Deeper into the duality between coalescent and drift

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A paradigm shift

1859 - ~1970
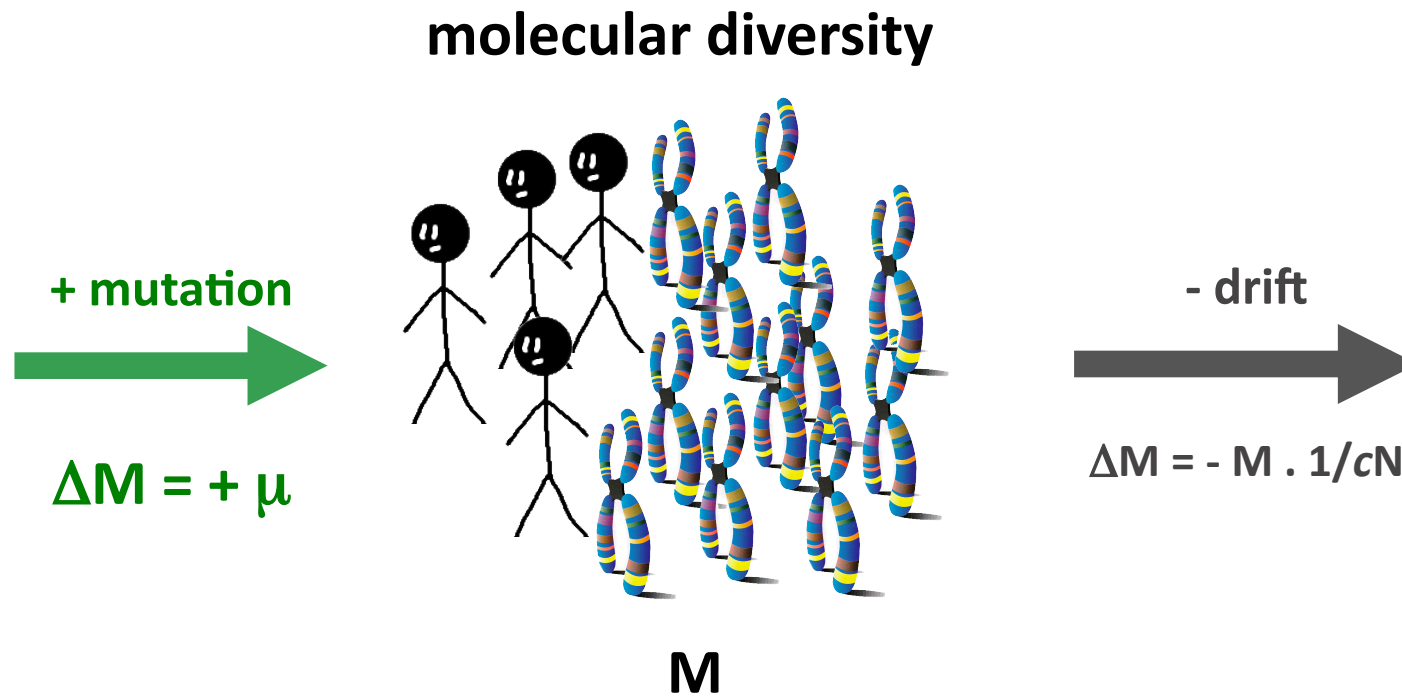   Evolution is driven by **adaptation**

1970 – Today
    Molecular evolution is driven by **genetic drift**

    Kimura (1968) *Evolutionary Rate at the Molecular level*
    Jukes and Kings (1969) *Non Darwinian Evolution*
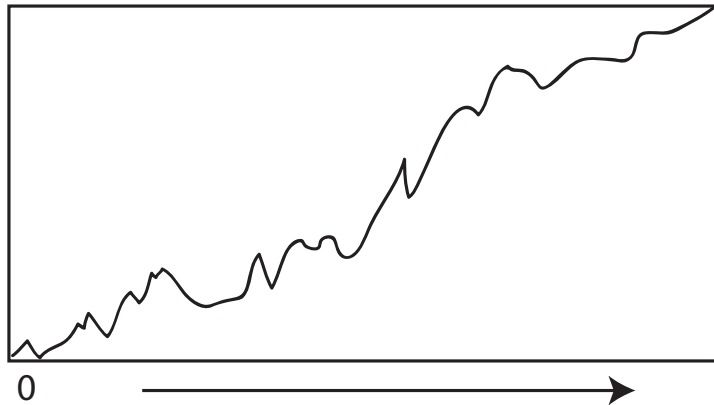
Today's H0 = a mutation-drift equilibrium

The mutation-drift equilibrium

molecular diversity

+ mutation

$\Delta M = + \mu$

- drift

$\Delta M = - M . 1/cN$

M

At equilibrium, $\Delta M=0 => M^* = c\, N\mu$

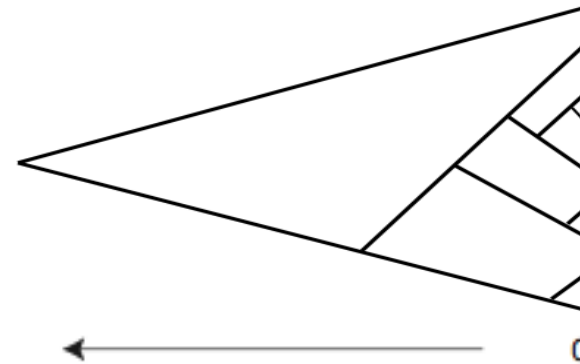# Two sides of the same coin

Genetic drift

Coalescent process

0
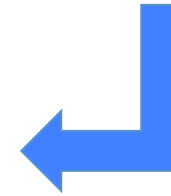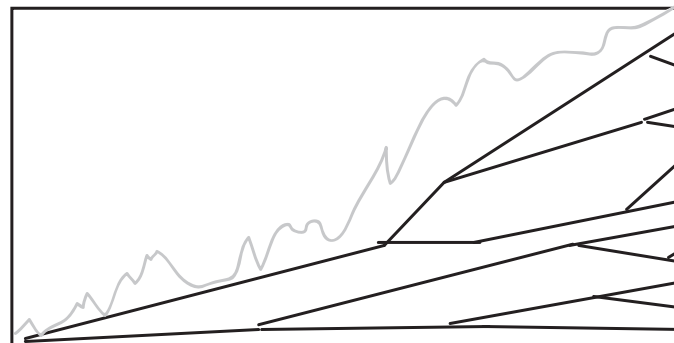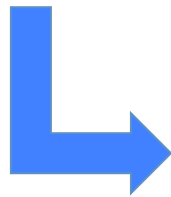
Forward time

Backward time

0

???

Fixation time / time to MRCA

Time is rescaled in the appropriate unit

(e.g. N for Wright-Fisher model; N(N-1)/2 for the Moran model)

Genetic drift   Coalescent process
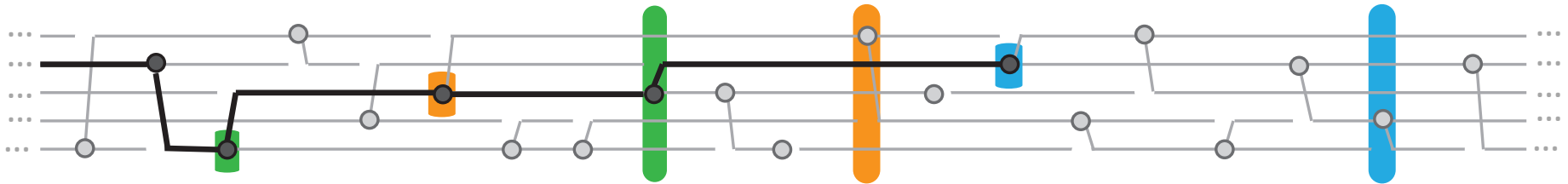
$E[T_{fix}] = 2$ $E[T_{MRCA}] = 2$
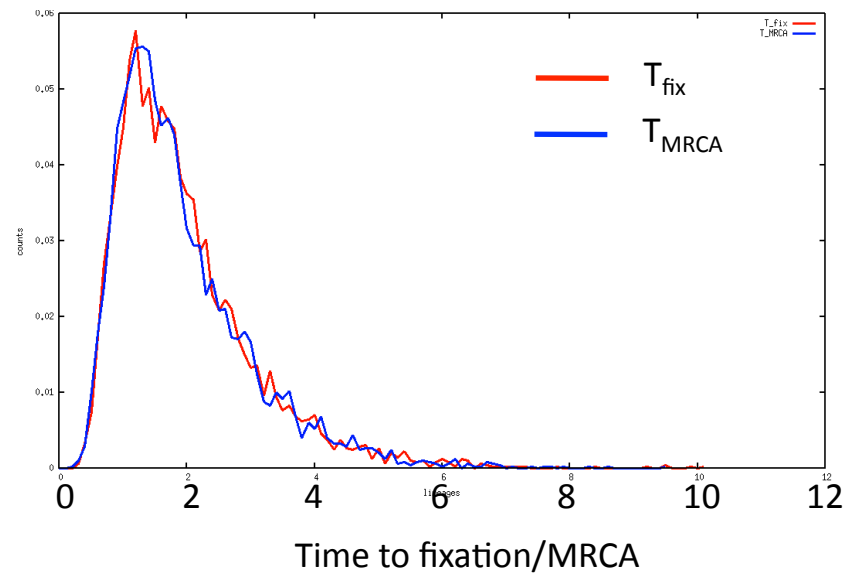
Are both distributions identical? If yes, why?

The Moran model (1958)

Following results hold for any exchangeable model (Cannings, 1972)
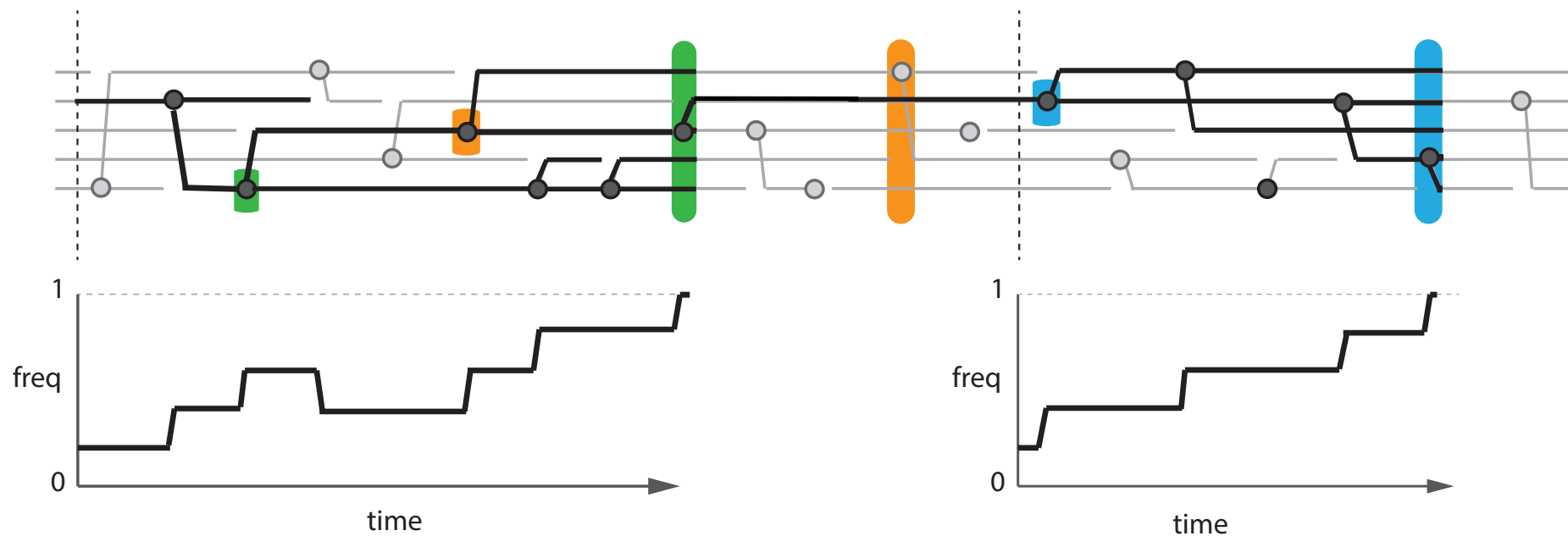(e.g. Wright-Fisher model)

1 generation = one new-born replaces one dead

Only one lineage survives indefinitely (here the thick black line)

(Griffiths, 1980)
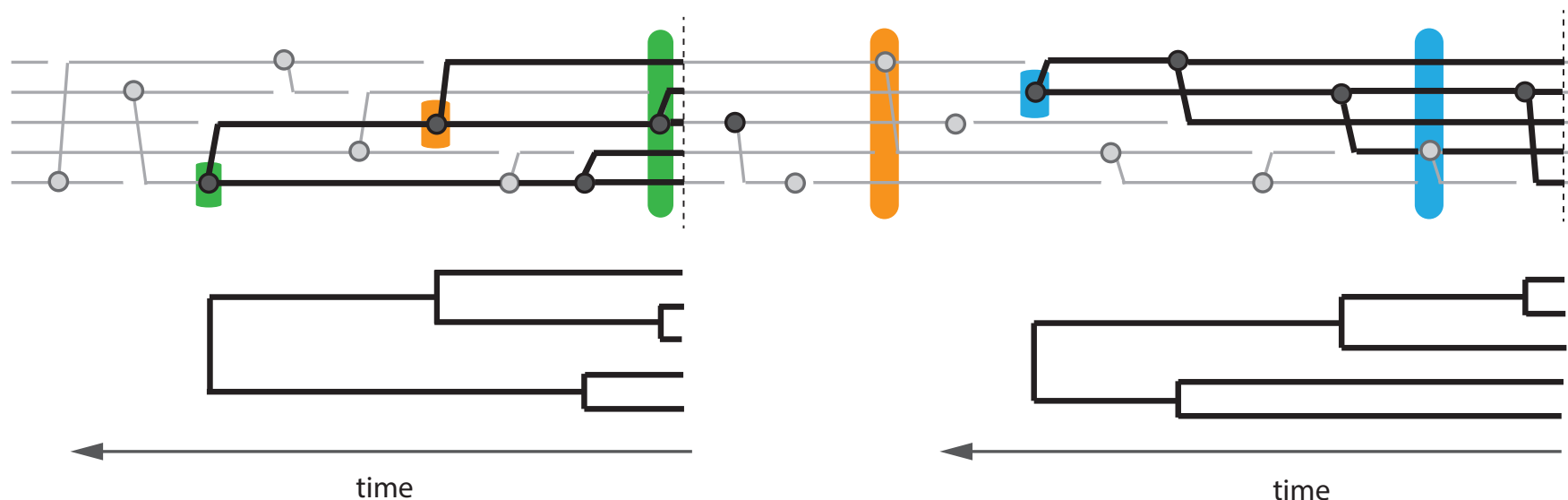
Forward drift in the Moran model

The fixation trajectories (forward perspective)

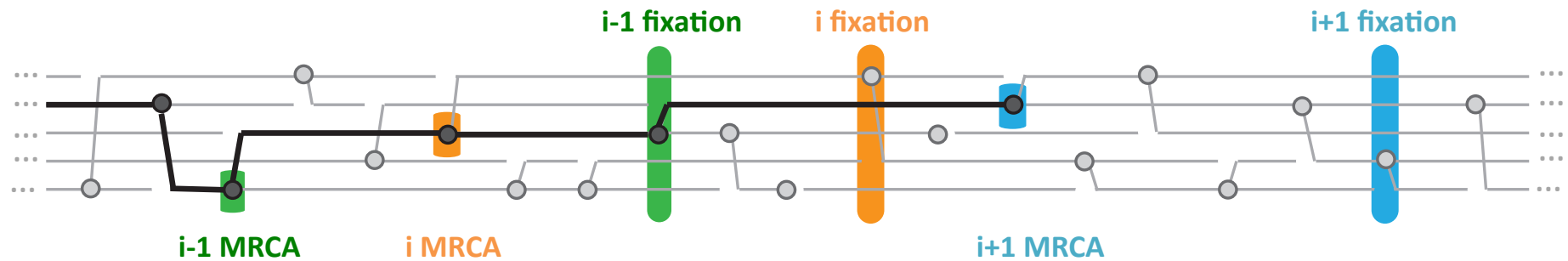Forward drift stops at successive fixation points

Backward coalescent in the Moran model

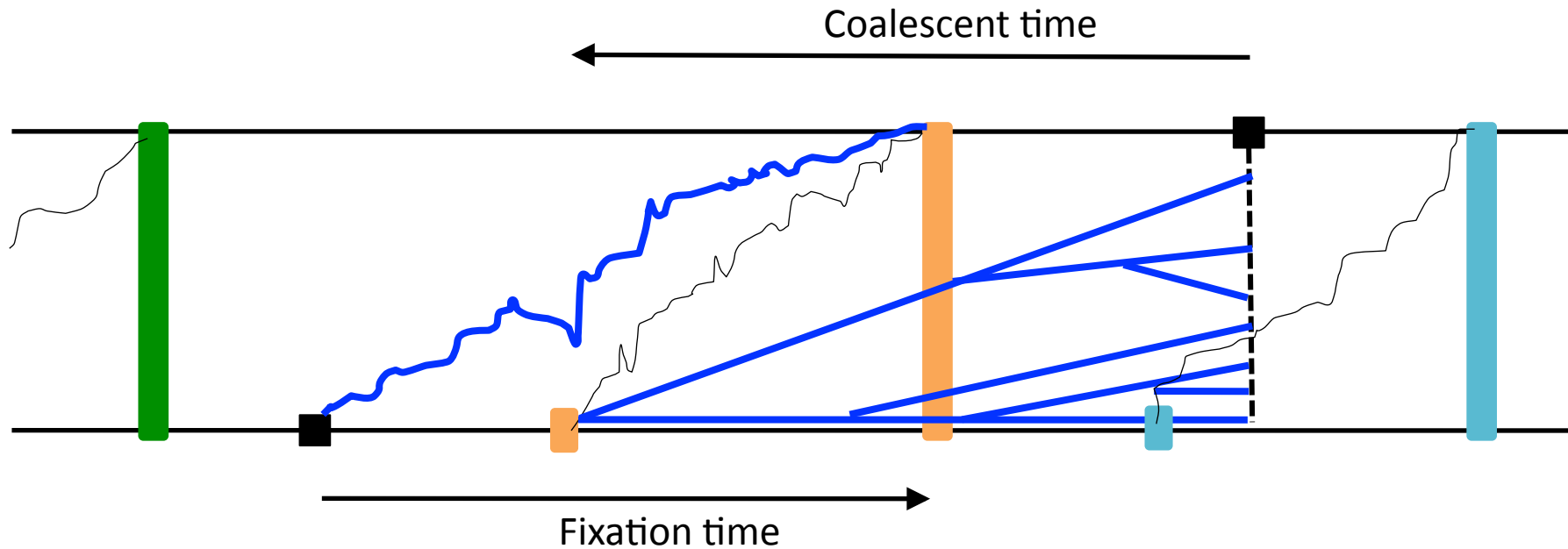The coalescent trees (backward perspective)

time

time

Backward coalescent stops at successive MRCA points

Milestones in the coalescent-drift duality

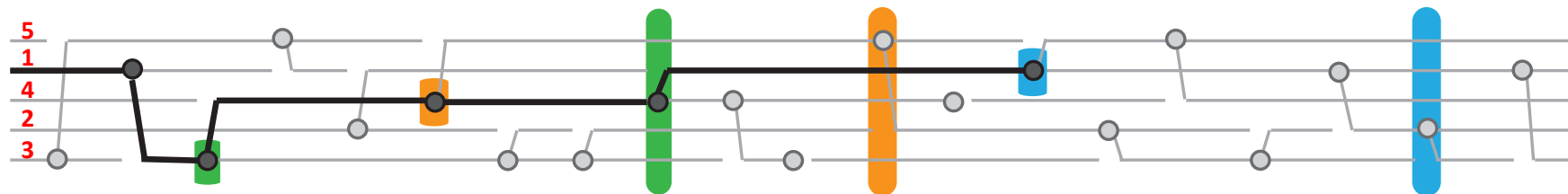The whole process has successions of pairs (MRCA -> fixation)

Forward – backward duality

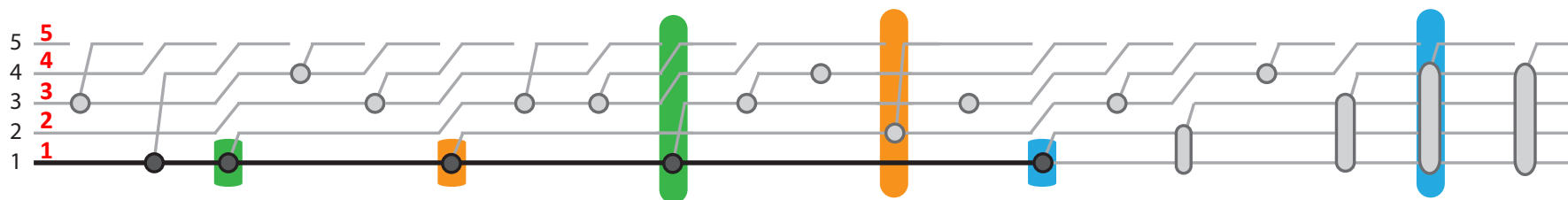
Forward drift and backward coalescent are only coupled between the MRCA and its fixation

Extracting analytical results
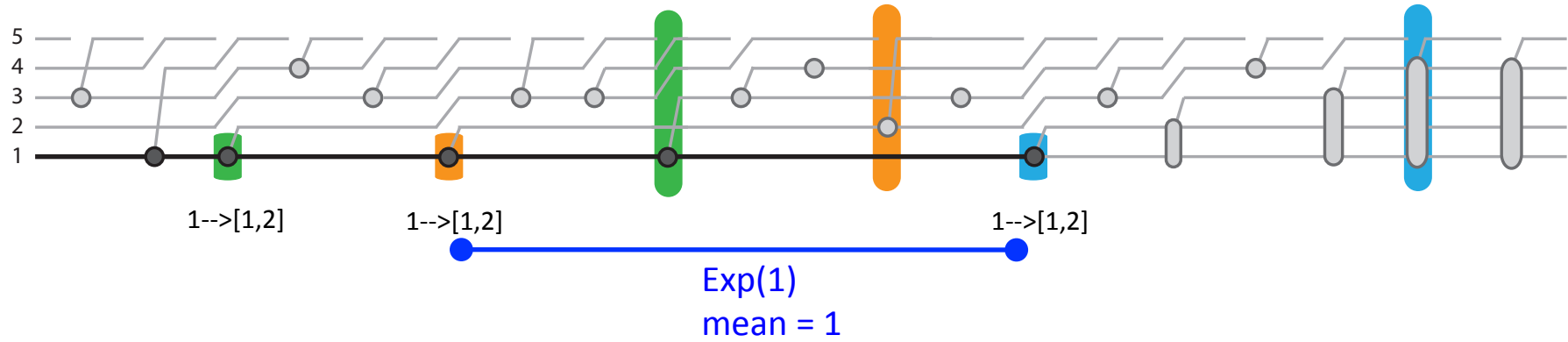
Classical representation

Lookdown dual representation (Donnely and Kurtz, 1996)

At every moments, lineages are ordered by their persistence

Waiting times

Time between two successive MRCAs

Waiting times

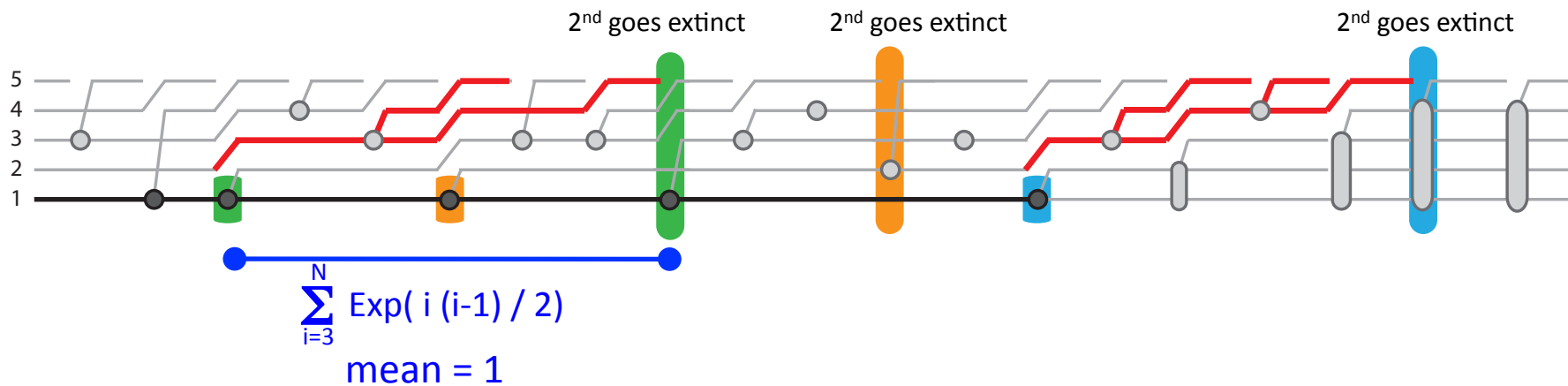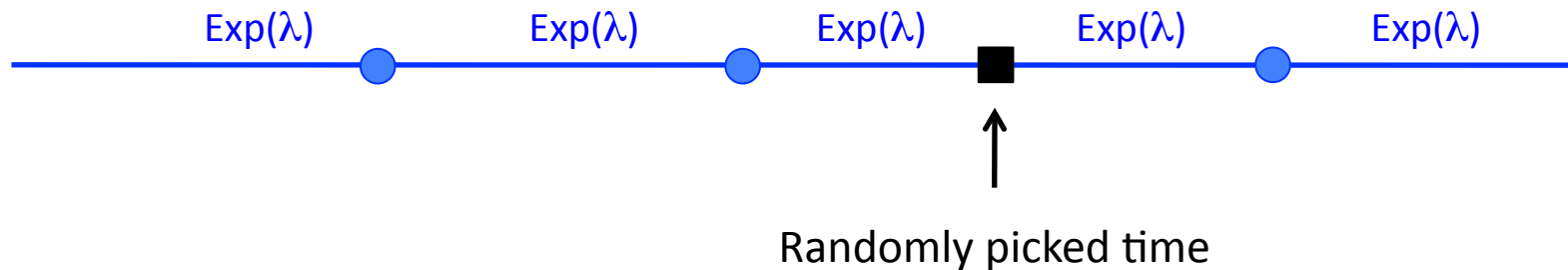## Time between two successive MRCAs

1-->[1,2]   1-->[1,2]   1-->[1,2]

$Exp(1)$
mean = 1
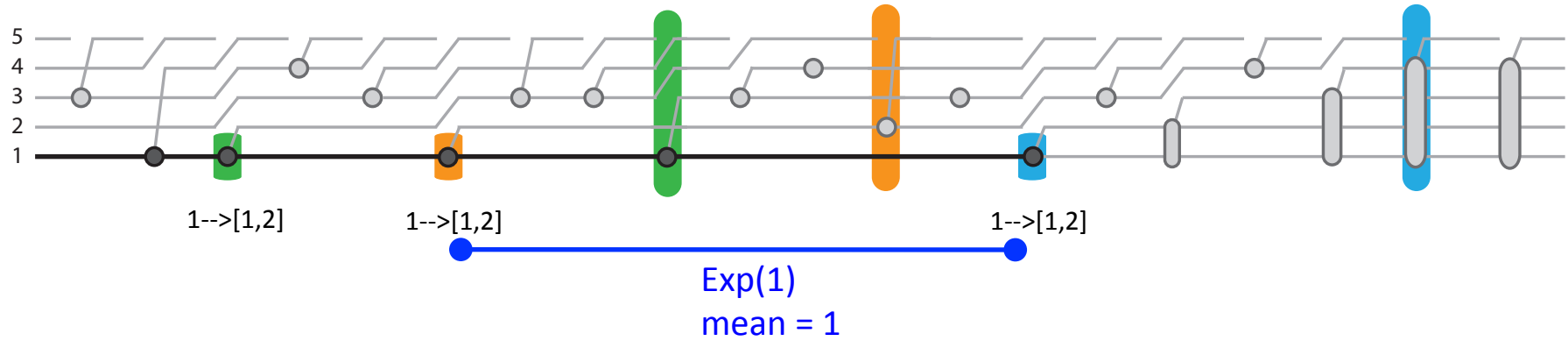
Time between MRCA and its fixation

2nd goes extinct   2nd goes extinct   2nd goes extinct

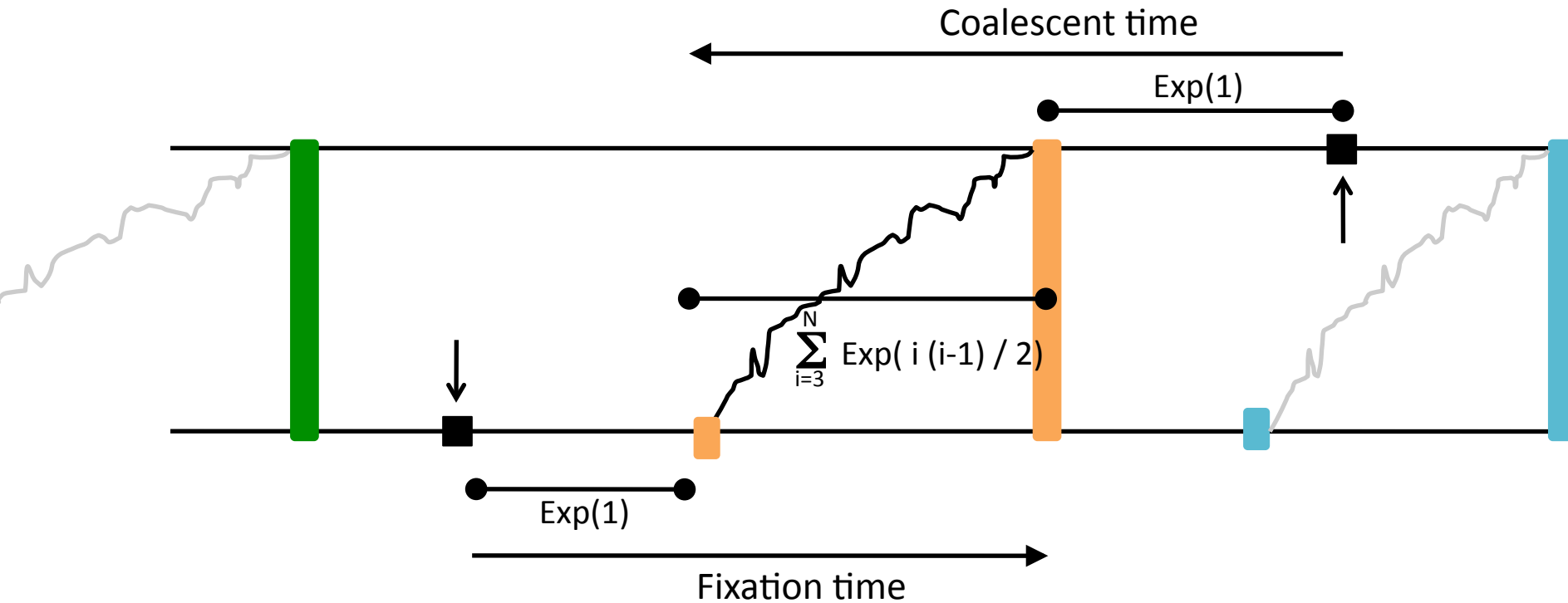$$\sum_{i=3}^{N} Exp(\,i\,(i-1)\,/\,2\,)$$

mean = 1

Picking a time point in Poisson process

On a Poisson process of rate λ (🔵)

A random fixed time (■) has a waiting time of rate λ to the previous and next event (🔵)

Exp(λ)   Exp(λ)   Exp(λ)   Exp(λ)   Exp(λ)

↑
Randomly picked time

The randomly picked time can be assimilated to an event

Coalescent and drift from a random point

Coalescent time

Exp(1)

$$\sum_{i=3}^{N} Exp(\ i\ (i-1)\ /\ 2)$$

Exp(1)

Fixation time

Fixation time (forward) and time to MRCA (backward) have the same distribution

$$\sum_{i=2}^{N} Exp(\ i\ (i-1)\ /\ 2)$$

Concluding remarks

Generally:

Even the most popular models have secrets to be unraveled

On the duality between drift and coalescent:

- Forward drift and backward coalescent are only <u>half</u> coupled

- Fixation time has the same distribution than coalescent TMRCA