Translation Control of a Fleet Circular Formation of AUVs under Finite Communication Range

CAR - 2011

Lara Briñón Arranz*,
Alexandre Seuret** and Carlos Canudas de Wit**

NeCS Team
*INRIA Rhône-Alpes
**CNRS - GIPSA-lab Automatic Department

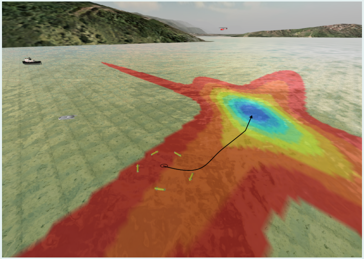
24th May 2011

Introduction

Main Objective

Coordinated Control of a Fleet Formation of AUVs under communication constraints to achieve a source seeking.

Projects: CONNECT and FeedNetBack in collaboration with IFREMER



Hostile Environment

- Deep-sea
- Underwater Communication

Needs

- Collaborative control under limited communication for AUVs formation motions

Previous Works

- Collaborative Control
  [Olfati-Saber et al. 2007], [Murray et al. 2004]

- Multi-agents Systems
  [Leonard et al. 2007], [Ögren et al. 2002]

- Formation Control
  [Sepulchre et al. 2007], [Marshall et al. 2004]

Previous Works

- Collaborative Control
  [Olfati-Saber et al. 2007], [Murray et al. 2004]

- Multi-agents Systems
  [Leonard et al. 2007], [Ögren et al. 2002]

- Formation Control
  [Sepulchre et al. 2007], [Marshall et al. 2004]

Contributions of this work

Redesign of the work [Leonard et al. 2007] to achieve tracking time-varying center of a multi-agents circle formation with uniform distribution.

Simulation

(Please wait while hopefully loading movie. . .)

AUV's Kinematic Model

Vehicle's model

Consider a set of N agents (vehicles), in which each agent $k = 1, \ldots, N$ has the following constrained dynamics:

$$\begin{aligned} \dot{\mathbf{r}}_k &= v_k e^{i\theta_k} \\ \dot{\theta}_k &= u_k \end{aligned}$$

where $\mathbf{r}_k$ is the position vector, θ_k the heading angle and v_k, u_k are the control inputs.

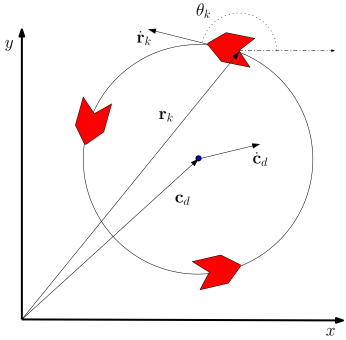


Figure: Problem formulation.

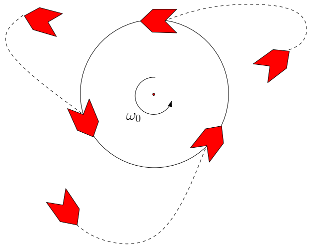
Preliminaries: Formation control for a fixed center

Previous works: *Leonard et al. 2007*

The control law:

$$\begin{aligned} v_k &= v_0 \\ u_k &= \omega_0(1 + \kappa\langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\theta_k}\rangle) \end{aligned}$$

where $v_0 = constant$, ensures the circular formation with a **fixed center** $\mathbf{c}_d$ and a **constant radius** is achieved.



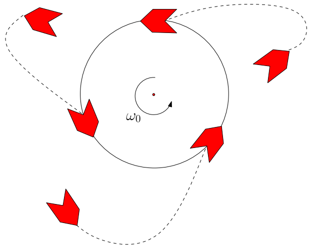
Preliminaries: Formation control for a fixed center

Previous works: *Leonard et al. 2007*

The control law:

$$
\begin{aligned}
v_k &= v_0 \\
u_k &= \omega_0(1 + \kappa\langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\theta_k}\rangle)
\end{aligned}
$$

where $v_0 = constant$, ensures the circular formation with a **fixed center $\mathbf{c}_d$** and a **constant radius** is achieved.

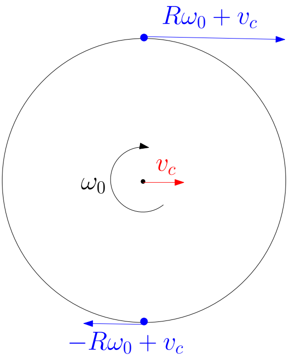


Our contributions

- **Translation of the circular formation**
- Contraction of the circular formation

Limitations of the previous theorem

The choice $v_k = v_0$ is not realistic for the translation of a circular formation.



Translation Control Design: Main Idea

Change of Variables

We introduce two new variables, ψ_k and the constant $v_0 > 0$. The change of variables shown in the figure corresponds to:

$$\tilde{\mathbf{r}}_k = \mathbf{r}_k - \mathbf{c}_d$$

$$\rightarrow \dot{\tilde{\mathbf{r}}}_k = \dot{\mathbf{r}}_k - \dot{\mathbf{c}}_d = v_0 e^{i\psi_k}$$

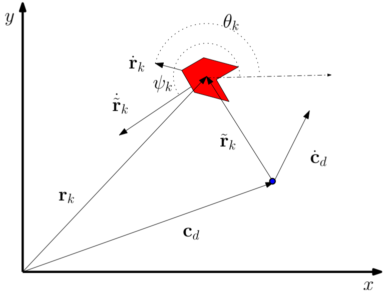


Figure: Change of variables.

Theorem: Translation Control

Leonard et al. 2007:

Formation Control law

$$
\begin{aligned}
v_k &= v_0 \\
u_k &= \omega_0 (1 + \kappa \langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\theta_k} \rangle)
\end{aligned}
$$

Translation Control law

$$
\begin{aligned}
v_k &= |v_0 e^{i\psi_k} - \dot{\mathbf{c}}_d| \\
u_k &= \left(1 - \frac{\langle \dot{\mathbf{r}}_k, \dot{\mathbf{c}}_d \rangle}{v_k^2}\right) \dot{\psi}_k - \frac{\langle \dot{\mathbf{r}}_k, i\ddot{\mathbf{c}}_d \rangle}{v_k^2} \\
\dot{\psi}_k &= \omega_0 (1 + \kappa \langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\psi_k} \rangle)
\end{aligned}
$$

with

$$
v_0 > \sup_{t \geq 0} \{ |\dot{\mathbf{c}}_d(t)| \}
$$

Theorem: Translation Control

Leonard et al. 2007:

Formation Control law

$$\begin{aligned} v_k &= v_0 \\ u_k &= \omega_0(1 + \kappa\langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\theta_k}\rangle) \end{aligned}$$

Translation Control law

$$\begin{aligned} v_k &= |v_0 e^{i\psi_k} - \dot{\mathbf{c}}_d| \\ u_k &= \left(1 - \frac{\langle \dot{\mathbf{r}}_k, \dot{\mathbf{c}}_d\rangle}{v_k^2}\right)\dot{\psi}_k - \frac{\langle \dot{\mathbf{r}}_k, i\ddot{\mathbf{c}}_d\rangle}{v_k^2} \\ \dot{\psi}_k &= \omega_0(1 + \kappa\langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\psi_k}\rangle) \end{aligned}$$

with

$$v_0 > \sup_{t\geq 0}\{|\dot{\mathbf{c}}_d(t)|\}$$

If

$$\dot{\mathbf{c}}_d = \ddot{\mathbf{c}}_d = 0 \quad \Rightarrow \quad \begin{aligned} v_k &= v_0 \\ u_k &= \dot{\psi}_k \end{aligned} \qquad \dot{\psi}_k = \omega_0(1+\kappa\langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\psi_k}\rangle)$$

Proof

ORIGINAL MODEL

$$\dot{r}_k = v_k e^{i\theta_k}$$

$$u_k = \dot{\theta}_k$$

Transformation L →

NEW MODEL

$$\tilde{r}_k = r_k - c_d$$

$$\dot{\tilde{r}}_k = \tilde{v}_k e^{i\psi_k}$$

$$\tilde{u}_k = \dot{\psi}_k$$

↓ *Leonard et al. 2007*

NEW CONTROL

Circular formation control law for $\tilde{r}_k$'s

$$\tilde{v}_k = v_0$$

$$\dot{\psi}_k = w_0(\kappa + < \tilde{r}_k, \dot{\tilde{r}}_k >)$$

← L^{-1} Inverse Transformation

CONTROL

$$v_k = |\dot{\tilde{r}}_k + \dot{c}_d|$$

$$\tan\theta_k = \frac{\langle \dot{r}_k, i\rangle}{\langle \dot{r}_k, 1\rangle}$$

$$\dot{\theta}_k = \frac{\langle \ddot{r}_k, i\dot{r}_k\rangle}{\langle \dot{r}_k, \dot{r}_k\rangle}$$

Proof: based on *Leonard et al. 2007*

The convergence to the formation is analyzed with the Lyapunov function:

$$S(\tilde{\mathbf{r}}, \psi) = \frac{1}{2} \sum_{k=1}^{N} |\dot{\tilde{\mathbf{r}}}_k - i\omega_0 \tilde{\mathbf{r}}_k|^2$$

The differentiation of $S(\tilde{\mathbf{r}}, \psi)$ leads to:

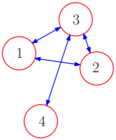
$$\dot{S}(\tilde{\mathbf{r}}, \psi) = \sum_{k=1}^{N} \langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\psi_k} \rangle (\omega_0 - \dot{\psi}_k)$$

Using the previous equation for $\dot{\psi}_k$, the derivative of the Lyapunov function thus satisfies $\dot{S}(\tilde{\mathbf{r}}, \psi) \leq 0$.

Cooperative Control Design: Communication Graph

Laplacian matrix **L** for a communication Graph

$$L_{k,j} = \begin{cases} d_k, & \text{if} \quad k = j \\ -1, & \text{if} \quad (k,j) \quad \text{are neigbors} \\ 0 & \text{otherwise} \end{cases}$$

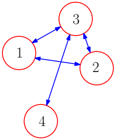


$$L = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

Cooperative Control Design: Communication Graph

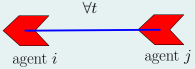
Laplacian matrix **L** for a communication Graph

$$L_{k,j} = \begin{cases} d_k, & \text{if} \quad k = j \\ -1, & \text{if} \quad (k,j) \quad \text{are neigbors} \\ 0 & \text{otherwise} \end{cases}$$

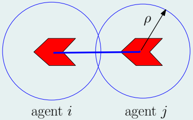


$$L = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

Fixed Graph



Time-varying Graph



Theorem: Translation Control with Uniform Distribution

Uniform Distribution Control Law

$$\dot{\psi}_k = \omega_0(1 + \kappa\langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\psi_k}\rangle) - \frac{\partial U}{\partial \psi_k}$$

where

$$U(\psi) = -\frac{K}{N}\sum_{m=1}^{[N/2]} \frac{1}{2m^2}\langle e^{im\psi}, \mathbf{L}e^{im\psi}\rangle$$

Theorem: Translation Control with Uniform Distribution

Uniform Distribution Control Law

$$\dot{\psi}_k = \omega_0(1 + \kappa\langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\psi_k}\rangle) - \frac{\partial U}{\partial \psi_k}$$

where

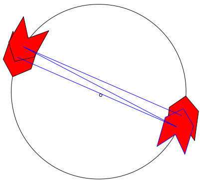
$$U(\psi) = -\frac{K}{N}\sum_{m=1}^{[N/2]} \frac{1}{2m^2}\langle e^{im\psi}, \mathbf{L} e^{im\psi}\rangle$$

- **Fixed Graph**

  Complet Graph $\Rightarrow$ Uniform Distribution

- **Time-varying Graph**

  $\rho > R\sin\frac{\pi}{N} \Rightarrow$ Uniform Distribution



Theorem: Translation Control with Uniform Distribution

Uniform Distribution Control Law

$$\dot{\psi}_k = \omega_0(1 + \kappa\langle \mathbf{r}_k - \mathbf{c}_d, v_0 e^{i\psi_k}\rangle) - \frac{\partial U}{\partial \psi_k}$$

where

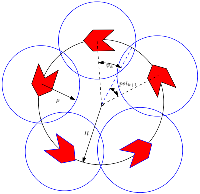
$$U(\psi) = -\frac{K}{N}\sum_{m=1}^{[N/2]} \frac{1}{2m^2}\langle e^{im\psi}, \mathbf{L} e^{im\psi}\rangle$$

- **Fixed Graph**

  Complet Graph $\Rightarrow$ Uniform Distribution

- **Time-varying Graph**

  $\rho > R \sin \frac{\pi}{N} \Rightarrow$ Uniform Distribution



Conclusions

Transformations of the circle formation

- **Translation Control:** stabilizes the agents to a circular formation tracking a time-varying center of the circle.
- Extension to Contraction Control and to Combined Motion Control
- Extension to more complex formations, not only circular

Conclusions

Transformations of the circle formation

- **Translation Control:** stabilizes the agents to a circular formation tracking a time-varying center of the circle.
- Extension to Contraction Control and to Combined Motion Control
- Extension to more complex formations, not only circular

Cooperative Control

- **Uniform distribution:** achieves the uniform distribution of the agents along the moving circular formation.

Conclusions

Transformations of the circle formation

- **Translation Control:** stabilizes the agents to a circular formation tracking a time-varying center of the circle.
- Extension to Contraction Control and to Combined Motion Control
- Extension to more complex formations, not only circular

Cooperative Control

- **Uniform distribution:** achieves the uniform distribution of the agents along the moving circular formation.

Work under progress

- $\mathbf{c}_d(t)$ and $R_d(t)$ obtained by agreement of all the agents
- Application to Gradient Search

Thank you for your attention