Optimal Design of a 4-dof Parallel Manipulator: From Academia to Industry
François Pierrot, Vincent Nabat, Olivier Company, Sébastien Krut, Philippe Poignet

Abstract—This paper presents an optimal design of a parallel manipulator aiming to perform pick-and-place operations at high-speed and high-acceleration. After reviewing existing architectures of high-speed and high-acceleration parallel manipulators, a new design of a 4-dof parallel manipulator is presented, with an articulated traveling plate, free of internal singularities and able to achieve high performances. The kinematic and simplified, but realistic, dynamic models are derived and validated on a manipulator prototype. Experimental tests show that this design is able to perform beyond the high targets, i.e. it reaches a speed of 5.5 m/s and an acceleration of 165 m/s². The experimental prototype was further optimized on the basis of kinematic and dynamic criteria. Once the motors, gear ratio, and several link lengths are determined, a modified design of the articulated traveling plate is proposed in order to reach a better dynamic equilibrium among the four legs of the manipulator. The obtained design is the basis of a commercial product offering the shortest cycle-times among all robots available on today’s market.


I. INTRODUCTION

MOST pick-and-place operations, including packaging, picking, packing and palletizing tasks; require four degrees of freedom (dof), i.e. three translations and one rotation around a vertical axis, namely SCARA or Schönflies motions [1]. For industrial profitability manipulators able to perform such motions in the shortest possible cycle time are required. Up to now, two main mechanism families have been used to perform such motions at high-speed (more than 4 m/s) and high-acceleration (more than 100 m/s²). First, the SCARA-type architecture uses a serial chain R-R-P-P (where R and P are, respectively, a revolute and a prismatic joint; while the underlines designate the joints that are actuated) with four parallel axes, as shown in Figure 1-a. Secondly, the Delta-type architecture uses three parallel chains R-(SS)² (where (SS)² designates a four-bar linkage made with four spherical joints, S) together with a fourth “telescopic” leg in order to provide rotation of the payload. The Delta robot (Figure 1-b) was proposed [2] by Clavel for high-speed pick-and-place operations and patented [3]. In order to reach high performances, the manipulator was designed with all actuators fixed on its base, thus minimizing the mass of moving parts. This truly innovative design has proven its efficiency in various application domains, including robotics for pick-and-place or medicine, machining and human-machine-interface. The Delta robot has opened a new field of research (known today as “lower-mobility parallel mechanisms” [4][5][6]) and a new segment in the robotics industry. Apart from these two main architectures, other parallel manipulators able to produce Schönflies motions have been developed in recent years. For example, Angeles [7] proposed a 4-dof parallel manipulator, EPFL researchers developed Kanuk and Manta manipulators [8] and a similar machine-tool called HITA STT [9], and more recently an innovative design was introduced by Zhao [10]; isotropic or (partially) decoupled 4-dof mechanisms have also been presented [11][12][13]. So far, none of these research prototypes has reached the robotics market.

Engineering experience and scientific reasoning agree on the strengths and weaknesses of serial and parallel architectures: small footprint, large workspace, simple modeling, but large moving masses, for serial manipulators; low moving masses, high dynamic capabilities, but large footprint and small workspaces, for parallel manipulators. In recent years, many research studies have been dedicated to the improvement of parallel mechanisms, but a large part of it was only devoted to kinematics; in this field, important results have been obtained regarding analysis and synthesis [14], [15], [16]. New results have also been obtained regarding efficient methods to derive dynamic models [17].

This scientific knowledge is of course a cornerstone in the development of new manipulators, but there is still often a gap between analysis and practical applications. This paper introduces a new design of a 4-dof parallel manipulator for
high-speed and high-acceleration pick-and-place operations and provides insight about the way this academic innovation was transformed into a commercially available robot. The approach and results presented here have been transferred from Academia to Industry to come up with a product whose capabilities are beyond the performances, in terms of cycle-time, of any existing robot on the market.

This paper first proposes a brief review of some existing designs (section II), before introducing a new design, Par4 (section III). Par4 kinematics (section IV) and dynamics (section V) are then described, followed by a glance at preliminary performance evaluation (section VI). The information provided in section IV and V gives the tools for understanding the design optimization described in section VI, before reaching the conclusions (section VII).

II. EXISTING DESIGN REVIEW OF HIGH-SPEED PICK-AND-PLACE ROBOTS

In recent years, authors have focused their efforts on parallel manipulators that could provide an alternative solution to the Delta-type manipulator especially for high-speed pick-and-place applications. Note that they had considered only lower mobility mechanisms without any passive leg, even though designing robots with passive legs could be an interesting option.

A. The Delta

Indeed, despite all the qualities of Delta (widely recognized by scientists and customers), one of its features could be reconsidered. With this manipulator, the rotational motion of the payload is obtained using an R-U-P-U-R leg (where U is a 2-dof universal joint), which may be hampered by a lack of stiffness at workspace boundaries (the chain can be considered as a long and thin beam supporting torsion stress), and from a short service life if it is not designed with great care (a light-weight passive prismatic joint supporting high acceleration motions is a design challenge). In fact, a reliable R-U-P-U-R chain able to sustain high acceleration increases the mass of moving components, especially if very large workspaces are of interest. Moreover, this chain may limit the workspace due to the mechanical limits of the prismatic joint. Therefore designs offering the required fourth dof without resorting to such a telescopic chain have been sought.

B. The H4, I4 Family

This is why the concept of articulated traveling plate has been proposed and this concept is briefly reviewed below before introducing the innovation design of this paper.

The major difference between most parallel manipulators and this family of mechanisms lies in the traveling plate design. Most parallel manipulators are designed with: (i) a base; (ii) kinematic chains; and (iii) a rigid traveling plate. Alternatively, the “H4” robot (see Figure 2 and [18]) introduced the idea of embedding joints into the traveling plate, expecting that such designs could increase the maximum range of motion in rotation. Three implementations of this concept have already been achieved: H4 firstly, and later I4L [19] and I4R [20], their traveling plates are shown in Figure 3. In H4, the traveling plate is a 3-body system with two R joints. In I4L, the traveling plate is a 4-body system with two P joints kinematically coupled (via a rack-and-pinion system) to an R joint. Finally, in I4R, the traveling plate is a 3-body system with one P joint kinematically coupled (via a cable-and-pulley system) to an R joint.

Figure 2. H4, one of the first robots with articulated traveling plate.

Figure 3. Some existing articulated traveling plates: H4; I4L; I4R.

Extensive studies and tests (which are not reviewed here because of the lack of space) have shown that none of these designs are optimal for very fast pick-and-place operations. In summary, we have found: (i) P joints on the traveling plate may have a low service life if they are light weight, while the behavior of R joints remains fine (then, I4R and I4L could be better suited for heavy equipment, such as machine-tools, where speed and acceleration are moderate and mass constraints not very marked); (ii) a symmetrical manipulator design is mandatory in order to obtain an homogenous distribution of effort (force and moment) along and between the kinematic chains, thus granting homogenous performances throughout the workspace, such as velocity and stiffness distribution [21] (this remark overlooks H4 which cannot be built with a symmetrical design); (iii) revolute actuation is compact and reliable; and finally, (iv) the internal singularities (also called constraint singularities) have to be carefully addressed. Before going further, let us explain the last remark.

C. Some Considerations on Constraint Singularities

For robotic manipulators, singularities, or singular postures, are manipulator configurations where it cannot be fully controlled, either because its end-effector is no longer able to produce all the necessary movements or because its stiffness decreases drastically (down to zero, theoretically). In general, manipulators may face three types of singularities: (i) serial;
(ii) parallel; and (iii) constraint [22, [23]. The first two types have been widely discussed since their introduction and the fundamental concepts can be found in the seminal papers of Merlet [24] or Gosselin [25]. They can be analyzed by resorting to a simple kinematic analysis that describes the input-output relationship between the actuated joint velocities and the user-space velocities. According to Gosselin, if the actuated joint velocity vector is denoted $\mathbf{q}$ and the task-space velocity vector is $\mathbf{v}$, the input-output relationship can be written as follows:

$$\mathbf{J}_q \mathbf{q} = \mathbf{J}_v \mathbf{v} \quad (1)$$

where $\mathbf{J}_q$ and $\mathbf{J}_v$ are two $n \times n$ matrices for an $n$-dimension task space (in a non-redundant case). Analyzing both matrices is sufficient to determine if a manipulator is in a serial-type or parallel-type singularity (Park showed [26] that a more mathematically consistent approach is better suited to understanding all implications of singular postures but the discussions in this paper can safely be made by resorting only to the classical scheme developed by Gosselin). However, when the manipulator kinematics includes an internal constraint, i.e. an arrangement of links and joints that creates a specific kinematic constraint on a sub-part of the whole mechanism, it is likely that the constraint may vanish, thus creating an improper behavior. Such constraints are not encompassed by the input-output relation (1); then any problem occurring at that level cannot be detected unless a more comprehensive kinematic model is established.

Most parallel manipulators based on articulated traveling plates resort to such internal constraints to create a specific kinematic relation between the various bodies that compose the traveling plate. As an example, the kinematic chains of H4 have to be arranged in such a way to ensure that the two sub-parts of the traveling plate remain parallel to each other. Such a constraint is not easy to understand at first sight, and it is necessary to establish a kinematic model that takes into account not only the task-space velocity (of dimension four in the case of H4: three translations and one rotation) but also the velocities of the passive joints of the traveling plate (that adds two more parameters), as well as the parameters of the remaining possible rotations in space for the traveling plate (that adds two more parameters, again). Such an analysis cannot be conveniently covered by an approach based on group theory [28], especially when $\mathbb{R}^4 \mathbb{S}^2 \mathbb{S}^2 \mathbb{2}$ chains are involved (these chains have 5 dof and there is no group that encompasses such chains), even though this approach is sometimes very helpful. Interested readers can refer to [20] for a detailed kinematic analysis; this analysis is similar to that proposed in [5], except it has to be extended to cope with passive joints embedded in the traveling plate, thus leading to a $8 \times 7$ Jacobian matrix for Par4 or a $8 \times 8$ Jacobian matrix for H4 (see also [27]), instead of a $6 \times 6$ matrix for simpler cases.

Alternatively, a simple 2-dof planar parallel manipulator, as depicted in Figure 4-a, may help to understand this problem. The traveling plate of this manipulator is kept at a constant orientation (horizontal in Figure 4). This constraint comes from the four-bar linkage located at the left hand side of the end-effector. The existence of this constraint is not captured by a simple input-output relationship, and the singular posture pictured in Figure 4-(b) is not understood as a singular position if only Eq. (1) is used. It has been shown that, for a given kinematic architecture, depending on the general arrangement of the various links (e.g. the location of each motor axis), a manipulator may or may not be hampered by internal singularities within its workspace. For example, such considerations make symmetrical designs of H4 impossible [18]. Consequently, it is of prime importance to address this problem right at the design level for a parallel manipulator that uses an articulated traveling plate. Note that some lower mobility PKM are free of constraint singularities, such as the “multi-pterons” [12][13], but they are not specifically suited for very high-acceleration motions (all moving links have to resist bending or torsion stress and consequently they tend to be heavy components).

III. NEW DESIGN: THE PAR4 ARCHITECTURE

The design specifications for the new parallel manipulator were as follows: (i) a symmetrical joint-and-loop-graph (i.e. all kinematic chains are identical; the traveling plate is symmetrical); (ii) a symmetrical placement of the actuators with respect to a vertical axis; (iii) four identical revolute actuators; and (iv) avoiding internal singularities within the workspace. In order to respect all of these specifications, and especially the fourth one, we created a new traveling plate (see Figure 5) by resorting to a planar four-bar linkage (equivalent of a 1-dof $\pi$ joint [1], i.e. a circular motion at constant orientation) between the two sub-parts of the traveling plate. Consequently, the Input/Output kinematic relations of the Par4 are identical to those of the H4, but the possibilities for rearranging the actuator locations are much better. Indeed, they can be placed in a symmetrical position, while avoiding internal singularities within the workspace, as it can be shown in Figure 6, and in the detailed kinematic analysis of [20].

The rotation of the traveling plate parallelogram allows for the fourth $dof$, but the range of this motion ($\pm45$ degrees) is not large enough for many practical tasks. Therefore, an
amplification system has to be added to increase the range to +/-180 degrees. This is done here through the belt and pulley device that is shown in Figure 5, but could be done with other technologies as well. The payload has to be attached to an axis located on one half-traveling plate. This choice makes the design, construction and assembly straightforward even though it may not be optimal in terms of dynamic behavior as will be shown later.

![Figure 5. Par4 prototype with details about its traveling plate.](image)

**IV. KINEMATICS**

In this section, the Inverse Kinematic Problem (IKP) is solved and some remarks are made regarding the Forward Kinematic Problem (FKP). The following notation is used (see Figure 6):

- \( \{O, e_x, e_y, e_z\} \) the base frame;
- \( i \) a number designating each chain \( (i = 1 \ldots 4) \);
- \( A_i \) and \( A_{i_1} \) the centers of the ball-joints on arm \( i \);
- \( B_i \) and \( B_{i_1} \) the centers of the ball-joints on the travelling plate (involved in chain \( i \));
- \( A \) the middle of \( [A_{i_1}, A_{i_2}] \); \( B \) the middle of \( [B_{i_1}, B_{i_2}] \);
- \( b_i = \overrightarrow{AB} \);
- \( q_i \) the actuated joint positions;
- \( n_i \) the ‘centre’ of the actuated joint, that is a point on the actuated axis, in the arm’s plane of symmetry; \( a_i = \overrightarrow{PA} \);
- \( L_i \) the arm length (between \( P_i \) and \( A_i \));
- \( u_i \) unit vectors along the actuated joints;
- \( l_i \) rod length (between \( A_i \) and \( B_i \));
- \( h_i \) and \( d_i \) distances between \( C_i \) and \( B_i \), respectively, along \( y \) axis, and along \( x \) axis;
- \( d \) the travelling plate length (along \( x \) axis);
- \( h \) the length of the travelling plate’s parallelogram bars;
- \( D \) the controlled point;
- \( x, y, z, \theta \) the task space coordinates (position of point \( D \); motion angle of the parallelogram);

The kinematic relationship is based on the following equation:

\[
\|\overrightarrow{AB}\|^2 = l_i^2 \quad (i = 1, \ldots, 4)
\]  

(2)

Expressions of points \( A_i \) can be easily derived (knowing \( P_i \) which are given by the robot construction) as functions of \( q_i \) and a few geometric constants (namely: \( L_i \)). On the other hand, the position of points \( B_i \) can be expressed as functions of the end-effector pose (i.e., \( x, y, z \) and \( \theta \)) and a few geometric constants (namely: \( d, h, d_i \) and \( h_i \)).

Finally, Eq. (2) can be written as a system of four equations:

\[
I i \sin q_i + J_i \cos q_i + K_i = 0 \quad (i = 1, \ldots, 4)
\]  

(3)

where \( I_i, J_i, K_i \) are functions of the robot geometry.

The IKP solutions are obtained by solving each equation of system (3) independently, as follows:

\[
q_i = 2 \tan^{-1} \left( \frac{-I_i \pm \sqrt{\Delta_i}}{K_i - J_i} \right) \quad (i = 1, \ldots, 4)
\]  

(4)

with \( \Delta_i = I_i^2 - K_i^2 - J_i^2 \).

It is worth noting that this model is extremely similar to the inverse kinematics of the Delta robot [29].

![Figure 6. Parameters and notations](image)

Solving the FKP is equivalent to solving system (3) for \( x, y, z \) knowing \( \theta \). However, this leads to an 8th order polynomial in \( \theta \) [29], contrary to the Delta robot where the polynomial is of 2nd order only [30]. For Par4, it is then more
efficient to solve the FKP with a numerical iterative scheme than to search for all solutions analytically. Note that no further research has been carried out so far regarding the FKP, such as the number of real roots of the polynomial; indeed the authors were mostly interested with practical considerations about the FKP and it turns out that it has only one solution over the workspace which is used for practical applications.

V. DYNAMICS

The dynamic model of PKM can be derived with systematic approaches such as that proposed by Khalil [31], where the contributions of all bodies can be accounted for. We propose here to use a simpler method based on a few realistic assumptions (these assumptions are hereafter verified and validated) and an Euler-like approach. The method is straightforward and the resulting model is compact.

A. Parameters and assumptions

In a nutshell, the torque necessary to perform a given motion is seen here as the sum of two major effects: 1) the effects at the traveling plate level (including the payload), and 2) the effects at the arm level (including the actuation system). One key assumption is to consider the rod inertia as small enough to be neglected and to consider their mass as two pinpoint masses located at each end, as shown in Figure 7.

An additional assumption is made to simplify the model when considering the contribution of the traveling plate: the bars of the traveling plate parallelogram are assumed to be equivalent to two pinpoint bodies located at each end (this is equivalent to stating that the bar inertia is neglected). The traveling plate can then be split (Figure 8) into two subparts (a) and (b) that are bodies moving in translation only and whose masses are, respectively, \( m_1 + m_3 + 2m_4 \) and \( m_2 + m_3 + 2m_5 \).

B. Contribution of the ‘arm level’

With these assumptions, the contribution of the actuation system (i.e. actuator, arm and pinpoint mass at the arm end) can be written as

\[
\tau_{\text{act}} = I_{\text{act}} \ddot{q} - g M_{\text{arm}} \cos(q) + F_v \ddot{q} \tag{5}
\]

where

\[
I_{\text{act}} = \text{diag}(\begin{bmatrix} I_{\text{eq}} & I_{\text{eq}} & I_{\text{eq}} & I_{\text{eq}} \end{bmatrix})
\]

\[
I_{\text{eq}} = n^2 I_{\text{act}} + I_{\text{arm}} + m_4 L^2
\]

\[
M_{\text{arm}} = \text{diag}(\begin{bmatrix} m_{\text{eq}} & m_{\text{eq}} & m_{\text{eq}} & m_{\text{eq}} \end{bmatrix})
\]

\[
m_{\text{eq}} = L_G m_4 + L m_5
\]

\[
\cos(q) = [\cos(q_1) \cos(q_2) \cos(q_3) \cos(q_4)]^T
\]

with \( g \) being the gravity acceleration, \( L_G \) the distance between the arm pivot axis and the arm gravity center, and \( m_4 \) the arm mass. Coulomb and viscous friction effects, \( f_c \) and \( f_v \) respectively, are chosen to be equal for each actuator:

\[
f_v = f_c \begin{bmatrix} \text{sign}(q_1) & \text{sign}(q_2) & \text{sign}(q_3) & \text{sign}(q_4) \end{bmatrix}^T
\]

\[
F_v = \text{diag}(\begin{bmatrix} f_c & f_c & f_c & f_c \end{bmatrix})
\]

C. Contribution of the ‘traveling plate level’

As shown in Figure 8, the two main bodies of the end effector are denoted by (a) and (b). The end effector velocity \( \mathbf{v} = [\dot{x} \dot{y} \dot{z} \dot{\theta}]^T \) leads to velocities of points \( D, v_B \) and \( D', v_{B'} \), expressed as follows:

\[
v_B = T_B \mathbf{v} \quad T_B = \text{diag}(\begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}) \tag{6}
\]

\[
v_{B'} = T_{B'} \mathbf{v} \quad T_{B'} = \begin{bmatrix} 1 & 0 & 0 & h \sin \theta \\ 0 & 1 & 0 & -h \cos \theta \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}
\]

These velocities are related to the actuated joint velocities \( \dot{q} \) as follows:

\[
J_{\text{v,a}} \dot{v}_B = J_{\theta} \dot{q} \quad \text{and} \quad J_{\text{v,b}} \dot{v}_{B'} = J_{\theta} \dot{q} \tag{8}
\]

where

\[
J_{\text{v,a}} = \begin{bmatrix} b_1^T & -h \cos \theta b_{1,x} & -h \sin \theta b_{1,z} \\ b_2^T & -h \cos \theta b_{2,x} & -h \sin \theta b_{2,z} \\ b_3^T & 0 \\ b_4^T & 0 \end{bmatrix}
\]

\[
J_{\text{v,b}} = \begin{bmatrix} b_1^T & 0 \\ b_2^T & 0 \\ b_3^T & h \cos \theta b_{3,x} + h \sin \theta b_{3,y} \\ b_4^T & h \cos \theta b_{4,x} + h \sin \theta b_{4,y} \end{bmatrix}
\]
\[ J_q = \begin{bmatrix} (b_x \times a_x) v_{i} & 0 & 0 & 0 \\ 0 & (b_x \times a_x) v_{i} & 0 & 0 \\ 0 & 0 & (b_x \times a_x) v_{i} & 0 \\ 0 & 0 & 0 & (b_x \times a_x) v_{i} \end{bmatrix} \]

From Eq. (6), the acceleration of the pinpoint mass at point D is given by \( \dot{v}_s = [\ddot{x} \ \ddot{y} \ \ddot{z} \ 0]^T \) and it produces an inertial force expressed as follows:
\[ f_s = M_s (\dot{v}_s + g) \]  
(9)

where
\[ M_s = \begin{bmatrix} m_s + m_i + 2m_s & 0 & 0 & 0 \\ 0 & m_s + m_i + 2m_s & 0 & 0 \\ 0 & 0 & m_s + m_i + 2m_s & 0 \\ 0 & 0 & 0 & m_s + m_i + 2m_s \end{bmatrix} \]

\( g \) is the gravity vector \( g = [0 \ 0 \ -9.81 \ 0]^T \) \{m/s²\}.

Once mapped into the joint space, this force gives the torque necessary to move the subpart (a), expressed as follows:
\[ \tau_s = J_s^T M_s (\dot{v}_s + g) \]  
(10)

where \( J_s = J_{s,v}^{-1} J_q \).

For subpart (a), the acceleration of the pinpoint mass at point D' is obtained by deriving Eq. (7) with respect to time:
\[ \ddot{v}_s = T_s \ddot{v} + T_s v \]  
(11)

Such acceleration creates on the subpart (a) an inertial force given by:
\[ f_s = M_s (\dot{v}_s + g) \]  
(12)

where
\[ M_s = \begin{bmatrix} m_s + m_i + 2m_s & 0 & 0 & 0 \\ 0 & m_s + m_i + 2m_s & 0 & 0 \\ 0 & 0 & m_s + m_i + 2m_s & 0 \\ 0 & 0 & 0 & m_s + m_i + 2m_s \end{bmatrix} \]

The torque necessary to move subpart (a) is then expressed as follows:
\[ \tau_s = J_s^T M_s (\dot{v}_s + g) \]  
(13)

Where \( J_s = J_{s,v}^{-1} J_q \).

Finally, a payload can be carried by the travelling plate and the torque necessary to move it is expressed as
\[ \tau_p = J_p^T M_p (\dot{v} + g) \]  
(14)

where \( M_p = \text{diag}(\{m_p \ m_p \ m_p \ I_p\}) \), \( m_p \) is the payload mass and \( I_p \) is the payload inertia.

It is worth noting that for the specific design at hand, the payload has to be attached at point D. Thus, \( J \) and \( J_q \) are equal. Moreover, all components of the \( M_4 \) fourth column are zeros, and multiplying \( M \) by \( \dot{v} \) or \( v \) makes no difference. Then, the total contribution of the travelling plate plus payload can be expressed as follows:
\[ \tau_{me} = J^T M (\dot{v} + g) + J_s^T M_s (\dot{v}_s + g) \]  
(15)

where \( M = M_s + M_p \).

D. Dynamic model and its validation

The contributions described in Eqs. (5) and (14) can be grouped together to obtain the overall (but simplified) dynamic model:
\[ \tau = \tau_{act} - g M_{\text{arm}} \cos(q) + f_\text{act} + F_q + J^T M (\dot{v} + g) + J_s^T M_s (\dot{v}_s + g) \]  
(16)

This simplified model was validated by simulating the robot dynamics using Adams® software and comparing the obtained results (motor torques) with those given by the model of Eq. (16). The aim was to verify if the modeling is correct, but also to determine the consequences of the simplifications: the robot was completely modeled using Adams, without any simplifications. The comparison has been carried out for several typical pick-and-place motions within the robot practical workspace.

This comparison shows that the proposed simplifications have very little impact on the final results. The error between our simplified dynamic model and the complete model is always less than 2%. Moreover, it is worth noting that the proposed model is suited for identification of parameters, as shown hereafter.

E. Identification of Dynamic Parameters

The dynamic parameters can be identified through the following classical scheme [32]:
\[ \tau_{act} = W \chi \]  
(17)

where \( \tau_{act} \) is the torque exerted by the actuators, \( W \) is the regressor matrix, while \( \chi \) is the vector of parameters to be identified. This identification was performed under the following conditions: (i) no payload was carried by the manipulator; and (ii) all chains were supposed to be identical. Under such assumptions, \( W \) and \( \chi \) are given as
\[ W = \begin{bmatrix} \dot{q} \ J_s^T v_s \ J_s^T v_s - \cos(q) \ & \ \text{sign}(q) \ \ & \ \dot{q} \end{bmatrix} \]

\[ \chi = \begin{bmatrix} \dot{q} \ \dot{q} \ \dot{q} \ \end{bmatrix} \]

Solving Eq. (17) for \( \chi \) implies that series of measurements are obtained by applying adequate commands onto the motor.
drives and measuring the actuated joint positions accordingly. This requires that:

- the relation between the command signal and the actual torque is known; this relation was evaluated experimentally by measuring the force exerted by the arm for various motor commands (under the assumption that this relation is linear);
- the actuated joint velocities and accelerations are known but only the positions are measured; numerical derivation (centered differentiation technique) and digital filtering (Butterworth filter) give velocity and acceleration estimations.

Identification results as well as the relative standard deviations are given in Table 1. The very low values of \( \% \sigma \) are indicators of a good identification.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>% ( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n^2 I_{d_e} + I_{arm} + m_p L^2 )</td>
<td>0.113 kg.m²</td>
<td>0.872</td>
</tr>
<tr>
<td>( m_1 + m_1 + 2m_s )</td>
<td>0.8043 kg</td>
<td>0.620</td>
</tr>
<tr>
<td>( m_x + m_1 + 2m_s )</td>
<td>0.8594 kg</td>
<td>0.596</td>
</tr>
<tr>
<td>( g (L_G m_x + L m_s) )</td>
<td>1.33 kg.m².s⁻²</td>
<td>1.261</td>
</tr>
<tr>
<td>( f_s )</td>
<td>2.43 N.m</td>
<td>0.488</td>
</tr>
<tr>
<td>( f_v )</td>
<td>20.4 N.m.s.rad⁻¹</td>
<td>0.372</td>
</tr>
</tbody>
</table>

Table 1 - Identification results

This fact was confirmed by an experimental validation: for a given motion, torques are measured on the robot on the one hand, and are estimated with the model and the identified parameters on the other. Plots in figure 9 confirm that the real and simulated data are extremely close.

**VI. PERFORMANCE EVALUATION**

The performance tests were carried out with a linear controller running on a CEREBELLUM™ control system (see Figure 10).

The first test involves a linear motion along the x axis with a range of 305 mm long, a peak velocity of 5.3 m/s and a peak acceleration of 158 m/s² (which is about 16 g). Performing this motion back-and-forth (thus, traveling 610 mm) takes only 0.236 s and the robot stabilizes in less than 8 ms. Figure 11 shows the behavior of joint No.1 during such motions: desired and measured positions (which are almost identical thanks to a very small tracking error), as well as tracking errors, are plot for 7 displacements of 305 mm, back and forth, which are equivalent to an arm motion of 27 deg. The tracking error is kept below ±0.15 deg which demonstrates the prototype’s ability to perform ultra high speed motions with good accuracy.

The second test is the industry standard “ADEPT cycle”. It represents a typical pick-and-place task and first requires a vertical motion (see Figure 12) at the picking location (up 25 mm), then a linear horizontal motion (305 mm) together with a 180 deg rotation, a vertical motion at the placing location (down 25 mm), and the same trajectory back: the overall time for performing this motion is measured. Two clothoids connect vertical and horizontal lines, so that the acceleration path does exhibit discontinuities. In this test, the peak linear acceleration is 165 m/s², the peak velocity 5.5 m/s, the peak rotational acceleration 1200 rad/s² and the peak rotational velocity is 26 rad/s: these data correspond to a cycle time of 0.243 s. This cycle time is shorter than those achieved by any existing commercial manipulator.

Note that the ADEPT cycle is more “aggressive” than a simple linear motion. Indeed, as shown in Figure 13, the non-linear kinematic transformation between task-space and joint-space transforms the quite smooth Cartesian trajectory into a more brutal joint trajectory; fast changes in joint direction of motion occur, resulting in very challenging joint accelerations.
and high joint torques. This is why the maximum joint error is larger (0.19 deg instead of 0.15 deg). However, the resulting behaviour of the parallel manipulator, in terms of tracking error and cycle time, remains more than satisfactory.

![Figure 13. Position and tracking error of joint no. 1 during an Adept cycle.](image)

VII. PROTOTYPE OPTIMIZATION

Transforming a conceptual design of a mechanism into an effective prototype able to achieve ultra-high performances requires a careful optimization that must consider not only the kinematics, but also the dynamics of the whole system, including the actuation system. These three aspects are briefly discussed below.

A. Kinematic Optimization

This first phase of the optimization is intended to search for a parallel manipulator geometry that is as compact as possible, able to reach a given Adept cycle time (in terms of ‘feasible velocity’), and exhibits homogeneous behavior throughout the workspace.

The parameters involved in the optimization process are (see Figure 14) the lengths of the arms and rods, \( L \) and \( l \), the radius of the base, \( R \), and the location of the workspace along the vertical axis, \( Z_0 \). The workspace is chosen as a cylinder of diameter \( D = 1 \text{ m} \) and height \( H = 0.25 \text{ m} \), based on typical customer requirements, especially in food packaging industries; note that the robot will be optimized for working in this workspace but that it will actually be able to access a much larger volume\(^1\). To fulfill these goals, an optimization scheme has been set up as follows:

Minimize the cost function \((I + L)\) under the constraints

\[
\max(\{\|q_i\|\}) < \text{Max Vel} \quad \text{for an Adept cycle time of} \quad \text{AdeptCycle Time}
\]

\[
\text{cond}(J) < \text{Worst Cond} \quad \text{throughout the workspace}
\]

where \( \text{Max Vel} \) is the maximum velocity that a given actuation system can provide; such a value is known when the type of actuation is known (e.g., brushless motor plus gears) and even the equipment category is chosen (e.g., high-end gears which accept high input velocities). Of course, this depends a lot on the gear ratio and several attempts may be necessary to cope with the commercially available gear ratios (e.g., a given supplier may offer 16:1, 21:1, 31:1 but not 26:1 …).

\( \text{AdeptCycle Time} \) is the duration of a 305 mm/25 mm Adept cycle; to ensure that this constraint is realistic, the actuator velocities are evaluated for such cycles at different locations throughout the workspace and with different orientations with respect to the base frame. \( \text{cond}(J) \) is the condition number of the Jacobian matrix defined as the ratio between its largest and smallest singular values. \( \text{Worst Cond} \) is chosen as a multiple of the smallest value of \( \text{cond}(J) \). This is a simple way to ensure that the robot will not be used too close to singular positions (where \( \text{cond}(J) \) goes to infinity) and also a way to ensure that the robot’s ability to perform motions along all directions is maintained.

![Figure 14. Parameters for the kinematic optimization](image)

As mentioned above, running such optimization depends on the kinematics (mainly: the size and shape of the workspace) and technological choices (the selection of the actuation system provider has a direct influence on the range of available gear ratios). The optimal gear ratio can roughly be estimated by balancing the inertia of motor plus gear on one side and the equivalent inertia of arm plus traveling plate on the other. Indeed, it is well known that for a system composed of a motor (inertia \( I_m \)), a gear (inertia \( I_g \); ratio \( n \)) and a constant load (inertia \( I_l \)) the gear ratio which provides the highest acceleration capability is given by:

\[
n^2 (I_m + I_g) = I_l
\]

In the case of non-constant load (on a robot, even if the load itself is constant, the equivalent load seen at the joint level is not constant), a rough estimate can be done by focusing on the equivalent load at the joint level created by the traveling plate at a specific location, e.g., the center of workspace. The preliminary prototype was designed with such an approach after selecting ALPHA GEAR DRIVES as actuator supplier. An
integrated motor plus gear system was selected with the following main features:

\[ n = 21 \]
\[ \text{Max. Vel} = 23 \, \text{rd/s} \]

Moreover, Worst Cond was chosen as four times larger than the condition number in the center of the workspace. The optimal dimensions of the preliminary industrial prototype (see Figure 15) are as follows: \( l = 0.825 \, \text{m}, L = 0.375 \, \text{m}, R = 0.275 \, \text{m}, z_0 = 0.58 \, \text{m} \) (these values are obtained with \( \dot{q}_{\text{max}} = 21.9 \, \text{rd/s} \)).

![Figure 15. Preliminary industrial prototype (FATRONIK QUICKPLACER) [33]](image)

**B. Dynamic Optimization**

In addition to this kinematic optimization, extensive tests have been carried out in order to observe the motor torques during several types of motion. One very interesting test was a 305 mm / 25 mm Adept cycle along the \( x \) axis under 8 g acceleration, while carrying a 1 kg payload.

Figure 16 shows two curves representing the torque of joints 2 and 3. With the displacement being performed along the \( x \) axis (i.e. in a plane of symmetry for this robot), it could be expected that joints 2 and 3, which are moving at exactly the same speed and acceleration, may provide the same torque. Figure 16 shows a 30% difference and it is worth noting that this is also true for motors 1 and 4. However, there is no significant difference when the motion is performed along the \( y \) axis.

![Figure 16 – Torque of joint 2 and 3 during an x-axis motion with payload](image)

Extensive tests have indeed shown that this phenomenon is exclusively related to the direction of the Cartesian acceleration. It is thus interesting to focus on elements directly related to \( \dot{v} \) in the dynamic model. Equation (15) shows that \( J^T \) is the only element which has a direct and straightforward interaction with \( \dot{v} \). Understanding the complete dynamics equation may be too complex, but analyzing the transpose of the Jacobian in a specific posture may be helpful. In fact, if the center of the workspace is considered, and the traveling plate placed such that \( \theta = 0 \), then all rods are arranged symmetrically, as shown in Figure 17.

![Figure 17. Components of vectors \( \mathbf{b}_i \) at a centered posture.](image)

Then, \( J^T \) has the following form:

\[
J^T = \begin{bmatrix}
-J_{q,1} & 0 & 0 & 0 \\
0 & J_{q,2} & 0 & 0 \\
0 & 0 & J_{q,3} & 0 \\
0 & 0 & 0 & J_{q,4}
\end{bmatrix}
\]

The two zeros in the transpose of the Jacobian matrix (first column) have the following consequence: if the payload is accelerated along the \( x \) axis, it produces no torque on motors 1 and 2 but it produces a torque on motors 3 and 4. Obviously, even if the demonstration is performed at a centered position, this is indeed the reason why the torque on motor 1 (or motor 2) is lower than that on motor 3 (or motor 4) for motions along the \( x \) axis. Therefore, we proposed to modify the design of the traveling plate in order to allow better load sharing between bodies (\( \alpha \) and \( \beta \)). The load may be fixed on one of the two rotating bars of the traveling plate. This can be done in a perfectly symmetric way, if a third bar is added to the parallelogram, as shown in Figure 18, but alternative designs are possible as well.
Figure 18. The proposed modified design for the traveling plate.

Figure 19. Torque of joint No 3 during an ADEPT cycle of 15 g while carrying a 2 kg payload

Under these conditions, the transpose of the Jacobian matrix has the following form:

\[
J^T = \begin{bmatrix}
-\frac{J_{q,1}}{4b_1} & -\frac{J_{q,2}}{4b_2} & -\frac{J_{q,3}}{4b_3} & -\frac{J_{q,4}}{4b_4} \\
\frac{J_{q,2}}{4b_2} & \frac{J_{q,3}}{4b_3} & \frac{J_{q,4}}{4b_4} & 2\alpha b_1 \\
\frac{J_{q,3}}{4b_3} & \frac{J_{q,4}}{4b_4} & \frac{J_{q,1}}{4b_1} & 2\alpha b_2 \\
\frac{J_{q,4}}{4b_4} & \frac{J_{q,1}}{4b_1} & \frac{J_{q,2}}{4b_2} & 2\alpha b_3
\end{bmatrix}
\]

In this case, the load acceleration effects are perfectly shared among the four motors (in the centered posture). This simple modification has a great effect on the torque demand during a pick-and-place motion. This can be demonstrated by comparing the torque required on joint No 3 (chosen here as an example) during an Adept cycle with 15 g acceleration and a 2 kg payload. As shown in Figure 19, the maximum torque is reduced by 30%. These optimization approaches were used to improve the preliminary industrial prototype shown in Figure 15 and to propose the first commercial version of Par4 shown in Figure 20.

VIII. CONCLUSIONS

With the first commercially available version of the high-speed pick-and-place parallel manipulator presented in this paper, a research cycle has been completed: starting with the architectural “articulated traveling plate” concept, an innovative manipulator architecture was proposed which is free of any singular postures within its workspace; the preliminary prototype has succeeded in achieving high speeds (more than 4 m/s) and high accelerations (more than 15 g); the kinematics, dynamics and actuation optimization has allowed us to further enhance these performances with better load sharing in the commercial version of the manipulator.

The technology transfer between Academia and Industry has required a lot of additional work that is not described in this paper: the world of pure engineering design which turns prototypes into products begins beyond mathematical models and numerical optimization. Tremendous attention has to be paid to “details” such as the material and surface treatment selection, stress analysis, fine control tuning, components integration, marketing … and above all, extensive tests.

Delta and Par4 architectures can be compared at two levels:
- As far as conceptual designs are concerned, the key differences are, obviously (i) four chains in parallel instead of three and (ii) an articulated traveling plate instead of a solid one. Having four chains for a 4-dof robot, Par4 is closer to the “parallel mechanism paradigm” and makes use of the four motors for all motions, offering a better load sharing among actuated chains and thus, better overall performance. The Delta simple traveling plate can be made smaller and lighter than the more complex traveling plate of Par4, but this is done at the cost of the ‘telescopic’ chain complexity and the corresponding added inertia. On the
other hand, the large size of the Par4 traveling plate becomes an advantage when bulky equipment has to be installed (cables, pneumatic components, sensors, grippers) which is almost always the case in practical applications.

- As far as commercial products are concerned, it is very complicated to draw a comparison because the raw technology and marketing will probably be as important as the "pure concepts" or "scientific optimization" for guiding customer choices. For example the quality of the control system, including the integration of vision control, or the stiffness of the actuation systems can make huge differences among products, and the efficiency of marketing and after-sales services can dramatically transform the market perception of a product. Such a comparison is obviously beyond the scope of this scientific paper.

Overall we believe that the work and results on this 4-dof parallel manipulator presented here offer a realistic competitor to SCARA and Delta robots.

REFERENCES


