Completeness of the Indexed ε-Calculus without equality for choice functions*

Hans Leiß
Centrum für Informations- und Sprachverarbeitung
Ludwig-Maximilians-Universität München
Oettingenstraße 67, 80539 München
leiss@cis.uni-muenchen.de

May 19, 2017

Abstract

The indexed ε-calculus of G.E. Mints and D. Sarenac (2003) was meant as a first-order version of the dynamic semantics of discourse of Egli and von Heusinger (1993), which models contexts by choice functions and evaluates (in)definite noun phrases by choice functions and a higher-order context update functional.

At the workshop Epsilon 2015 I raised doubts on the completeness result of G.E. Mints and D. Sarenac (2003) for the indexed ε-calculus. Here, I will show that the completeness fails, but holds for a version without equality between choice functions.[1]

The language $L\varepsilon$ of the indexed ε-calculus extends a first-order language L by quantifiers $\exists, \forall$ over "contexts" interpreted as choice functions, an equality predicate $\doteq$ for contexts, and "indexed ε"-terms $\varepsilon_i x\alpha$ interpreted as the object chosen by choice function i from the set of objects x satisfying α, if this set is non-empty.

G.E. Mints and D. Sarenac (2003) claimed a completeness theorem for the indexed ε-calculus with respect to a class of generalized higher-order structures $(A, \mathbb{F}, \dots)$ with a set $\mathbb{F} \subseteq \mathscr{P}(A) \to A$ of choice functions for the universe A of objects. The proposed proof extends a consistent set Γ of sentences to a maximal consistent set Δ and then defines the universe A as the set of Δ-congruence classes $[t]$ of closed object terms t and $\mathbb{F}$ as the set of choice functions $\Phi_{[a]}$ for Δ-congruence classes $[a]$ of closed context terms a, where $\Phi_{[a]}$ is defined by

$$\Phi_{[a]}(S) \quad := \quad \begin{cases} [\varepsilon_a x\alpha], & \text{if } S = \{[t] \mid \alpha[x/t] \in \Delta\} \text{ for some formula } \alpha(x), \\ f(S), & \text{otherwise,} \end{cases}$$

using is a fixed choice function f for A. In the models constructed this way, two choice functions are equal iff, for each definable set, both choose the same element. We refute the completeness result by showing that the theory

$$\Gamma := \{a \neq b\} \cup \{\, \forall\vec{y}\forall\vec{i}(\varepsilon_a x\alpha \doteq \varepsilon_b x\alpha) \mid \alpha(x, \vec{y}, \vec{i}) \text{ an } L\varepsilon\text{-formula}\,\}$$

*Presented at the ESSLLI 2017 workshop *QUAD: Quantifiers and Determiners* (Retoré, Steedman)

[1] These results meanwhile are published in Leiß (2017), but have not yet been presented at a workshop.

of two different choice functions a,b that agree on all definable sets is consistent, but has no model in the sense of Mints/Sarenac. To show that Γ is consistent, one starts with a countable structure $\mathscr{A} = (A, \mathbb{F}, a^{\mathscr{A}}, \ldots)$ with $a^{\mathscr{A}} \in \mathbb{F}$. If there is no $b^{\mathscr{A}} \in \mathbb{F}$ such that Γ holds, there is a set $B \subseteq A$, undefinable in $\mathscr{A}$, and a choice function $b^{\mathscr{A}} \in \mathscr{P}(A) \to A$ that differs from $a^{\mathscr{A}}$ just on B. We then consider $\mathscr{A}' = (A, \mathbb{F}', a^{\mathscr{A}}, b^{\mathscr{A}}, \ldots)$ where $\mathbb{F}' = \mathbb{F} \cup \{b^{\mathscr{A}}\}$ and show that B is undefinable in $\mathscr{A}'$ as well, so that Γ is consistent. The subtle part of the argument is that all sets definable in $\mathscr{A}'$ are definable already in $\mathscr{A}$, though generally with different defining formulas and parameters: extending $\mathbb{F}$ to $\mathbb{F}'$ changed the meaning of quantification over choice functions.

On the positive side, when the equality between choice functions is removed, a modification of Mints/Sarenac's construction indeed provides a model for each consistent set of sentences. This also holds if one uses structures $(A, I, \Phi, \ldots)$ with a sort I of contexts as *indices* and a possibly non-injective mapping Φ from indices to choice functions for A, so that $a \doteq b$ is interpreted as equality of indices, not as equality of their associated choice functions.

Once the equality for choice functions is removed, one can go on and remove first-order quantifiers $\exists x\alpha$ to context quantifiers $\exists i.\alpha[x/\varepsilon_i x\alpha]$ and context quantifiers $\exists i\beta$ to formulas $\beta[i/\varepsilon i\beta]$ using an *ε-operator for contexts*, following Hilbert's idea to eliminate quantifiers. With a weak form of extensionality for this context choice operator,

$$\forall i(\alpha \leftrightarrow \beta) \to \varepsilon_{\varepsilon i\alpha} x\varphi \doteq \varepsilon_{\varepsilon i\beta} x\varphi, \quad \text{if } x \notin \mathit{free}(\alpha \leftrightarrow \beta), i \notin \mathit{free}(\varphi)$$

saying "the choice functions chosen for equivalent formulas agree on all definable object sets", a completeness theorem for a quantifier-free variant of the indexed epsilon calculus can be shown.

The dynamic semantics of definite and indefinite noun phrases using choice functions, both in its functional version of von Heusinger (1995) and its relational version of Peregrin and von Heusinger (2005), is based on the second-order notion of updates of choice functions. It is open whether indexed ε-calculi with update relations and equality between *indices* can be complete *and* useful to study dynamic semantics using choice functions.

References

G.E. Mints and Darko Sarenac. Completeness of indexed ε-calculus. *Archive for Mathematical Logic*, 42(7), 617–625, 2003.

Klaus von Heusinger. Reference and salience. In A. von Stechow F.Hamm, J.Kolb, editors, *The Blaubeuren Papers. Proc. Workshop on Recent Developments in the Theory of Natural Language Semantics*, SfS-Report 08-95, 149–172, 1995.

Klaus von Heusinger. Der Epsilon-Operator in der Analyse natürlicher Sprache. Teil I: Grundlagen. Arbeitspapier 59. Fachbereich Sprachwissenschaft Universität Konstanz, 1993.

Urs Egli and Klaus von Heusinger. Definite Kennzeichnungen als Epsilon-Audrücke. In G. Lüdi, C.-A. Zuber, editors, *Akten des 4. regionalen Linguistentreffens*. Romanisches Seminar, Universität Basel, 105–115, 1993.

Hans Leiß. On Equality of Contexts and Completeness of the Indexed ε-Calculus. IfCoLog Journal of Logics and their Applications 4(2), 347–366, March 2017. http://www.collegepublications.co.uk/journals/ifcolog/?00011

Jaroslav Peregrin and Klaus von Heusinger. Dynamic Semantics with Choice Functions. In: H. Kamp and B. Partee (eds.) Context Dependence in the Analysis of Linguistic Meaning, 309–329. Elsevier, Amsterdam, 2005.