A Trivalent Logic for Plural Predication and Quantification*

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Abstract

We describe a trivalent algebraic semantics for a typed language with plural predication and plural quantification designed to capture the peculiar behaviour of sentences with plural noun phrases in natural language. In this system, a notion of parthood is defined for the domains of all types, and a single algebraically formulated constraint applies to all of them. This constraint enforces the right pattern of trivalence for plural predication when applied to the domain of relations, and the right behaviour for (trivalent) plural quantification when applied to the domain of quantifiers.

1 Introduction

It is standardly assumed in natural language semantics that the denotations of definite plural noun phrases are pluralities of entities, and that properties can be ascribed to these pluralities (Link, 1983). Such predications are known to be systematically trivalent (Löbner, 2000; Križ, 2015). For example, in a situation where Mary read only some of the books, (1a) has the third truth value, and so does, correspondingly, its negation (1b).

(1)

	a. Mary read the books.	b. Mary didn't read the books.
all books	1	0
only some	#	#
none	0	1

This raises the question of what the trivalent logic it is that this phenomenon obeys, in particular once quantifiers enter the picture.

2 Desiderata

There are a number of desiderata that a logical characterisation of the trivalent quantifiers of natural language ought to fulfil.

1. It allows for quantification over pluralities. This is necessary in order to account for the compatibility of plural quantifiers with collective predicates.

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(2) Two boys carried the piano upstairs (together).
'There is a plurality x consisting of two boys such that x carried the piano upstairs.'

2. Distributive predicates are predicates that are true of a plurality in virtue of being true of all the atomic individuals that make up the plurality. With such predictes, quantification over pluralities is equivalent to quantification over atoms.

   (3) There is a duality x of boys such that x smiled $\equiv$ There are boys x, y, $x \neq y$ such that x smiled and y smiled.

3. FO-definable quantifiers over atoms behave as in Strong Kleene logic (experiments by Križ and Chemla (2015)). For example, the universal quantifier is true if the scope is true of all members of the restrictor, and false if the scope is false of at least one member of the restrictor, otherwise undefined.

   (4) Every student read the books.
   1 iff every student read all the books.
   0 iff at least one student read no book.
   # otherwise.

   Together with 2, this also constrains quantifiers over pluralities to some extent.

4. Falsity conditions do not need to be stated lexically, but are systematically derivable from truth conditions, since natural language does not seem to have any (lexical) predicates or quantifiers with identical truth conditions but different falsity conditions.

3 Illustration

A good case to illustrate these desiderata, and why they are somewhat non-trivial to fulfill, is the quantifier *all the students*.

1. *All the students* is a quantifier over pluralities, since it is compatible with collective predicates.

   (5) All the students carried the piano upstairs together / met in the hallway.

   We therefore cannot analyse *all the students* as a simple universal quantifier over (atomic) students. Instead, what we have to say is that *all the students* is true of a predicate if and only if that predicate is true of the plurality denoted by *the students* (and then, in accordance with Desideratum 4, derive falsity conditions from that).

2. For distributive predicates, the quantifier over pluralities *all the students* is equivalent to the quantifier over atoms *every student*.

   (6) All the students arrived. $\equiv$ Every student arrived.

   Here it becomes apparent that while the truth conditions of $||\text{all the students}||(P)$ are the same as those of $P(||\text{the students}||)$, but their falsity conditions are different: while

$P(\|\text{the students}\|)$ is false only if P is false of every individual student (the original homogeneity fact), $\|\text{all the students}\|(P)$ is false as soon as P is false of at least one student.[1]

3. Križ and Chemla (2015) experimentally tested both *every* and *all* (on distributive predicates that contained a definite plural in object position) and found that both behaved like the universal quantifier in FO Strong Kleene logic.

4. There is no quantifier in natural language with the same truth conditions as *all the students*, but different falsity conditions.[2] For example, a quantifier that is true of P iff P is true of the plurality of all students, and false otherwise (and so never has the third truth value) is not a possible natural language quantifier.

Crucially, we have seen that the truth value of $\|\text{all the students}\|(P)$ depends not only on the truth value of P for the plurality of all students. In the case where P is undefined[3] of the plurality of all students, the truth value of $\|\text{all the students}\|(P)$ depends on, intuitively speaking, *why* P is undefined of that plurality. If it is undefined because it is true of some students and false of others, *all the students* must return falsity. If, however, P is undefined of the plurality of all students because it is undefined of at least some of the students (while not being false of any of them), then *all the students* must return the third truth value.

As a toy example, assume that our ontology contains only the two students a and b. Then *all the students* ought to be false of P, but true of P':

$$P = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto 0 \\ a \oplus b & \mapsto \# \end{bmatrix} \quad P' = \begin{bmatrix} a & \mapsto 1 \\ b & \mapsto \# \\ a \oplus b & \mapsto \# \end{bmatrix}$$

4 Proposal

We present a trivalent algebraic semantics for one-sorted type theory (without identity) that fulfils these desiderata and may serve as a language of analysis for a fragment of natural language including plural noun phrases.

Domains for all types come with an ordering relation $\preceq$. The domain of individuals is a Boolean algebra without the bottom element as per Link (1983), so that $\preceq$ is the individual parthood relation that relates individuals to pluralities of individuals. The domain of truth values contains # as the third truth value, which is the supremum of 0 and 1 (which are unordered with respect to each other), so that # effectively functions as the plurality consisting of 0 and 1. The order is generalised to the functional domains as shown below.[4] We call it parthood despite the fact that the functional domains are not in general models of classical mereology.

Generalised Parthood. For all $f, g \in B^A$, $f \preceq g$ iff for all $x \in A$, $f(x) \preceq g(x)$.

[1] We employ indirect interpretation and write $\|\cdot\|$ for the function that translates natural language into the logical language that we are eventually going to give a semantics for.

[2] This is because *the students* denotes an entity, not a quantifier. We might want to type-lift *the students* to obtain the corresponding Montagovian individual, but that quantifier is not lexical in the same way as *all*.

[3] For convenience, we use *undefined* as a technical term so that *undefined* : *the third truth value* :: *true* : *truth*. No further connotations are intended.

[4] This approach to treating trivalence in type theory, including the imposition of constraints on domains formulated in terms of the order, is heavily inspired by Lepage (1992), who, however, is not interested in the connection with pluralities.

We can now define a property of functions, which we call homogeneity for historical reasons of linguistic nomenclature, but which is, in fact, simply preservation of overlap.

Homogeneity. A function $f : A \to B$ is homogeneous iff for all $x, y \in A$: if there is a $z \in A$ such that $z \preceq x$ and $z \preceq y$, then there is a $z' \in B$ such that $z' \preceq f(x)$ and $z' \preceq f(y)$.

We then postulate two constraints on models. First, we constrain domains in the ontology to contain only homogeneous functions: $D_{\sigma\tau}$, for any types σ and τ, is the set of all homogeneous functions from D_σ to D_τ.

Second, we put a constraint on the interpretation function for lexical constants. What this constraint does it the following: taking as given the (sequences of) arguments for which the denotation of a relation or quantifier constant is true, it must, in all other cases, be false whenever it can without violating homogeneity. Formally, this can be stated as minimality with respect to the ordering $\preceq_0$, where $f \preceq g$ iff for all x, if $g(x) \neq \#$, then $f(x) = g(x)$, and if $g(x) = \#$, then $f(x) = 0$ or $f(x) = \#$. This is needed so that truth conditions uniquely determine falsity conditions. Imposing the constraint on the domain itself instead of the interpretation function for lexical constants would, however, undesirably eliminate some legitimate denotations for complex expressions.

With these constraints in place, we can then prove that all possible denotations of quantifier constants fulfil the desiderata we have set out. Furthermore, these results extend to determiners, in that unary quantifiers that can be obtained by applying a determiner to a restrictor predicate have the same properties. It is therefore not necessary to analyse *all the students* as a lexical constant—we can separate out *all* on its own as a determiner.

5 Conclusion

Constraining predicates to preserve overlap, with both truth and falsity being proper parts of the third truth value, is a direct implementation of Križ's (2016) Generalised Homogeneity:

Generalised Homogeneity (Križ 2016)**.** A predicate P cannot be true of x and false of y if x and y have a part in common.

The addition of $\preceq_0$-minimality as a condition on the denotation of lexical constants then makes the falsity conditions of a predicate derivable from its truth conditions (it is false whenever homogeneity allows it to), which is desirable because we do not seem to find observe lexically arbitrary falsity conditions in natural language.

Crucially, the very same constraints, when applied to quantifiers and determiners instead of predicates, enforce a very particular behaviour that fulfils the four desiderata we have set out and makes additional fine-grained and non-trivial predictions about the truth and falsity conditions of sentences with plural noun phrases and distributive as well as collective predicates, which appear to be borne out.

In this system, we restrict out attention to the parthood relation that holds between atomic individuals and pluralities thereof, such as between a book and a plurality of books. Other parthood relations, such as the one between a book and its chapters, do not feature in the system. However, the Generalised Homogeneity constraint holds with respect to these notions of parthood as well: *Mary read the book* has the third truth value when Mary read only some of the chapters of the book. If we simply took this parthood relation to be an instance of $\preceq$, our system would make wrong predictions for quantifiers. We hope that an extension of our system to a multi-sorted ontology along the lines of Asher (2011) or Retoré (2014) will be able to account for these facts, but this remains to be worked out.

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