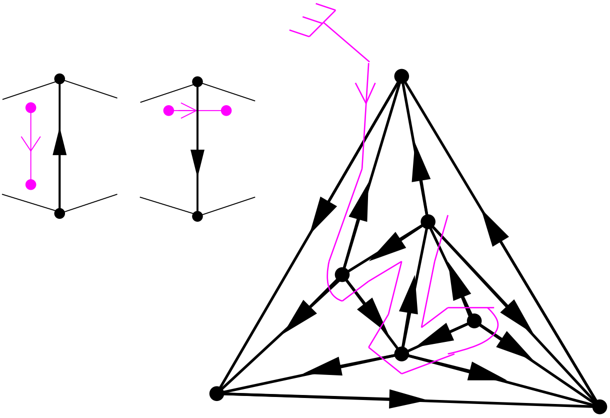

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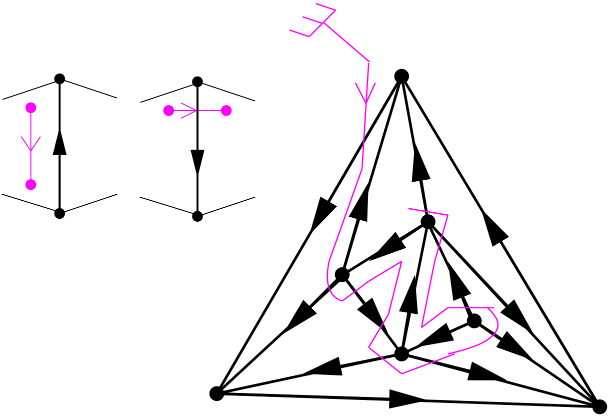

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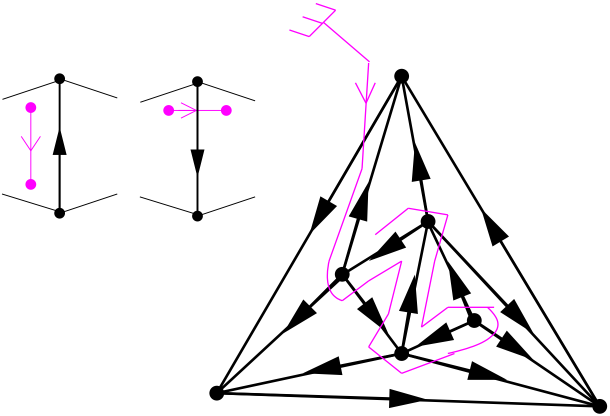

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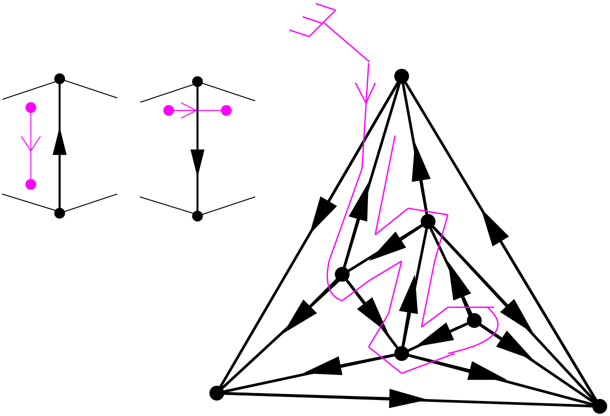

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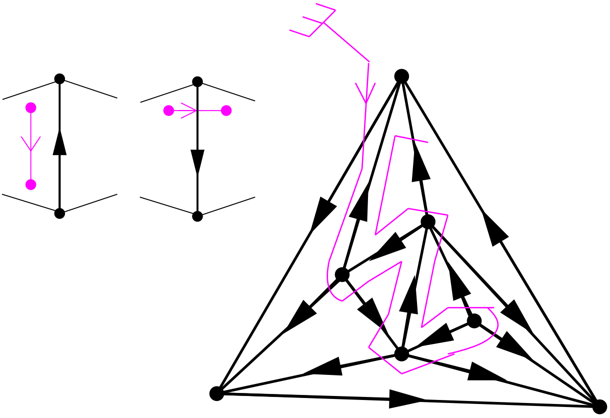

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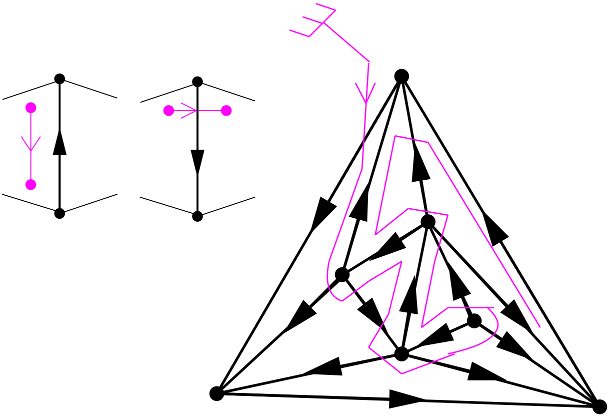

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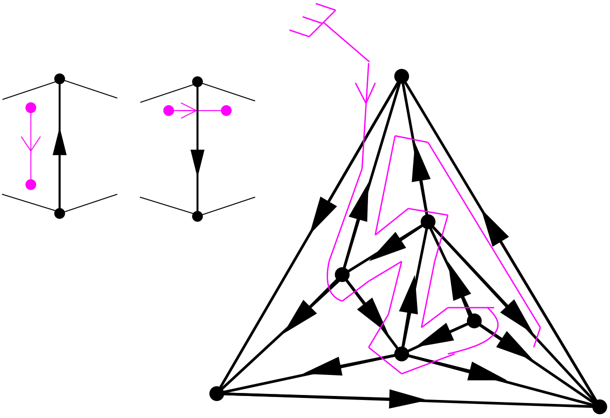

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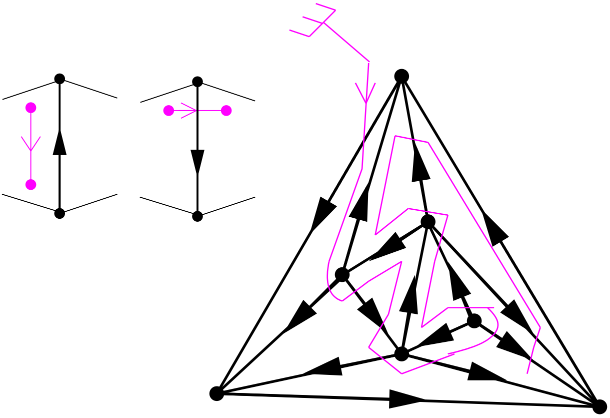

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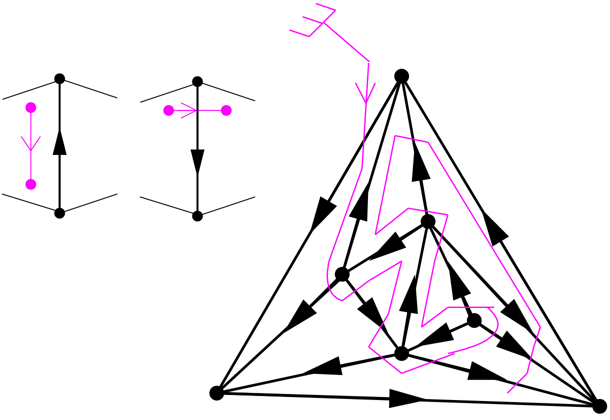

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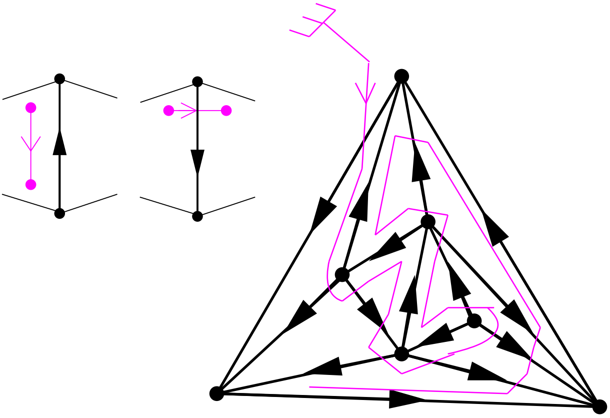

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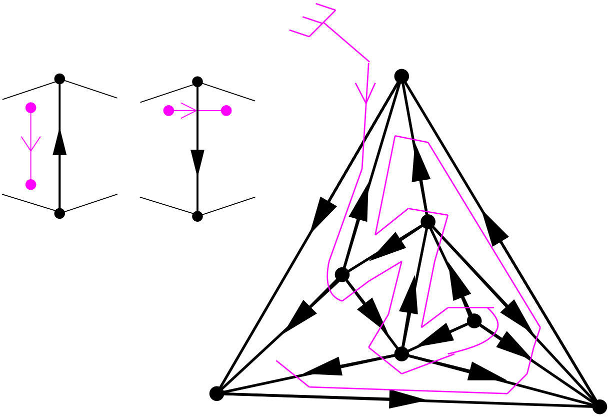

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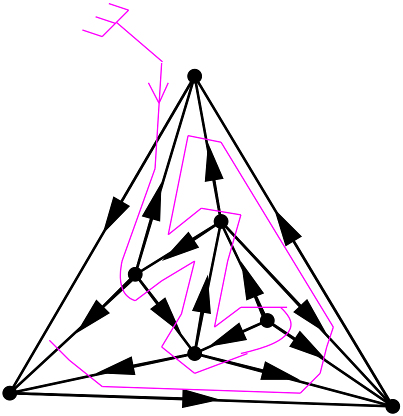

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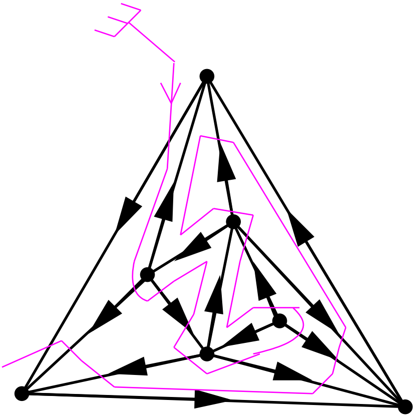

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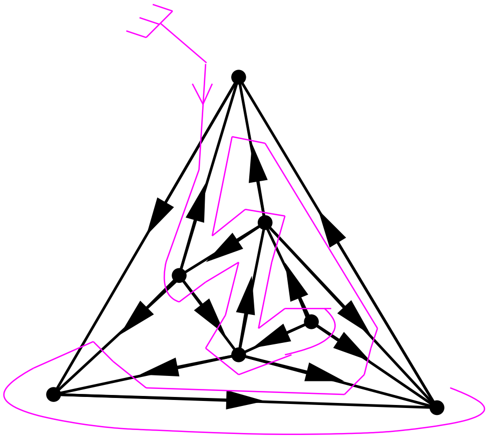

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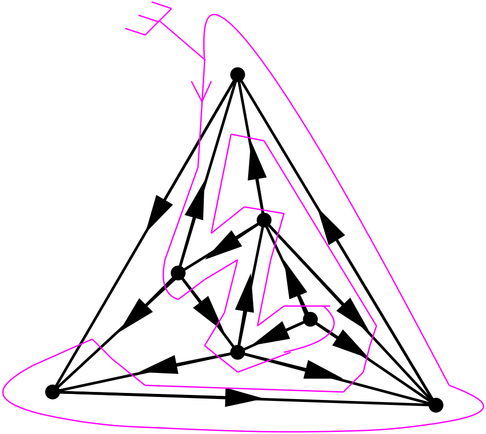

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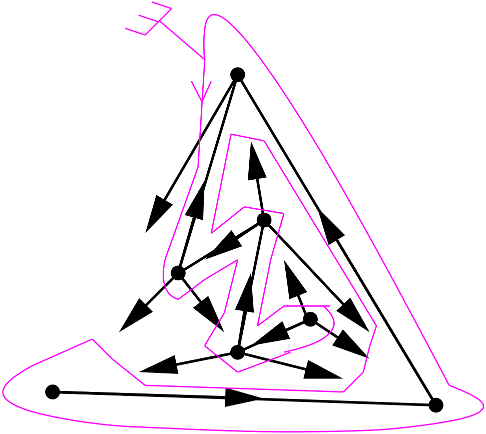

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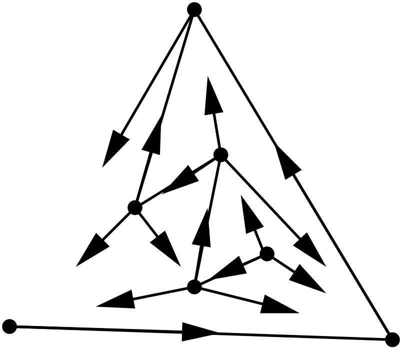

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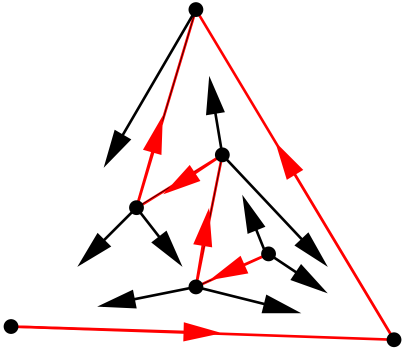

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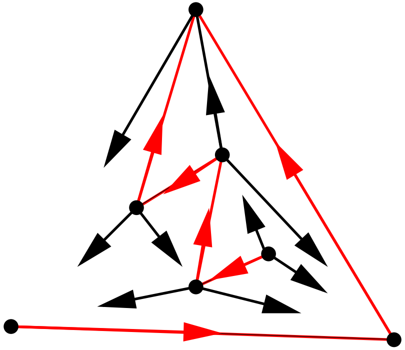

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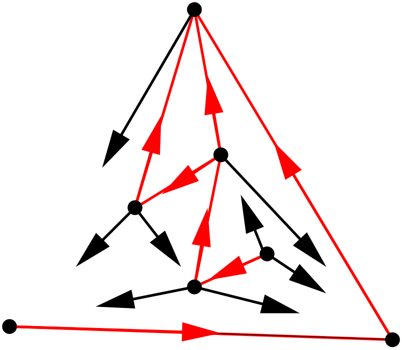

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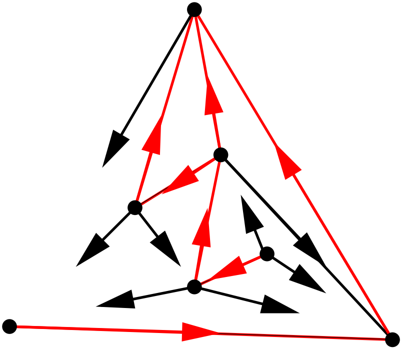

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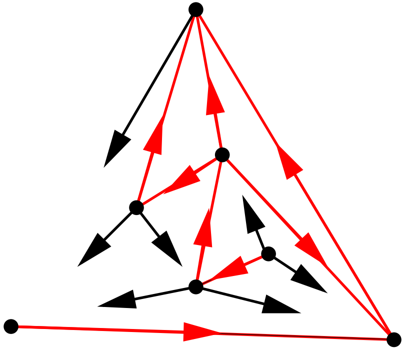

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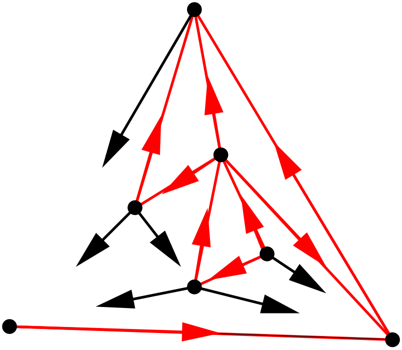

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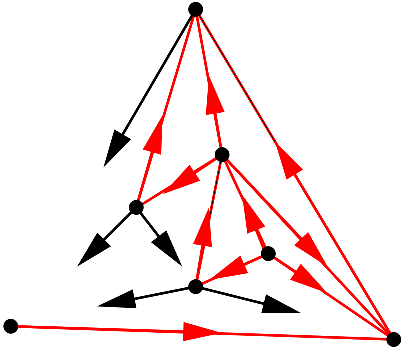

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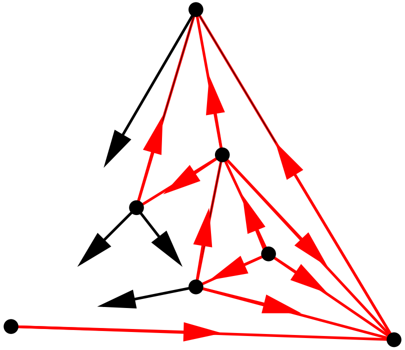

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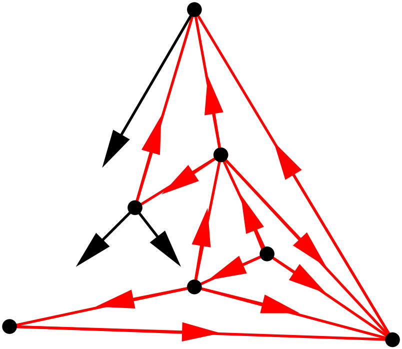

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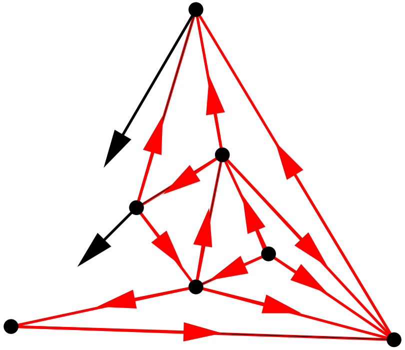

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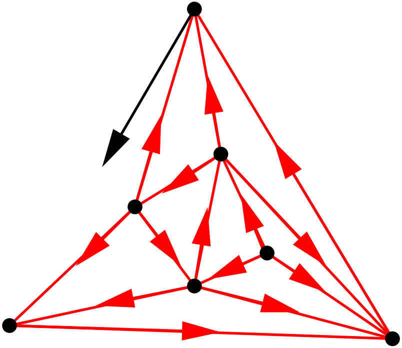

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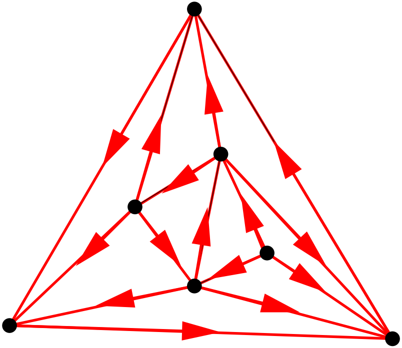

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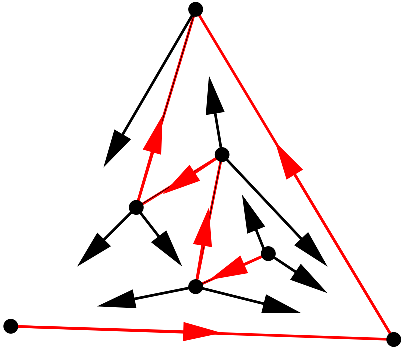

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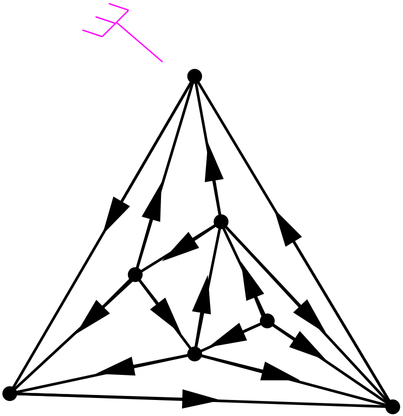

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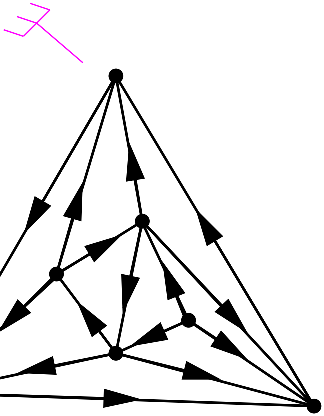

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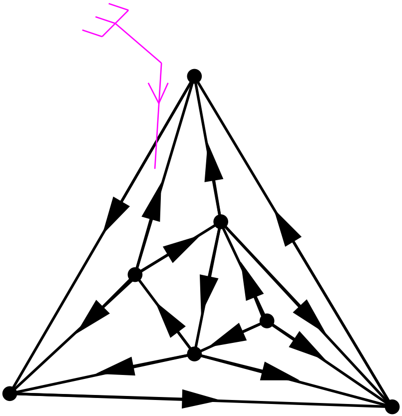

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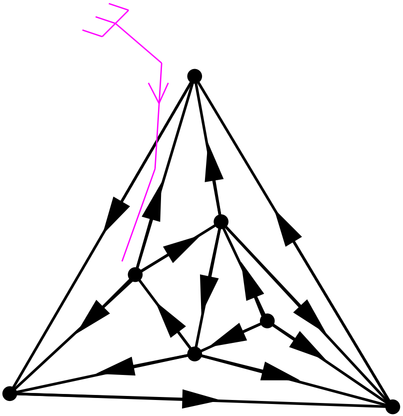

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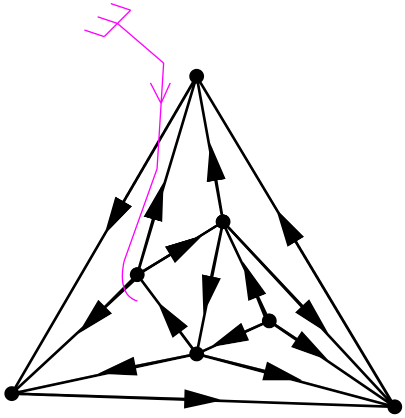

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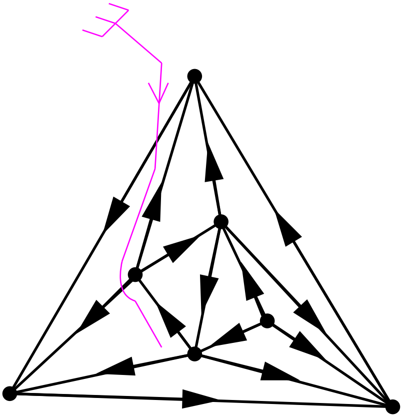

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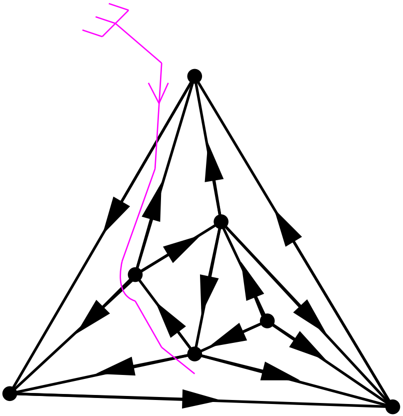

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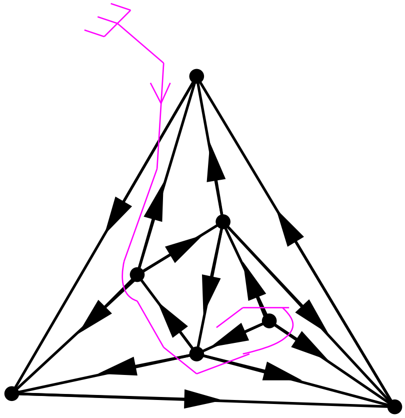

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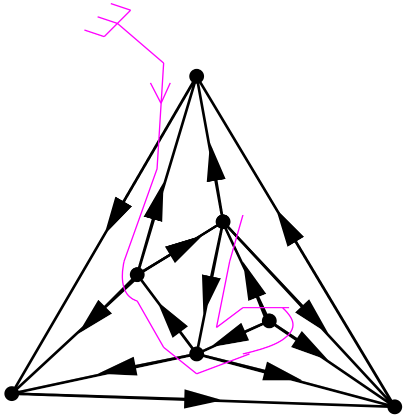

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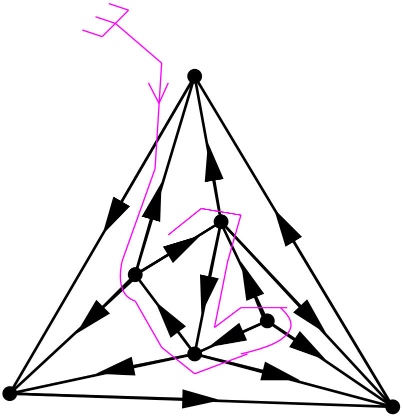
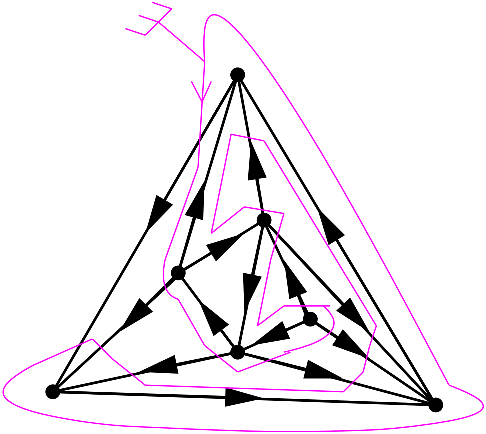


Encoding - Poulalhon, Schaeffer (2003)



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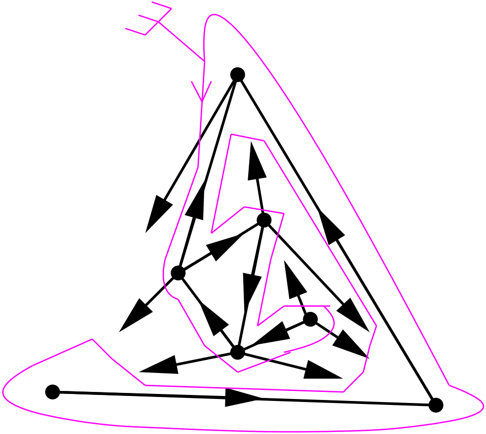

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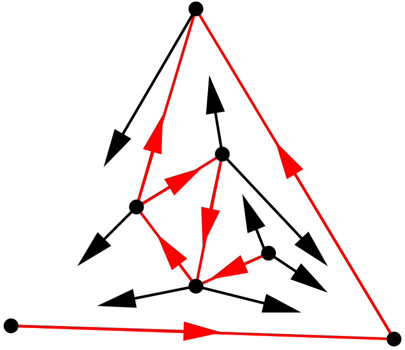
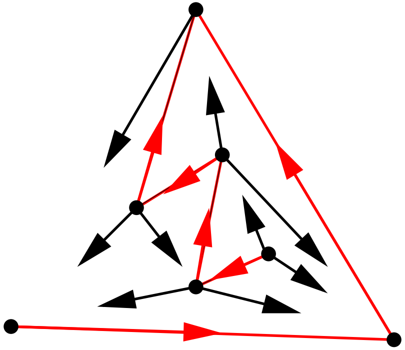
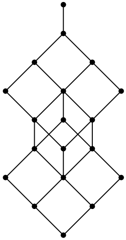


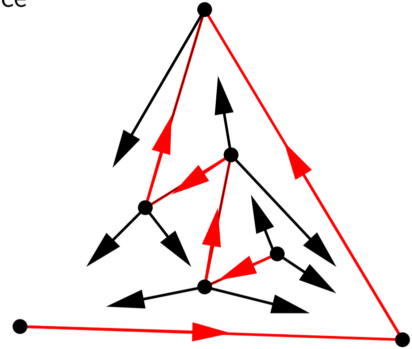
Encoding - Poulalhon, Schaeffer (2003)



Encoding - Poulalhon, Schaeffer (2003)

Distributive lattice





Encoding - Poulalhon, Schaeffer (2003)

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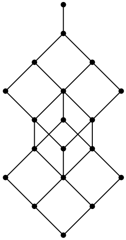

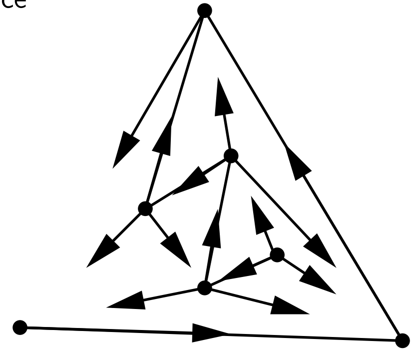

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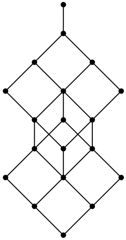
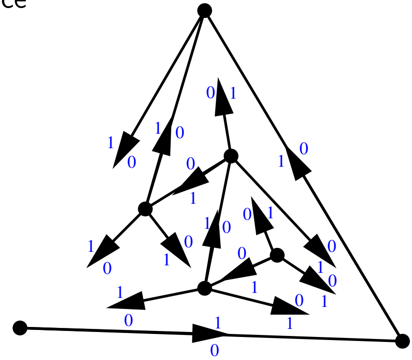
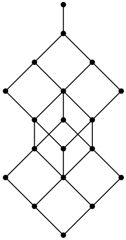


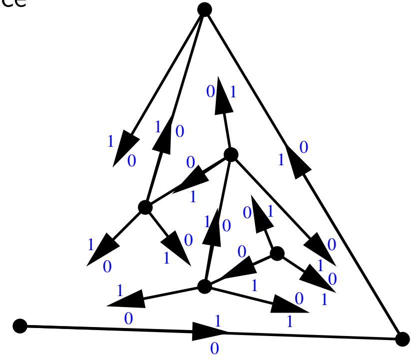


101101011101011010001010001100

Encoding - Poulalhon, Schaeffer (2003)

Distributive lattice





1011010111010110100010100011 00 ⇝ 6n bits

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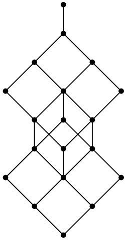
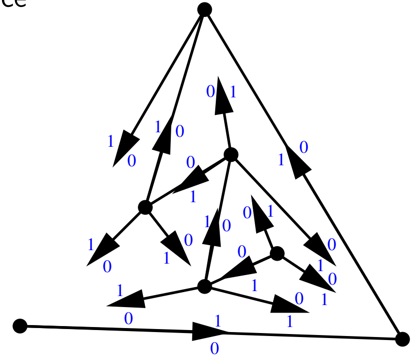


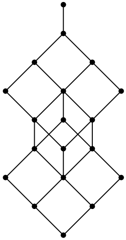


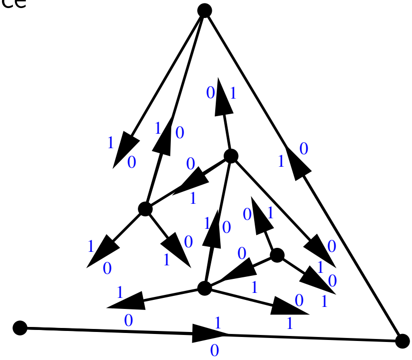
1011010111010110100010100011100 $\rightsquigarrow$ 6n bits

...1w10w10w0...

Encoding - Poulalhon, Schaeffer (2003)

Distributive lattice



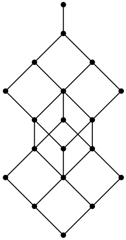


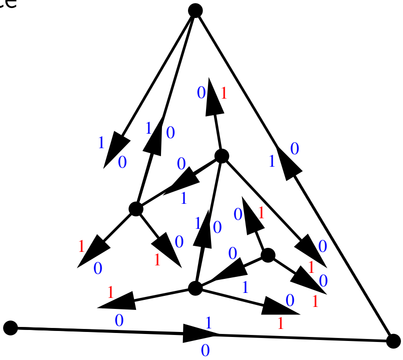
10110101110101101000101000110 0 ⇝ 6n bits

...1w10w10w0...

Encoding - Poulalhon, Schaeffer (2003)

Distributive lattice





1011010111010110100010100011000 $\rightsquigarrow$ 6n bits

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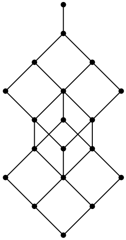

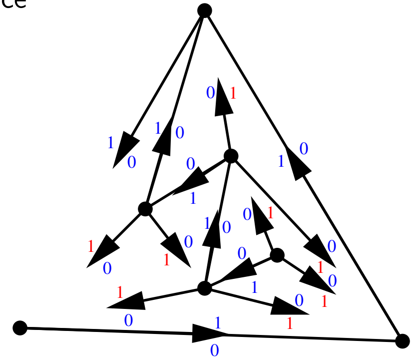

101101011101011010001010001100

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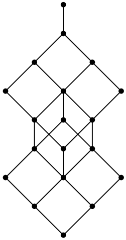
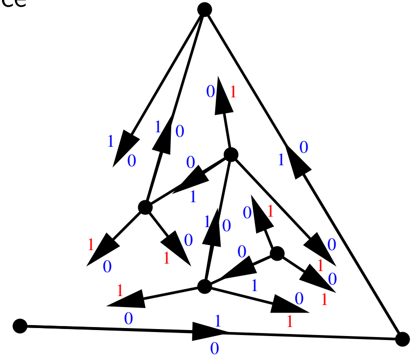


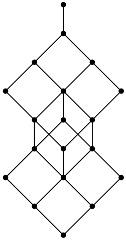


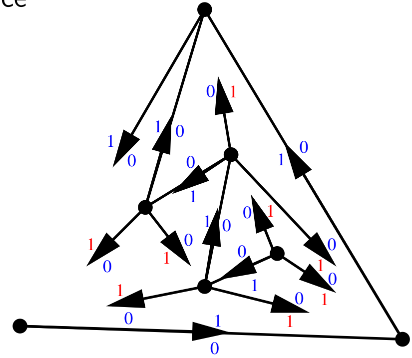
101001100100000001100

...1w10w10w0...

Encoding - Poulalhon, Schaeffer (2003)

Distributive lattice



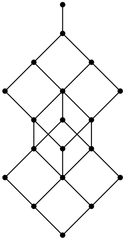


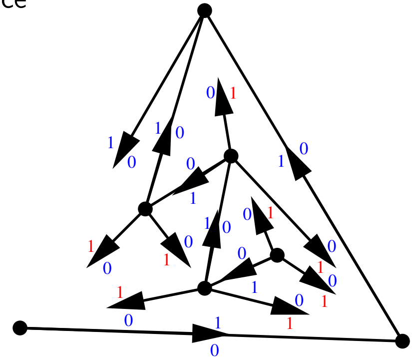
1010011001000000001100 ⇝ 4n bits

...1w10w10w0...

Encoding - Poulalhon, Schaeffer (2003)

Distributive lattice



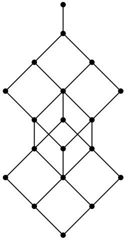


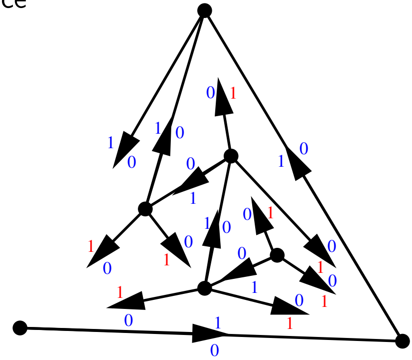
10100110010000000001100 $\rightsquigarrow$ 4n bits (n bits 1)

...1w10w10w0...

Encoding - Poulalhon, Schaeffer (2003)

Distributive lattice



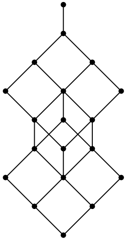


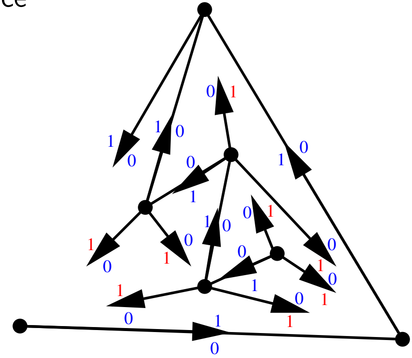
101001100100000001100 ⇝ 4n bits (n bits 1) ⇝ 3,25n bits

...1w10w10w0...

Encoding - Poulalhon, Schaeffer (2003)

Distributive lattice





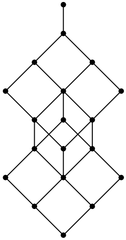
101001100100000001100 ⇝ 4n bits (n bits 1) ⇝ 3,25n bits

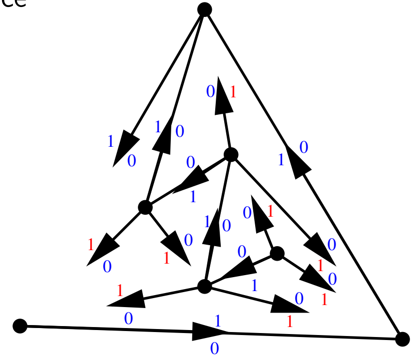
...1w10w10w0...

OPTIMAL !

Encoding - Poulalhon, Schaeffer (2003)

Distributive lattice





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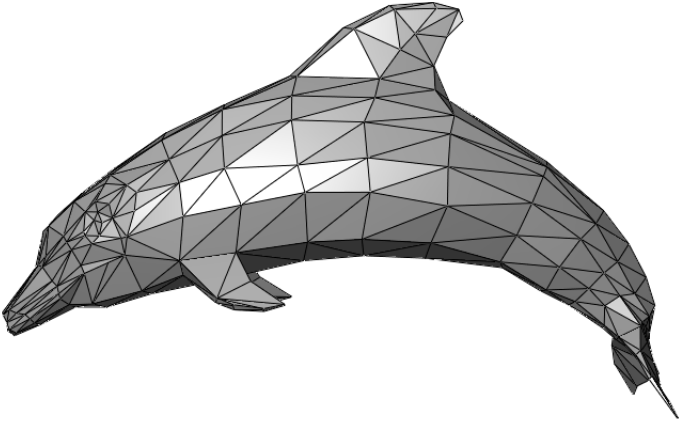
...1w10w10w0...

OPTIMAL !

Also : linear, bijective, counting, sampling

Encoding - Poulalhon, Schaeffer (2003)



10100110010000000001100 ⇝ 4n bits (n bits 1) ⇝ 3,25n bits

...1w10w10w0...

OPTIMAL !

Also : linear, bijective, counting, sampling

What about more complex object ?

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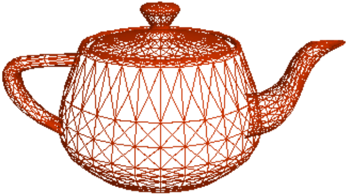

$\rightsquigarrow$ Generalization to higher genus triangulated surfaces

Triangulations on oriented surfaces

Triangulations on oriented surfaces

Euler's formula in genus g : $n - m + f = 2 - 2g$

Triangulations on oriented surfaces

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	Genus	Triangulation
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Triangulations on oriented surfaces

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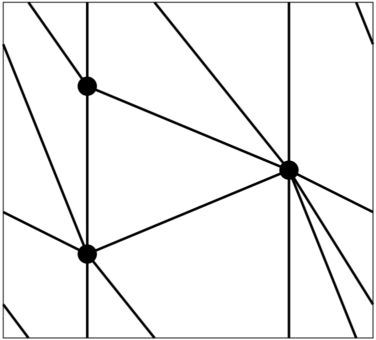

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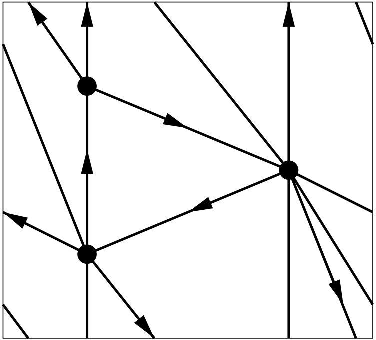

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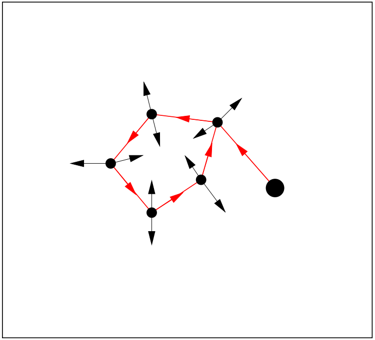

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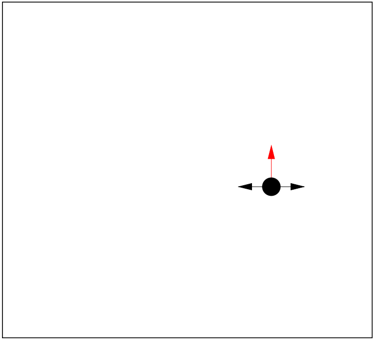

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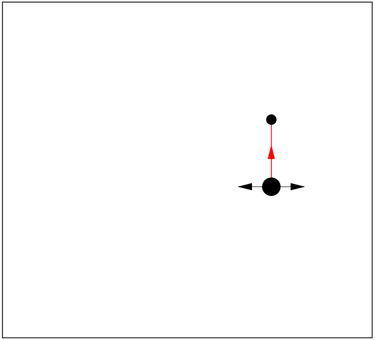

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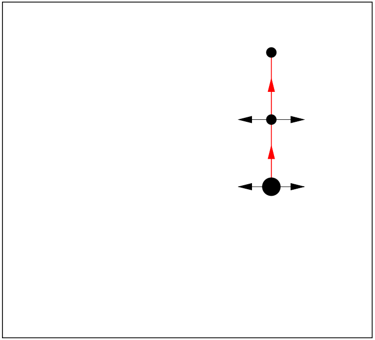

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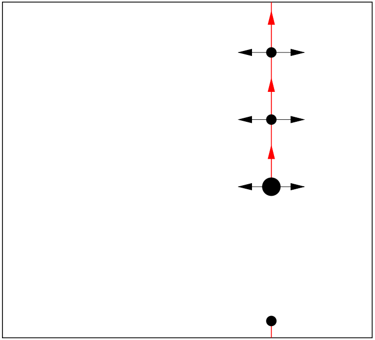

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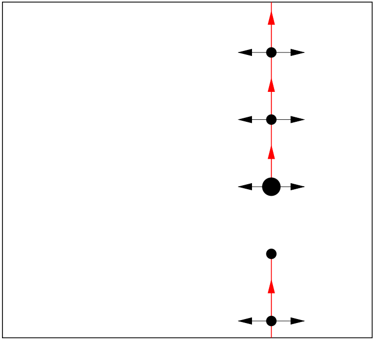

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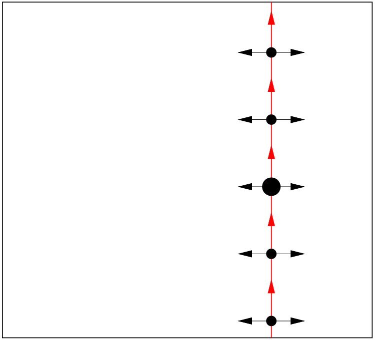

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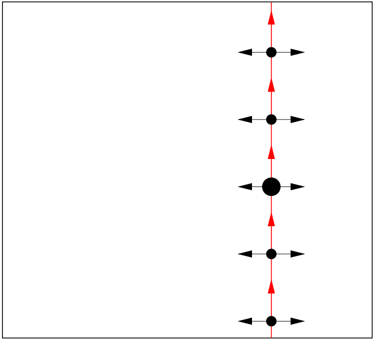

"Middle paths" creates non-contractible cycles

Triangulations on oriented surfaces

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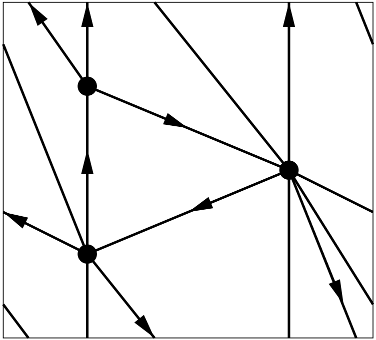

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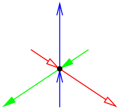

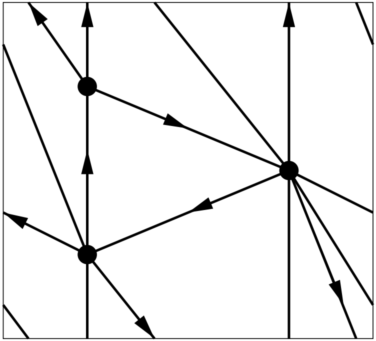

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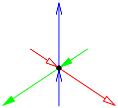

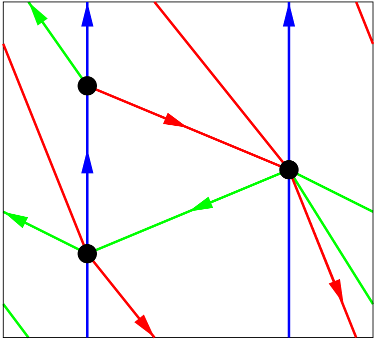

Triangulations on oriented surfaces

Euler's formula in genus g : $n - m + f = 2 - 2g$

	Genus	Triangulation
Plane	0	$m = 3n - 6$
Torus	1	$m = 3n$
Double torus	2	$m = 3n + 6$

Triangulations on oriented surfaces

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	Genus	Triangulation
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...	g	$m = 3n + 6(g - 1)$

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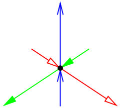

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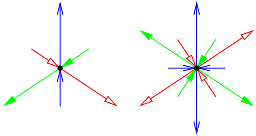

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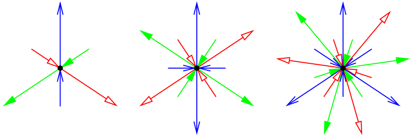

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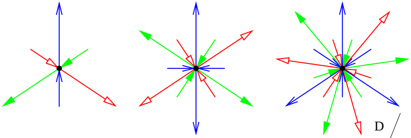

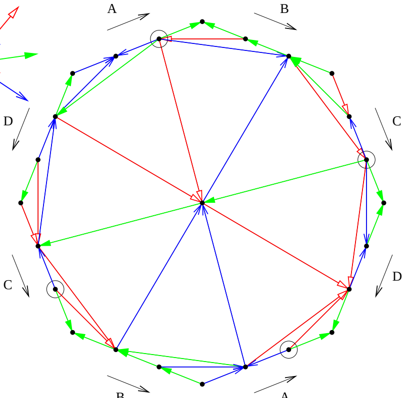

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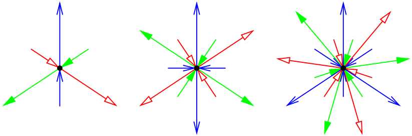

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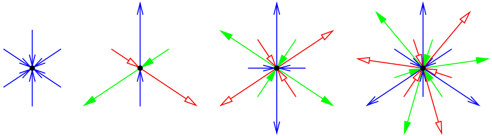

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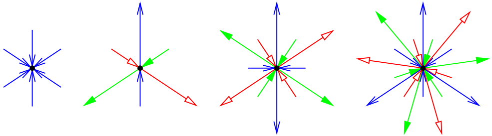

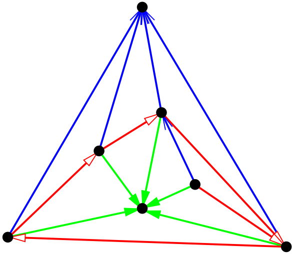

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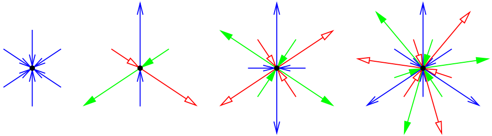

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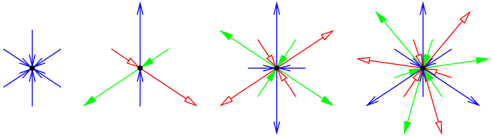

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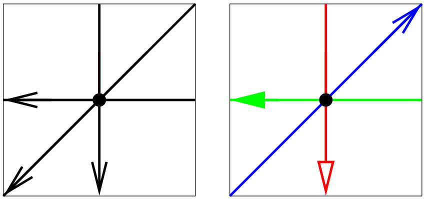

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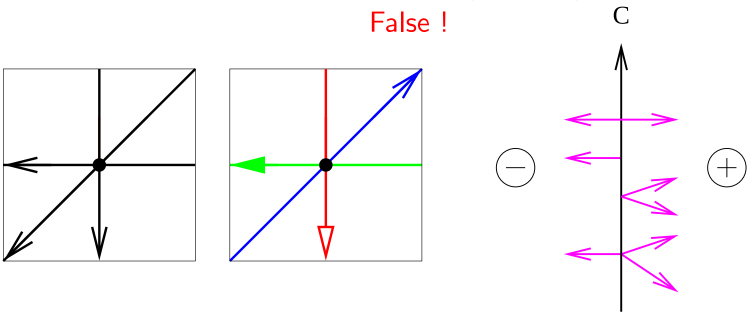

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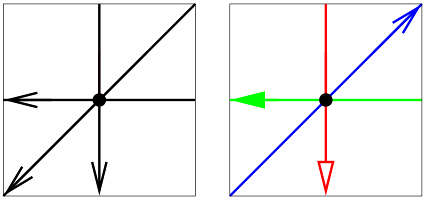

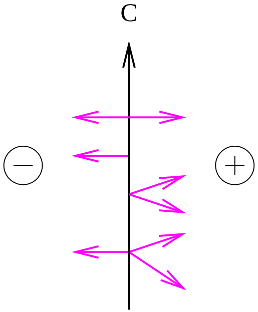

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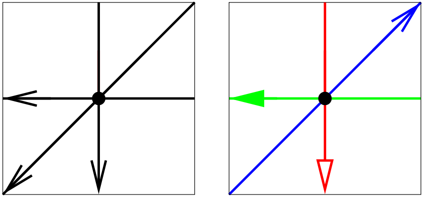

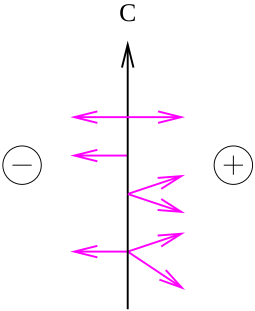

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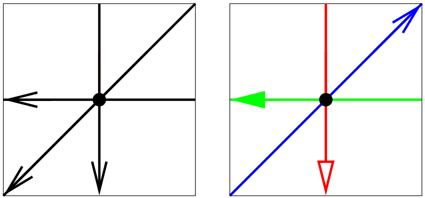

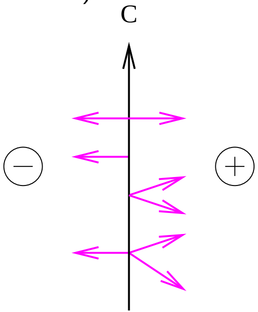

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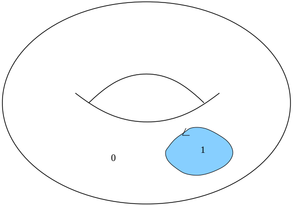

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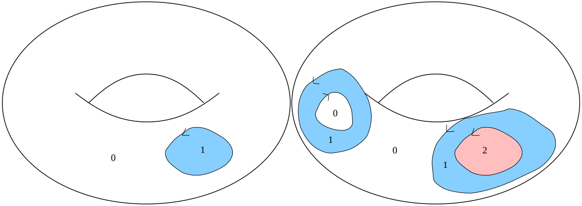

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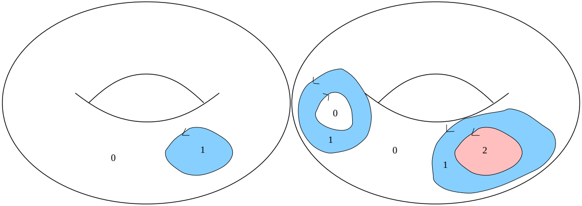

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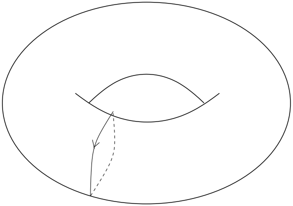

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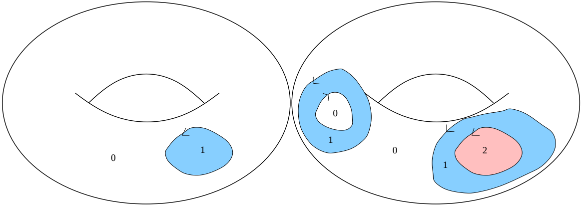

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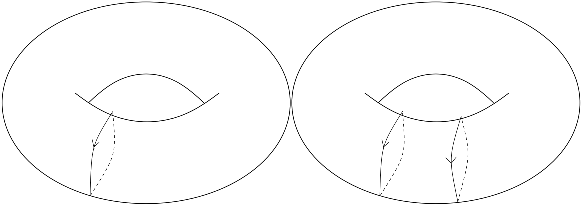

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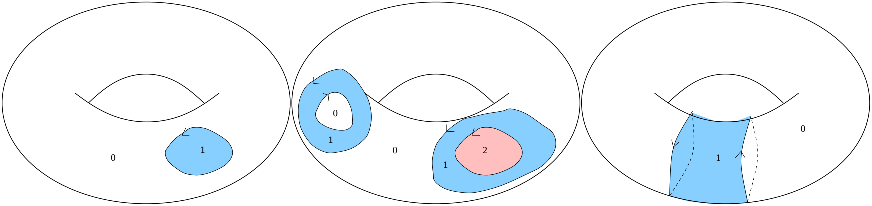

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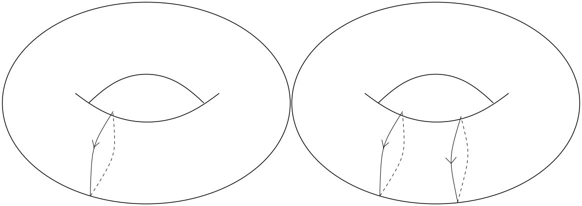

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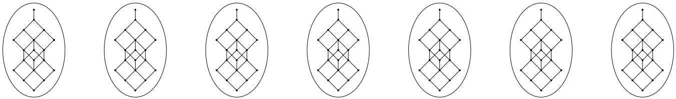

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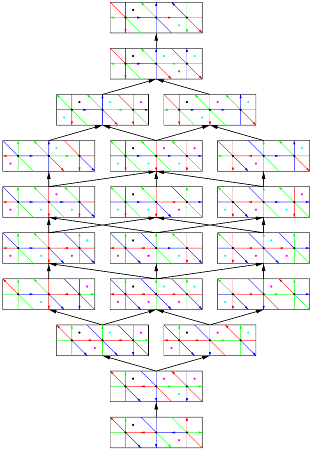

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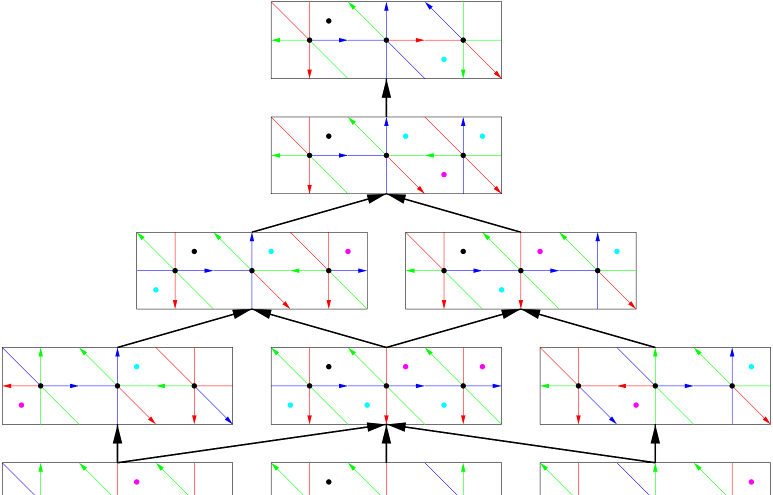
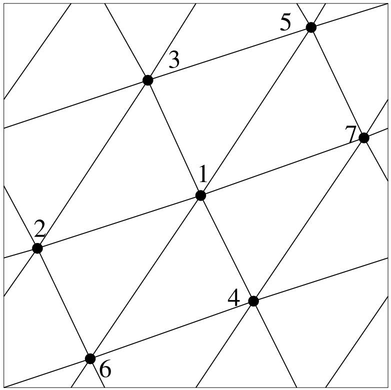


Generalization of Poulalhon-Schaeffer to the torus



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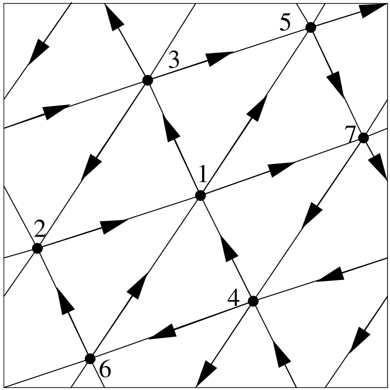

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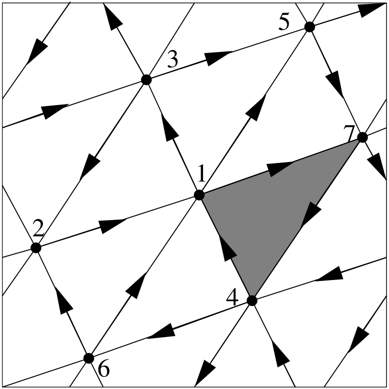

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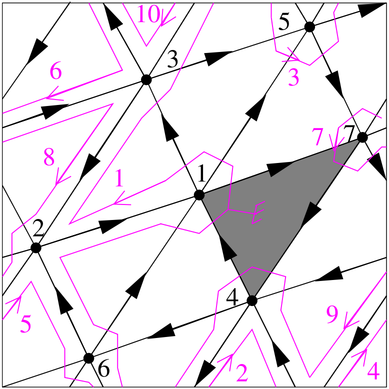

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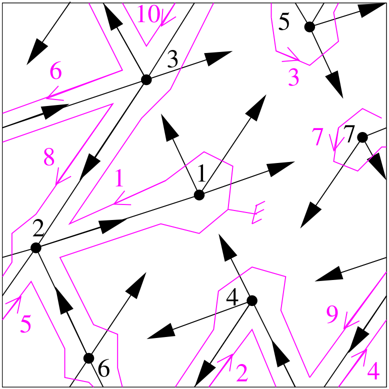

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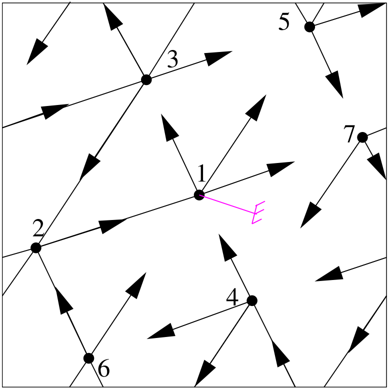

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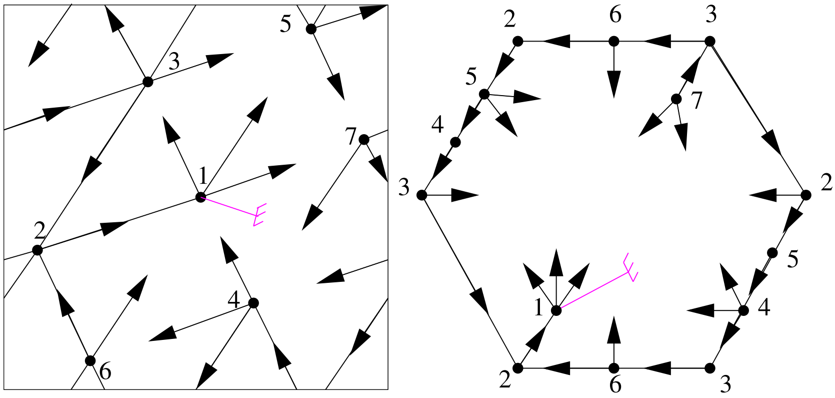

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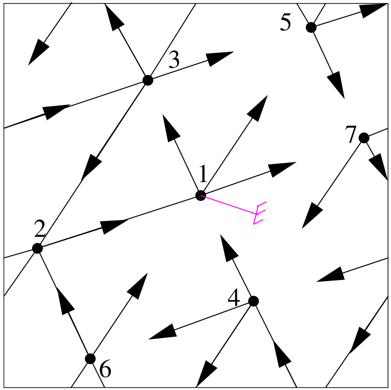

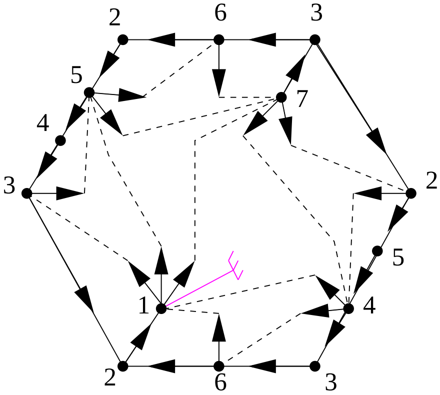

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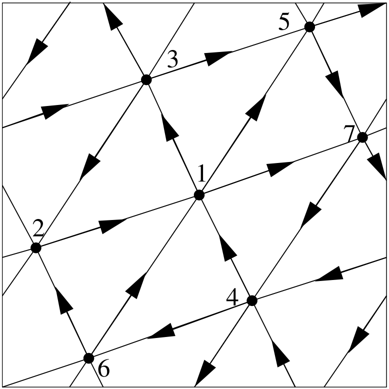

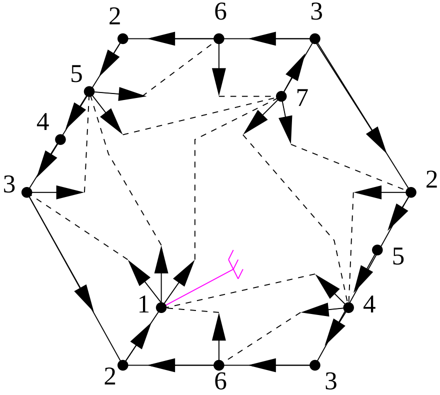

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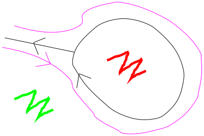

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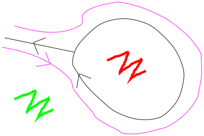

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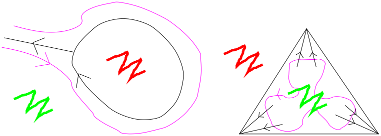

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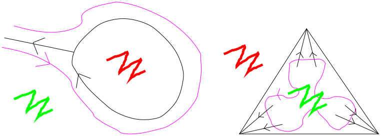

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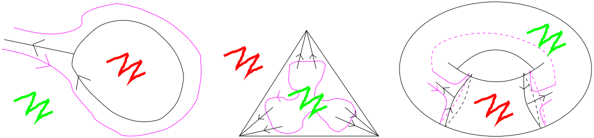

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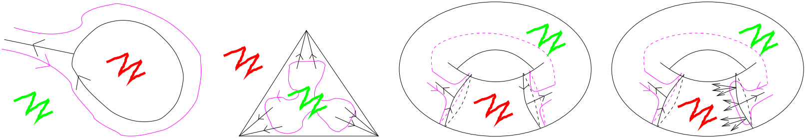

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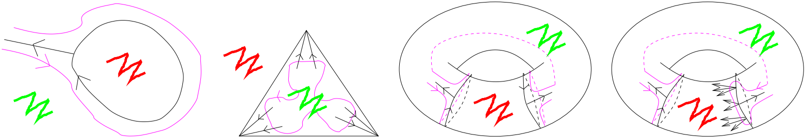

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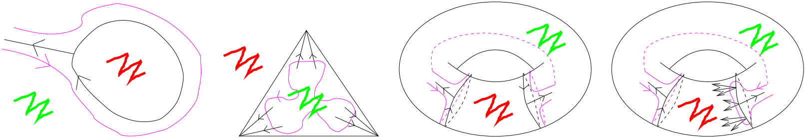

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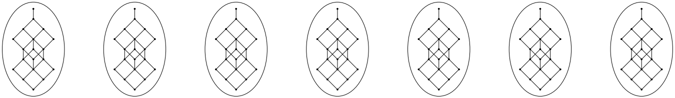

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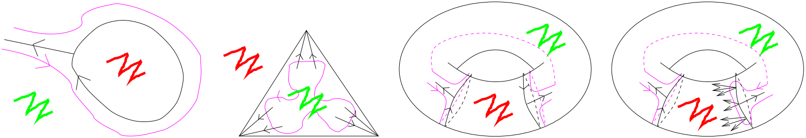

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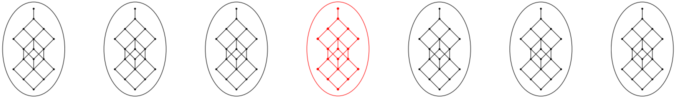

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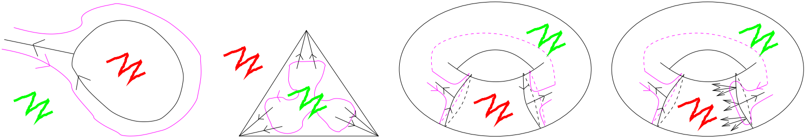

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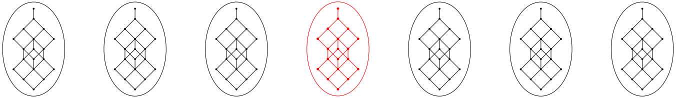

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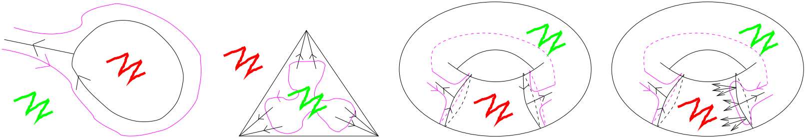

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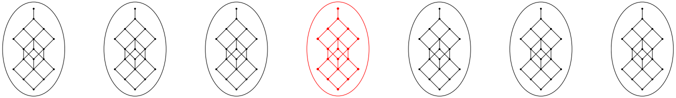
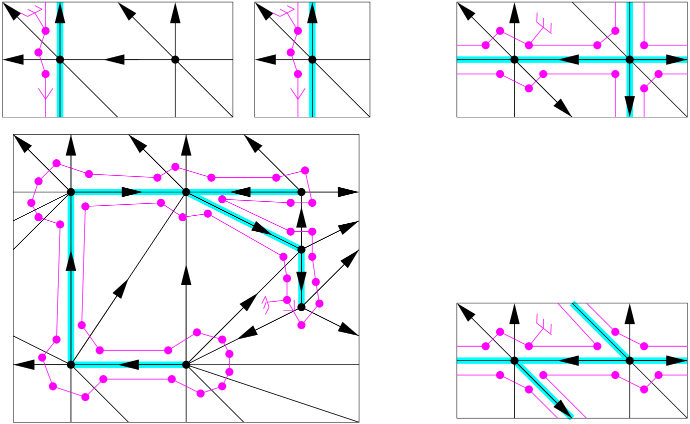
- ⇝ Orientation with no oriented non-contractible cycle in the dual

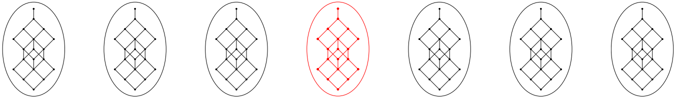


$\gamma = 0$ for any non-contractible cycle

Theorem Despré, Gonçalves, Lévêque (2015)
Applied on the minimal γ_0-Schnyder wood, Poulalhon-Schaeffer algorithm outputs a toroidal spanning unicellular map.

Examples





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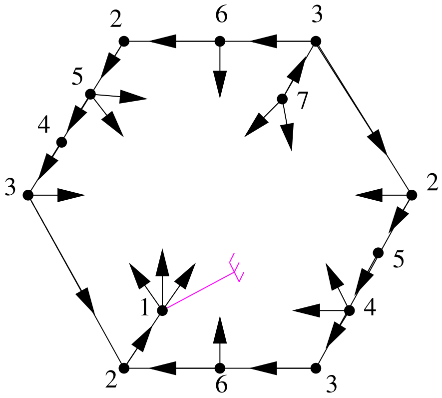

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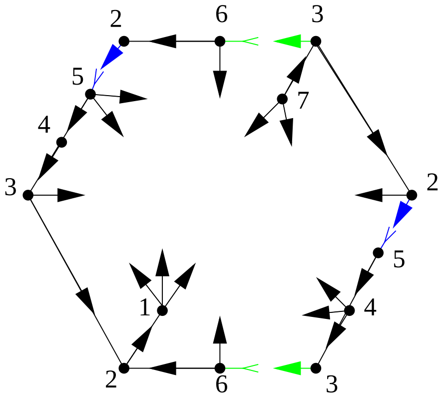

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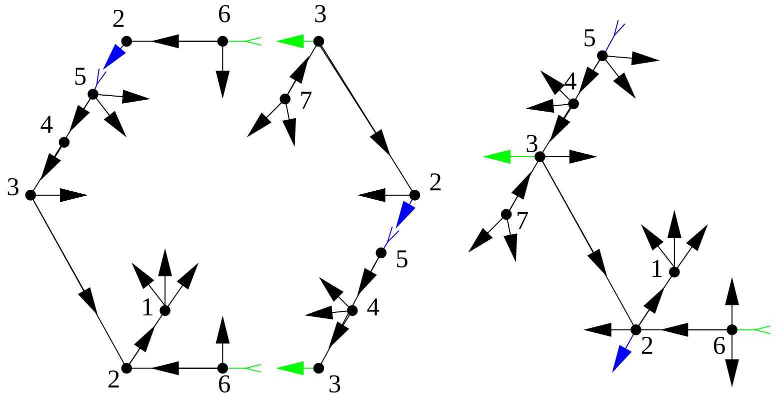

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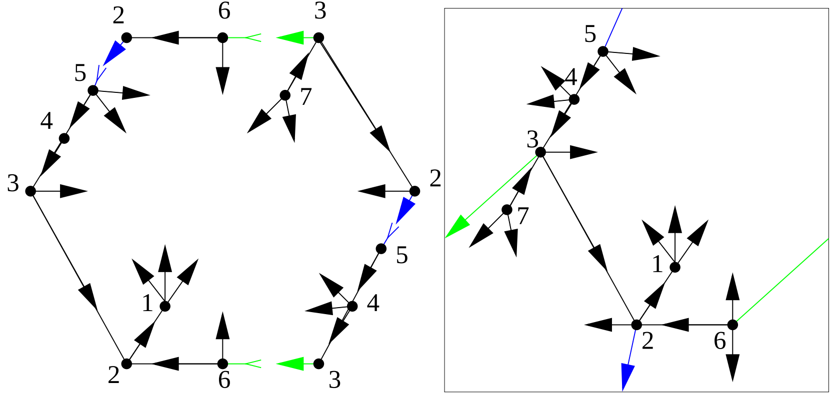

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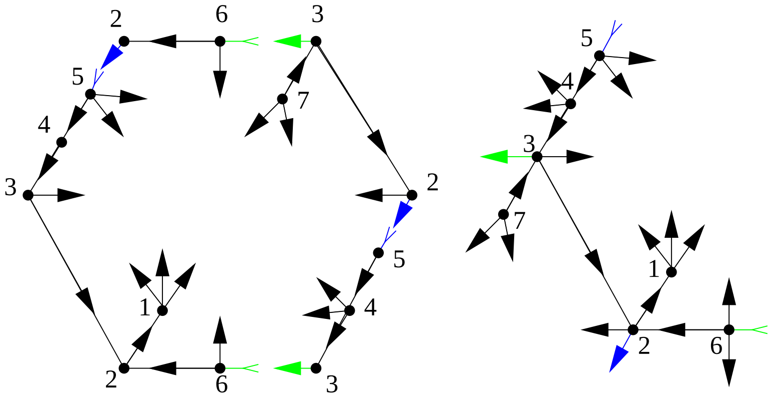

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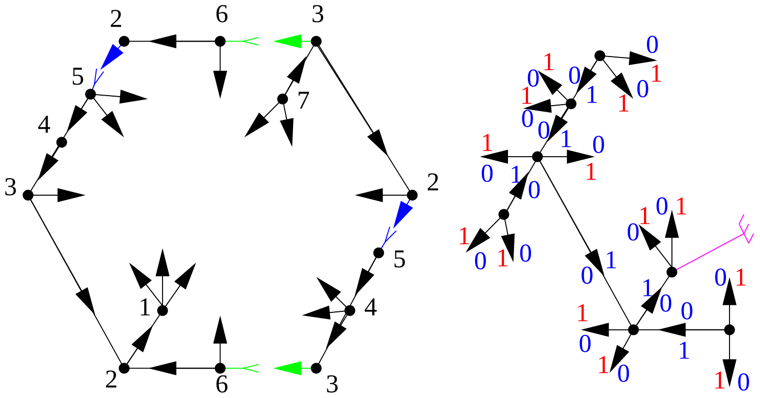
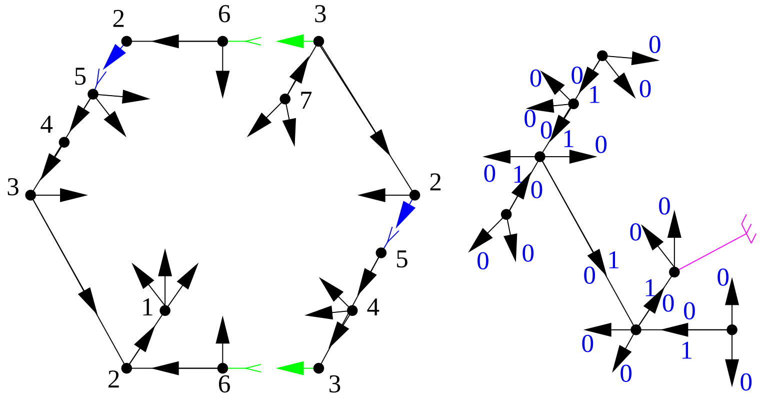


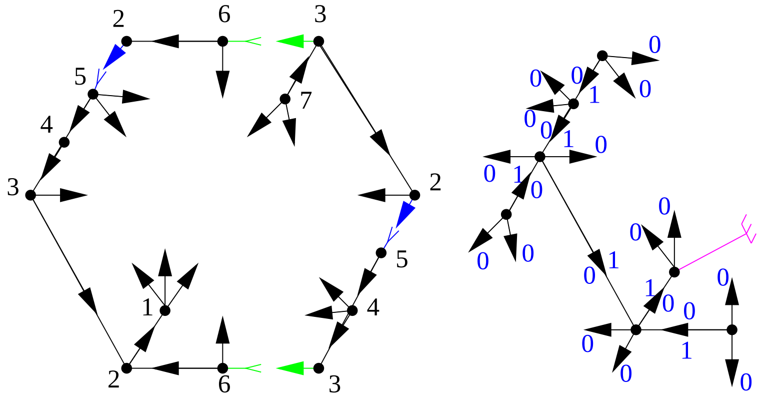
10101110111010010100101101000101011101000 ⇝ 6n bits

Encoding



0011011000000001000000010000⇝ 4n bits (n bits 1) ⇝ 3,25n bits

Encoding



001101100000000100000010000 ⇝ 4n bits (n bits 1) ⇝ 3,25n bits

Optimal, linear and bijective !

To do

- Counting, sampling, etc.

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- Bijections for other toroidal maps : d-angulations, 3-connected maps, 4-connected triangulations, etc.

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- Higher genus : What is the generalization of γ_0 property ?

Conjecture
Triangulation on a surface $g \geq 1 \implies$
orientation of the edges such that $d^+(v) = 0 \mod 3$, $d^+(v) > 0$
and no oriented non-contractible cycle in the dual.