Translation Control of a Fleet Circular Formation of Vehicles under Communication Constraints

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Abstract

This work proposes a control algorithm to stabilize a circular formation of vehicles tracking a time-varying center. We also consider the problem of uniform distribution of all the agents along the circle from two approaches: all-to-all and limited communication. We tackle with this communication constraint using a cooperative control strategy which includes the Laplacian matrix of the communication graph (fixed or distance-dependent). The system was implemented in computer simulation, accessible though Web[1].

1 Introduction

This paper treats the problem of formation translation in multi-agent control under limited communication range. In particular, we propose an extension of the control proposed in [1, 2] to the case where the center of the cycle formation is time-varying. This is studied under different set-ups: all-to-all, fixed, and range-dependent communication graphs.

This problem is pertinent to some applications where the agents should perform collaborative tasks requiring the formation to displace towards an a priori unknown direction [3]. For instance, in source seeking applications, the formation is displaced in the source gradient direction (which is computed on-line, and instrumented as an additional outer loop) [1]. Translations of the formation can be seen here as a first step toward more general formation transformations including rotation and scaling.

Formation control has been extensively studied in [1, 2, 4, 5, 6, 7, 8, 9] among many others. These studies concern circular and parallel formations [1, 2, 7, 8], but also motions of formation induced by flocking [10, 11]. One strategy to produce formation motions (i.e. flight formations) is the virtual-leader approach [4], where an agent is designed as being the leader, and then a suitable inter-distance (and orientation) is set between agents. The motion of the formation results from the motions of the leader. Although it is possible to create circular formations via a particular pursuit graph as suggested in the work [5], it seams less appropriate to apply these methods to the problem of a moving circular formation if the formation is desired to be keep "rigid" while moving.

Moreover, in the context of the source seeking problem of underwater vehicles advocated here, it is necessary to keep the AUVs (autonomous underwater vehicles) in an uniformly distributed formation during the source search to avoid unnecessary energy waist, and to produce efficient search motions. To the knowledge of the authors, the design of a control law to keep a rigid circular formation around a time-varying (almost arbitrarily) center, has not bee addressed so far.

Another difficulty in the underwater fleet formation problem is due to the transmission of information in a marine media. Communication between agents is confronted with several

[1] Animated simulations are accessibles in the CONNECT project web at:
http://www.lag.ensieg.inpg.fr/connect/

difficulties such as signal distorsion and interference, doppler effect, etc. Communication in shallow water amplifies these limitations in particular. In this work, we assume that the communication between each agent is "good enough" within a particular range specific to the application. Therefore, the previously described moving circular formation control will be studied under such a limited-range communication assumptions, where the communication graph depends the agent location [10, 12, 13].

In the present work we first show that tracking a moving circle is not possible with constant linear velocities. We thus relax this assumption by using one additional control input, and we show that after a suitable change of coordinates, this problem can be solved by a new feedback law yielding global asymptotic stability. We show that the stability conditions not only hold for all-to-all communication but also for fixed limited communication graph yielding uniformly distributed formation. We also devise a control law for the case of range-dependent graph, and provide some simulation showing the asymptotic convergence.

Notation. A complex number $\mathbf{z}$ is written in boldface and is expressed as $\mathbf{z} = x_z + iy_z$ where $i^2 = -1$ and where $x_z = Re\{z\}$ and $y_z = Im\{z\}$ correspond to the real and the imaginary part of $\mathbf{z}$. For compactness in the notation, we use the following operator $\langle \mathbf{z}_1, \mathbf{z}_2 \rangle = Re\{\overline{\mathbf{z}}_1^T \mathbf{z}_2\}$ where $\overline{\mathbf{z}}_1^T$ represents the conjugate transpose of $\mathbf{z}_1$. Note that the real part (respectively the imaginary part) of a complex number $\mathbf{z}$ can be written as $\langle \mathbf{z}, 1 \rangle$ (and respectively $\langle \mathbf{z}, i \rangle$). Thus, for any complex numbers $\mathbf{z}_1$ and $\mathbf{z}_2$, the equality $\langle \mathbf{z}_1, i \rangle \langle \mathbf{z}_2, 1 \rangle - \langle \mathbf{z}_1, 1 \rangle \langle \mathbf{z}_2, i \rangle = \langle \mathbf{z}_1, i\mathbf{z}_2 \rangle$ holds. Also, the notation $|\mathbf{z}| = \langle \mathbf{z}, \mathbf{z} \rangle^{1/2}$ and $\angle \mathbf{z}$ denotes the magnitude and the argument of the complex number $\mathbf{z}$. The derivative of this operator can be expressed as $\frac{d}{dt}\langle \mathbf{z}_1, \mathbf{z}_2 \rangle = \langle \dot{\mathbf{z}}_1, \mathbf{z}_2 \rangle \langle \mathbf{z}_1, \dot{\mathbf{z}}_2 \rangle$.

2 Problem formulation and Previous works

2.1 Problem formulation

Consider a set of N agents (vehicles), in which each agent $k = 1, ..., N$ has the following constrained dynamics:

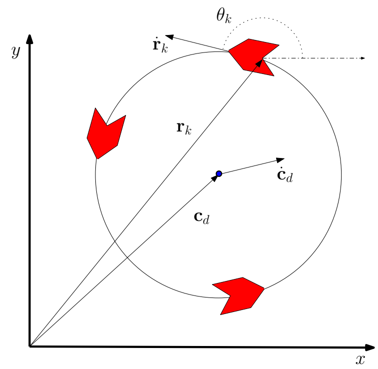


Figure 1: Illustration of the problem formulation.

$$\dot{\mathbf{r}}_k = v_k e^{i\theta_k} \tag{1a}$$
$$\dot{\theta}_k = u_k \tag{1b}$$

where $\mathbf{r}_k$ is the position vector, θ_k the heading angle and v_k, u_k are the control inputs, as illustrated in Figure 1. This is the standard agent model commonly used the literature to model AUVs (autonomous underwater vehicles) restricted kinematics, among many others vehicles as Dubin's cars or UAVs (unmanned air vehicles), see [1, 2, 3, 5, 7, 8, 14, 15] and [16]. It corresponds to a kinematic unicycle fitting with model properties subject to a simple nonholonomic constraint, adequate for the underwater vehicles.

The problem is to design a control law such that the group of AUVs forms a circle that tracks the time-varying center motion $\mathbf{c}_d(t)$ as described in Figure 1. $\mathbf{c}_d(t)$ is considered here as an external reference. The circle radius R and the rotation velocity ω_0 are given parameters. Moreover, an additional objective is to achieve a uniform distribution of all the agents along the circle (i.e. the difference between headings of adjacent vehicles is $2\pi/N$), under two different cases:

1. Fixed communication graph
2. Limited range time-varying communication graph

2.2 Previous works

Some previous works on the field of coordinated control and specifically on planar collective motions, use the kinematic model in which each vehicle moves in the plane subject to planar steering control, which is our model (1) with constant velocity $v_k = v_0 = 1$. In [1], the authors suggest a control law for stabilization to a circular formation center at a particular and constant $\mathbf{c}_d^0$. It corresponds to the center of mass and is obtained by solving a consensus algorithm. The control law uses the relative position vector from the center to vehicle k defined as $\tilde{\mathbf{r}}_k = \mathbf{r}_k - \mathbf{c}_d^0 = \frac{1}{N}\sum_{j=1}^{N}(\mathbf{r}_k - \mathbf{r}_j)$. For such a formation, the authors propose the following theorem:

Theorem 1 *(Leonard et al. [1]) Consider the vehicle model (1) with $v_k = v_0 = 1, \forall k$. Then the control law:*

$$u_k = \omega_0(1 + \kappa\langle\tilde{\boldsymbol{r}}_k, \dot{\boldsymbol{r}}_k\rangle) \tag{2}$$

where $\kappa > 0$ is a scalar gain, ensures that all the agents converge to a circular formation centered at $\boldsymbol{c}_d^0$ and of radius $\rho_0 = v_0/|\omega_0|$

Proof 1 *The proof is based the Lyapunov function:*

$$S(\boldsymbol{r}, \theta) = \frac{1}{2}\sum_{k=1}^{N}|v_0 e^{i\theta_k} - i\omega_0\tilde{\boldsymbol{r}}_k|^2$$

The details can be found in [7].

Remark 1 *Note that when $S(\boldsymbol{r}, \theta) = 0$, the dynamics of the agents satisfy the differential equation $\dot{\boldsymbol{r}}_k - i\omega_0\tilde{\boldsymbol{r}}_k = 0$. As the center $\boldsymbol{c}_d^0$ is fixed, $\dot{\boldsymbol{r}}_k = \dot{\tilde{\boldsymbol{r}}}_k$, the previous equation means that $\dot{\tilde{\boldsymbol{r}}}_k - i\omega_0\tilde{\boldsymbol{r}}_k = 0$. It corresponds to a circular motion around $\boldsymbol{c}_d^0$ with an angular velocity ω_0.*

2.3 Fundamental limitations

This previous result is only applicable to the case of a fixed formation center, $\mathbf{c}_d$. In this situation, it is sufficient to design a control law such that the velocity of all the agents is constant (i.e. $v_k = 1, \forall k$). However, when it comes to the case of a time-varying center $\mathbf{c}_d(t)$, the mechanical equation for the combined motion of a rotation and a translation of the rigid body leads to a contradiction with the choice of constant velocity of the agents (see for instance the example of the wheel motion).

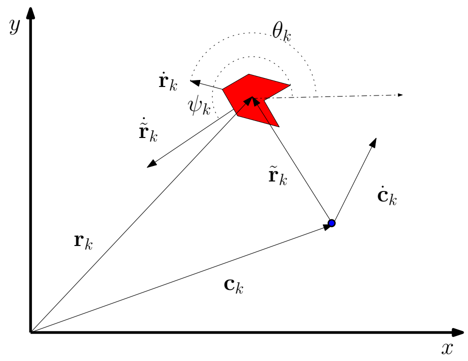


Figure 2: Model of the vehicles

Hence a new strategy which tackles the objectives needs to be developed. The velocity v_k becomes a new and necessary control input to overcome this mechanical constraint. Then, in the sequel, the variables (v_k, u_k) and $(\mathbf{r}_k, \theta_k)$, respectively, are the inputs and the state of the agents. In the latter, the notations $\mathbf{r}$ and θ denote the vectors containing the position and headings of all the agents.

3 Translation Control for a Moving Circle

3.1 Translation Control Design

Inspired the Theorem 1, to keep the circular formation, all the agents must satisfy the equation $\dot{\tilde{\mathbf{r}}}_k - i\omega_0\tilde{\mathbf{r}}_k = 0$, where $\tilde{\mathbf{r}}_k = \mathbf{r}_k - \mathbf{c}_d$. We assume that the first and second time-derivative of $\mathbf{c}_d$ are known and bounded. However since the center is moving, the velocities $\mathbf{r}_k$ and $\tilde{\mathbf{r}}_k$ are not the same anymore. The previous Lyapunov function will thus not be useful for reaching the time-varying formation.

For the sake of simplicity, we introduce the change of variables, shown in Figure 2, to express the relative velocity in a moving frame centered in $\mathbf{c}_d$. This allows designing a control law along the ideas of Theorem 1. The new variable ψ_k and the constant $v_0 > 0$ are defined such that:

$$\dot{\tilde{\mathbf{r}}}_k = \dot{\mathbf{r}}_k - \dot{\mathbf{c}}_d = v_0 e^{i\psi_k}, \tag{3}$$

and ψ is given as:

$$\psi_k = \arctan \frac{\langle \dot{\mathbf{r}}_k - \dot{\mathbf{c}}_d, i\rangle}{\langle \dot{\mathbf{r}}_k - \dot{\mathbf{c}}_d, 1\rangle} + \epsilon\pi$$

where $\epsilon = 0$ if $\langle \dot{\mathbf{r}}_k - \dot{\mathbf{c}}_d, 1\rangle > 0$ and 1 otherwise. The control input $\dot{\psi}_k$ allows stabilizing the relative position to the circular motion. This new system of coordinates allows us to consider a circular formation centered at 0. We are now in the situation of a fixed center which corresponds to the previous problem solved in Theorem 1. A new control law is proposed in the following theorem that is the main paper contribution:

Theorem 2 *Consider a twice differentiable function $\mathbf{c}_d : \mathbb{R} \to \mathbb{C}$, with bounded first and second time-derivatives and the radius of desired formation $R > 0$. Let the control parameters be such that $|\omega_0| = v_0/R$, $\kappa > 0$ and the following condition is satisfied:*

$$v_0 > \sup_{t\geq 0}\{|\dot{\mathbf{c}}_d(t)|\} \tag{4}$$

Then the control law:

$$v_k = |v_0 e^{i\psi_k} + \dot{\mathbf{c}}_d| \tag{5a}$$

$$u_k = \left(1 - \frac{\langle \dot{\boldsymbol{r}}_k, \dot{\mathbf{c}}_d\rangle}{\langle \dot{\boldsymbol{r}}_k, \dot{\boldsymbol{r}}_k\rangle}\right)\dot{\psi}_k - \frac{\langle \dot{\boldsymbol{r}}_k, i\ddot{\mathbf{c}}_d\rangle}{\langle \dot{\boldsymbol{r}}_k, \dot{\boldsymbol{r}}_k\rangle} \tag{5b}$$

$$\dot{\psi}_k = \omega_0(1 + \kappa\langle \boldsymbol{r}_k - \mathbf{c}_d, v_0 e^{i\psi_k}\rangle) \tag{5c}$$

with the initial conditions $\psi_k(0)$ as:

$$\psi_k(0) = \arctan \frac{\langle \dot{\boldsymbol{r}}_k(0) - \dot{\mathbf{c}}_d(0), i\rangle}{\langle \dot{\boldsymbol{r}}_k(0) - \dot{\mathbf{c}}_d(0), 1\rangle} + \epsilon_k\pi \tag{6}$$

where $\epsilon_k = 0$ if $\langle \dot{\boldsymbol{r}}_k(0) - \dot{\mathbf{c}}_d(0), 1\rangle > 0$ and 1 otherwise, makes all the agents defined by (1) converge to a circular motion of radius R, and of center the time-varying reference $\mathbf{c}_d$. The direction of rotation is determined by the sign of ω_0.

Proof 2 *The convergence to the formation is analyzed with the Lyapunov function:*

$$S(\tilde{\boldsymbol{r}}, \psi) = \frac{1}{2}\sum_{k=1}^{N} |\dot{\tilde{\boldsymbol{r}}}_k - i\omega_0\tilde{\boldsymbol{r}}_k|^2$$

Evaluating the derivative of $S(\tilde{\boldsymbol{r}}, \psi)$ along the solutions of the resulting closed-loop system (3) with (2) leads to:

$$\begin{aligned}\dot{S}(\tilde{\boldsymbol{r}}, \psi) &= \sum_{k=1}^{N}\langle \dot{\tilde{\boldsymbol{r}}}_k - i\omega_0\tilde{\boldsymbol{r}}_k, i\dot{\tilde{\boldsymbol{r}}}_k\dot{\psi}_k - i\omega_0\dot{\tilde{\boldsymbol{r}}}_k\rangle \\ &= \sum_{k=1}^{N}\langle \omega_0\tilde{\boldsymbol{r}}_k, v_0 e^{i\psi_k}\rangle(\omega_0 - \dot{\psi}_k) \\ &= -\kappa\sum_{k=1}^{N}\langle \omega_0\tilde{\boldsymbol{r}}_k, v_0 e^{i\psi_k}\rangle^2 \leq 0\end{aligned}$$

Note that when $S(\tilde{\boldsymbol{r}}, \psi) = 0$ the dynamics of agents satisfy:

$$\dot{\boldsymbol{r}}_k = \dot{\mathbf{c}}_d + i\omega_0\tilde{\boldsymbol{r}}_k$$

which is the kinematic relation for the combined motion of a translation and a rotation of the rigid body. Therefore $S(\tilde{\boldsymbol{r}}, \psi)$ is an suitable Lyapunov candidate for this system and the agents asymptotically reach the circular formation centered at $\mathbf{c}_d$ and of radius $R = v_0/|\omega_0|$.

The next step of the proof concerns the design of the control inputs of the original system. Considering (1a), it is easy to see that v_k and θ_k are given by:

$$v_k = |\dot{\boldsymbol{r}}_k| \text{ and } \tan\theta_k = \frac{\langle \dot{\boldsymbol{r}}_k, i\rangle}{\langle \dot{\boldsymbol{r}}_k, 1\rangle}$$

The control input v_k is thus straightforwardly given by (5a). A more particular attention is

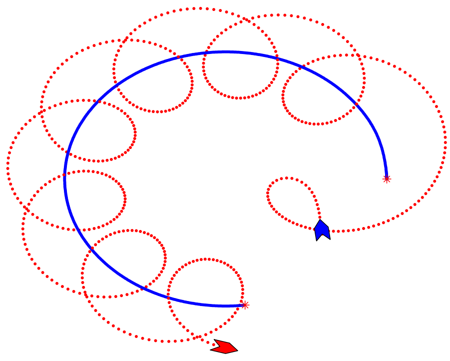


Figure 3: Simulation of an agent with the control law (2). The trajectories of $\mathbf{c}_d$ and of the agent are respectively the continuous and the dashed lines.

addressed to the input u_k*. To fit with the model (1), we derive the expression* $\tan(\theta_k)$

$$\dot{\theta}_k = \frac{1}{1+\frac{\langle \dot{\boldsymbol{r}}_k, i\rangle^2}{\langle \dot{\boldsymbol{r}}_k, 1\rangle^2}} \frac{d}{dt}\left(\frac{\langle \dot{\boldsymbol{r}}_k, i\rangle}{\langle \dot{\boldsymbol{r}}_k, 1\rangle}\right)$$

Using the properties of the operator $\langle \cdot, \cdot \rangle$*, described in the notation, the following relation is obtained:*

$$\dot{\theta}_k = \frac{\langle \ddot{\boldsymbol{r}}_k, i\dot{\boldsymbol{r}}_k\rangle}{\langle \dot{\boldsymbol{r}}_k, \dot{\boldsymbol{r}}_k\rangle}$$

To express this equation in terms of the previous control variable $\dot{\psi}_k$ *(change of coordinates), note that:*

$$\ddot{\boldsymbol{r}}_k = \ddot{\tilde{\boldsymbol{r}}}_k + \ddot{\boldsymbol{c}}_d = i\dot{\tilde{\boldsymbol{r}}}\dot{\psi}_k + \ddot{\boldsymbol{c}}_d = i(\dot{\boldsymbol{r}}_k - \dot{\boldsymbol{c}}_d)\dot{\psi}_k + \ddot{\boldsymbol{c}}_d$$

the control law (5b) *is obtained. We have designed a control law for the agents to follow a time-varying circular formation. Note that this control law has singular points when* $v_k = |\dot{\boldsymbol{r}}_k|$ *is zero. To understand this singularity, consider the example of the cycloid whose first derivative is not defined at some instants. This constraint fits with the choice of underwater vehicles. This singular point occurs if there exists a time* t_c *such that:*

$$\angle \dot{\boldsymbol{c}}_d(t_c) = \psi_k(t_c) + \pi \quad \textit{and} \quad |\dot{\boldsymbol{c}}_d(t_c)| = v_0$$

However condition (4) *of Theorem 2 ensures that this situation is avoided since* $|\dot{\boldsymbol{c}}_d(t_c)| \neq v_0 \quad \forall t_c > 0$.

Remark 2 *Consider the vehicle model (1) with the fixed center, the the angles* ψ_k *and* θ_k *are equal, and the control law* (2) *is the same control as in Theorem 1:*

$$\begin{aligned} v_k &= v_0 \\ u_k &= \dot{\psi}_k = \omega_0(1 + \kappa\langle \tilde{\boldsymbol{r}}_k, v_0 e^{i\psi_k}\rangle) \end{aligned}$$

4 Cooperative Control Design under Communication Constrains

This section is dedicated to the problem of homogenizing the distribution of the agents along the circle. The desired control law is decentralized, i.e. the use of a global controller who organizes the distribution of the agents around the circle is not permitted. As a first step of research, this section tackle the problem of limited communication range. Considering limited communication means that each agent may receive information from only some of the other agents [12]. It is known that designing collaborative controllers leads to more difficulties than in the case of all-to-all communications.

4.1 Preliminaries on Graph Theory

This paragraph presents some basic tools of graph theory. When an agent k communicates with an agent j both agents are called neighbors. The set of neighbors of agent k is denoted by $\mathcal{N}_k$. The communication topology for the groups of agents can be represented by means of a graph $G(V, E)$ where $V = \{1, 2, ..., N\}$ is the set of vertices (agents) and $E = \{(k, j) : j \in \mathcal{N}_k\}$ the set of edges (communication links) such that $(k, j) \in E$ if agent k communicates with agent j. The adjacency matrix $\mathbf{A} = [a_{kj}]$ is the $N \times N$ matrix given by $a_{kj} = 1$ if $(k, j) \in E$ and $a_{kj} = 0$ otherwise. The *degree* d_k of vertex k is defined as the number of its neighboring vertices. Let Δ be the $N \times N$ diagonal matrix of d_k's. The *Laplacian* of G is the matrix $L = \Delta - A$. For an undirected graph, (j is a neighbor of k if and only if k is a neighbor of j), the Laplacian matrix is symmetric positive semidefinite [7].

4.2 Fixed communication graph

This paragraph is an application of the result of [1] to the case of a time-varying formation center. A potential function is added to the formation control law to achieve the uniform distribution. In the case of a fixed communication structure, G is assumed undirected because the case of bidirectional communication between two AUVs is considered. Then a constant Laplacian matrix $\mathbf{L}$ describes the communication links between agents. In this way, the previous circular law is combined with a potential function as:

Theorem 3 *Consider a twice differentiable function* $\boldsymbol{c}_d : \mathbb{R} \to \mathbb{C}$*, with bounded first and second time-derivatives and the radius of desired* *formation $R > 0$. Let the control parameters be such that $|\omega_0| = v_0/R$ and $\kappa > 0$ and the following condition is satisfied:*

$$v_0 > \sup_{t \geq 0}\{|\dot{\boldsymbol{c}}_d(t)|\}$$

Let G be circulant, see [2], and $\boldsymbol{L}$ be the corresponding Laplacian matrix. Then the control law (2) *now with:*

$$\begin{cases} \dot{\psi}_k = \omega_0(1 + \kappa\langle \boldsymbol{r}_k - \boldsymbol{c}_d, v_0 e^{i\psi_k}\rangle) - \frac{\partial U}{\partial \psi_k} \\ U(\psi) = -\frac{K}{N}\sum_{m=1}^{[N/2]} \frac{1}{2m^2}\langle e^{im\psi}, \boldsymbol{L}e^{im\psi}\rangle \end{cases} \quad (8)$$

and initial conditions $\psi_k(0)$ as:

$$\psi_k(0) = \arctan \frac{\langle \dot{\boldsymbol{r}}_k(0) - \dot{\boldsymbol{c}}_d(0), i\rangle}{\langle \dot{\boldsymbol{r}}_k(0) - \dot{\boldsymbol{c}}_d(0), 1\rangle} + \epsilon\pi \quad (9)$$

where $\epsilon = 0$ if $\langle \dot{\boldsymbol{r}}_k(0) - \dot{\boldsymbol{c}}_d(0), 1\rangle > 0$ and 1 otherwise, makes all the agents defined by (1) *converge to a circular motion of radius $R = v_0/|\omega_0|$ and of center the time-varying reference $\boldsymbol{c}_d$. Moreover the curve-phase arrangement is a critical point of $U(\psi)$. For $K > 0$, the set of curve-phase arrangements that are synchronized modulo $2\pi/N$ is locally exponentially stable.*

Proof 3 *The proof uses the La Salle Invariance principle*[2] *applied to the function*

$$\langle e^{im\psi}, \boldsymbol{L}e^{im\psi}\rangle / 2m^2$$

which is zero for ψ synchronized modulo $2\pi/m$ and positive otherwise. In this way, is combined the previous circular law with a gradient control term which leads to (8)*. The stability is analyzed by the following composed Lyapunov function:*

$$V(\tilde{\boldsymbol{r}}, \psi) = \kappa S(\tilde{\boldsymbol{r}}, \psi) + U(\psi)$$

The details of the proof can be found in [1].

Remark 3 *Theorem 3 does not exclude convergence to formations which corresponds to other critical points of $U(\psi)$ [15].*

Remark 4 *If the graph G is complete, then the set of curve-phase arrangements that are balanced modulo $2\pi/N$ is a global maximum of $U(\psi)$ in the reduced space of relative curve-phases; this is asymptotically stable for $K > 0$ [15]. Moreover if $K < 0$ the control law of Theorem 3 forces convergence to the synchronized circular formation [7].*

[2] Due to change of coordinates (3) the dynamic closed-loop equation is time-invariant, hence LaSalle principle can be applied.

4.3 Range-dependent communication graph

In the previous subsection, a control law ensures that the agents reach a circular formation centered at the time varying position $\mathbf{c}_d$. It also distributes the agents in a particular way. However as shown in [15], there is no guarantee that the formation is uniform along the circle in the case of fixed communication graphs. Moreover, in practice, considering fixed communication graphs is not realistic since the two linked agents could be very far away from one another. As in the case of underwater communication, the quality of the links is strongly affected by the distance between two agents [4, 17, 18, 19], it might be more interesting to consider distance-dependent communication graph.

Hence, a communication area is introduced. Assume this area is the same for all agents and is defined by a parameter. This parameter corresponds to ρ which is the critical communication distance given by the characteristics of the communication devices and of the environment of the AUVs. The condition to get a communication between k and j is expressed as:

$$k \in \mathcal{N}_j \text{ and } j \in \mathcal{N}_k \quad \Longleftrightarrow \quad \frac{|\mathbf{r}_k - \mathbf{r}_j|}{2} \leq \rho$$

Based on the definitions presented in section 4.1, a time-varying Laplacian matrix $\mathbf{L}(t)$ is defined as:

$$L_{k,j} = \begin{cases} d_k, & \text{if} \quad k = j \\ -1, & \text{if} \quad |\mathbf{r}_k - \mathbf{r}_j| \leq 2\rho \\ 0 & \text{otherwise} \end{cases} \quad (10)$$

where d_k is the *degree* of vertex k. In such a situation, the following theorem holds.

Theorem 4 *Consider a twice differentiable function $\boldsymbol{c}_d : \mathbb{R} \to \mathbb{C}$, with bounded first and second time-derivatives and the radius of desired formation $R > 0$. Let the control parameters be* *such that* $|\omega_0| = v_0/R$ *and* $\kappa > 0$ *and the following condition is satisfied:*

$$v_0 > \sup_{t \geq 0}\{|\dot{\mathbf{c}}_d(t)|\}$$

The following assumption is also considered:

$$v_0/|\omega_0| < \rho \qquad (11)$$

Then the control law (2) with (8) and (9) ensures that all agents reach the circular formation centered at $\mathbf{c}_d(t)$ *of radius* R*. Moreover the uniform distribution of the agents along the circle is achieved.*

Proof 4 *Consider that all the agents asymptotically reach the circle centered at* $\mathbf{c}_d$ *and of radius* $v_0/|\omega_0|$ *and a positive scalar* $\epsilon > 0$ *such that* $v_0/|\omega_0| + \epsilon < \rho$*. Thanks to theorem 2, there exits a time* t_L *such that the distance between all agents is less than* $v_0/|\omega_0| + \epsilon$ *and consequently less than* ρ*. The communication graph is thus complete. By vertue of the potential function* $U(\psi)$*, the formation is uniformly distributed along the circle.*

Remark 5 *Note that condition (11) is restrictive since the uniform distribution can also obtained for smaller radius* $v_0/|\omega_0|$*. However this Theorem constitutes a first result on the case of uniform circular formation around a time-varying center with range dependent communication graph. Besides these limitations described above, Theorem 4 allows obtaining a unform distribution whatever the critical distance* ρ *by managing with the ratio of the relative velocity of each agent* v_0 *and angular velocity of the formation* ω_0*.*

Remark 6 *From the geometric constraints, the minimal distance between two agents* k *and* j *lying in the circle is given by* $d = 2R\sin\frac{\psi_k - \psi_j}{2}$*. In the case of uniform distribution, the minimal value of* $\psi_k - \psi_j$ *is given by* $2\pi/N$ *and so a necessary condition for the agents to communicate in such situation is* $\rho > R\sin\frac{\pi}{N}$*. If not, the formation is not uniform all over the circle but only on a section of the circle as it will be shown in the simulations.*

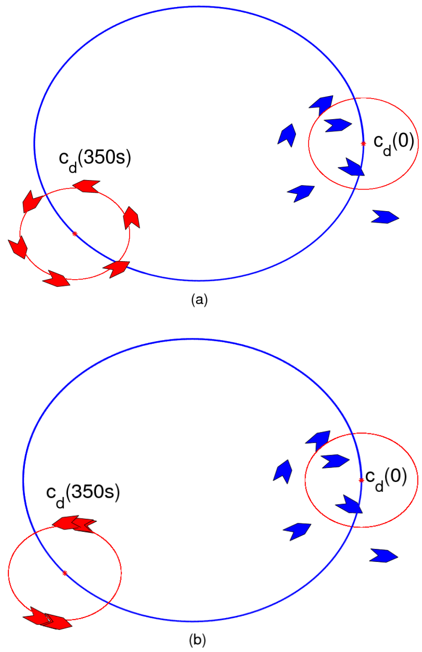


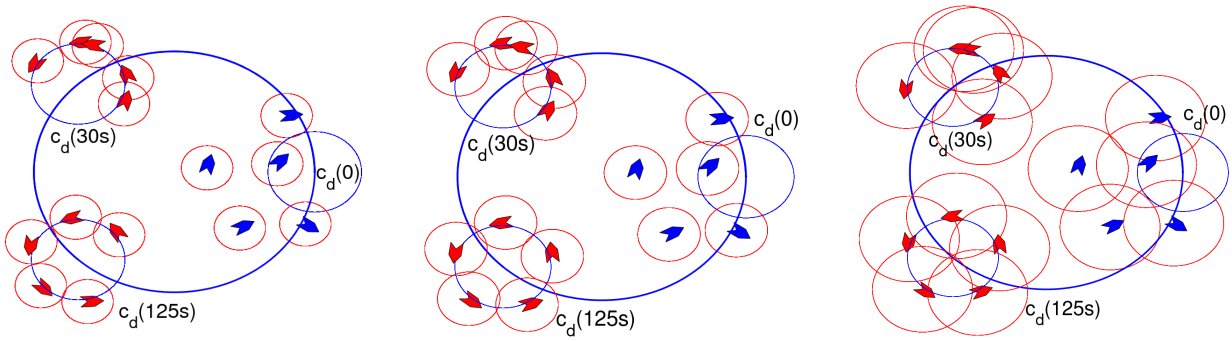
Figure 4: Simulation of six agents with the controller of Theorem 3 in the case of two fixed communication graph : (a) all-to-all (b) ring communication. Each figure shows two snapshots. The blue one represents the initial position of the agents and the red one shows the obtained formation.

5 Simulation results

This section presents the simulation of the AUVs whose dynamics are defined in (1). The time-varying center of the formation describes a circle around the origin. The vector $\mathbf{c}_d$ is taken as $\mathbf{c}_d^0 e^{\omega_1 t}$ where $\mathbf{c}_d^0 = 3$ and $\omega_1 = 27.7e^{-3}$ and $|\dot{\mathbf{c}}_d| = 83.3e^{-3}$ satisfying the assumption of Theorems 2, 3 and 4. In the simulation, the controller parameters are $\kappa = 1$, $v_0 = 1$ and $\omega_0 = 1$. The control parameter to achieve the uniform distribution is $K = 0.1$.

Figure 3 shows the trajectory of only one agent governed by the control law defined in Theorem 2. The tracking of the moving circle is achieved for all random initial conditions (position and heading of agents).

Figure 4 shows the trajectories of all agents tracking the time-varying formation centered at $\mathbf{c}_d$ in the cases of a complete communication graph (a) and of a ring communication graph (b). One can see that in (a) the formation



(a) $\rho = 0.55 < \frac{v_0}{|\omega_0|}\sin(\pi/N)$ (b) $\rho = 0.65 > \frac{v_0}{|\omega_0|}\sin(\pi/N)$ (c) $\rho = 1.1 > v_0/|\omega_0|$

Figure 5: Simulations of five agents with the controller designed in Theorem 4 and range-dependent communication. Each figure shows three snapshots. The blue one represents the initial condition and the reds one represent a intermediate state and the stable final state.

is uniformly distributed along the circle of radius $R = v_0/|\omega_0|$. This is not the case in (b). The agents converge to a formation which corresponds to a local minimum of the potential function but not to the global one.

Figure 5 shows the trajectories of agents under range-dependent communication. The three simulations start from the same random initial conditions of positions and headings of all the agents. They are represented in blue ($\mathbf{c}_d(0)$). After $30s$ all simulations show that all the agents converge to the circular formation. At this time, in (a), the geometric constraint $R\sin(\pi/N) < \rho$ is not satisfied then the agents do not achieve the uniform distribution all over the circle. For the second one, the previous geometric constraint is satisfied then the agents achieve the uniform distribution along the circle. In the last one, there exists all-to-all communication since condition (11) is satisfied. Thanks to the time-varying communication graph is complete, from a certain instant, the uniform distribution is also achieved ($\mathbf{c}_d(125s)$).

6 Conclusions

This article proposes a translation control law that stabilizes a circular formation tracking a time-varying center. The center of the circle is a given reference which is known for all the agents. A cooperative control algorithm is also considered to achieve the uniform distribution of the agents along the moving circular formation. This algorithm integrates with the translation control a potential function which reaches its minimum in the desired uniform configuration. This potential function is designed to take into account the communication constraints between agents. The result of this combination is a cooperative control of a planar particle model under limited communication to track a time-varying reference.

At this time, it is assumed that all agents have perfect knowledge of the position of the center $\mathbf{c}_d$ and its first and second derivatives. One can consider this assumption as a very restrictive one. However this constitues a first step of our research. Further developments would consider a cooperative algorithm which will avoid this assumption. To include collaborative algorithms in the control design would be a step towards source tracking.

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