A Puzzle about Long-Distance Indefinites and Dependently Typed Semantics with Generalized Quantifiers

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Abstract. Indefinites (e.g. *a woman*, *some problem*) have given rise to a number of puzzles concerning their scopal and dynamic behavior. One such puzzle about long-distance indefinites seems to be unsettled in the literature ([2], [14], [13]). In this paper we show how the puzzle of long-distance indefinites can be handled in Dependently Typed Semantics with Generalized Quantifiers (DTSGQ). The proposal builds on our formal system combining generalized quantifiers ([9], [5], [1]) with dependent types ([8], [11], [7], [6]) in [3].

Keywords: quantifier scope, long-distance indefinite, dependent type

1 Background

The puzzle about long-distance indefinites arises in connection with sentences such as

(1) Every linguist has studied every solution that <u>some problem</u> might have.

Sentence (1) allows the so-called long-distance intermediate scope reading saying that for every linguist there is a possibly different problem such that he/she has studied every solution that this problem might have. This reading is considered exceptional, for the indefinite *some problem* takes scope out of its syntactic island (unlike standard quantifiers). Kratzer in [4] credits the problematic reading to the presence of a hidden pronoun/functional element, i.e. the apparent long-distance intermediate readings are in fact bound/functional readings and they only become available when there is a contextually salient pairing each of the linguists with some particular problem/function present. The intended readings can be expressed by the following paraphrases

(1a) Every linguist has studied every solution that a certain problem that intrigued him/her might have.

(1b) Every linguist has studied every solution that some problem that intrigued him/her most might have.

To capture the readings, Kratzer uses the mechanism of 'Skolemized choice functions (CF)'. Chierchia in [2] observes that there is a second kind of long-distance readings that cannot be reduced to bound/functional readings

(2) Not every linguist has studied every solution that some problem might have.

Sentence (2) is intuitively true in a situation where: for some linguist there is no problem such that he/she has studied every solution that this problem might have. These are the truth-conditions for the negated long-distance intermediate reading: for every linguist there is some problem such that he/she has studied every solution that this problem might have (and not for the negated Kratzer's bound/functional reading). Chierchia's proposal is to capture the long-distance intermediate reading using the mechanism of the intermediate existential closure of the CF variable. So the puzzle is that we need two mechanisms to account for the behavior of long-distance indefinites: Skolemized CF (as pointed out by Schlenker in [12], Skolemized CF are needed to account for some clear-cut cases of functional readings) and the intermediate existential closure of the CF variables. Moreover, the two mechanism are problematic on both theoretical and empirical grounds (see e.g. [10]).

2 Our Proposal

Our Dependently Typed Semantics with Generalized Quantifiers (DTSGQ) combines two semantic approaches to account for natural language quantification: Generalized Quantifier Theory familiar from Montague-style semantics ([9], [5], [1]) and type-theoretic approach ([8], [11], [7], [6]). Like in the classical Montague-style analysis, DTSGQ makes essential use of generalized quantifiers (GQs). But in the spirit of the type-theoretic framework we adopt a many-typed analysis (in place of a standard single-sorted analysis). Like in the standard type-theoretic approaches, we have type dependency in our system. But our semantics is model-theoretic (with truth and reference being basic concepts), and not proof-theoretic (where proof is a central semantic concept).

Combining GQs with dependent types allows us to handle in a uniform manner a number of semantic puzzles concerning natural language quantifiers. In our previous work we have defined a new interpretational algorithm to account for a wide range of anaphoric (dynamic) effects associated with natural language quantification ([3]). In this paper we will show how the puzzle of long-distance indefinites can be handled in DTSGQ. We propose to credit the problematic readings to the presence of (possibly hidden) dependencies or functions, i.e. the apparent long-distance intermediate readings involve in fact either dependent types or functions.

2.1 Dependently Typed Semantics with Generalized Quantifiers (DTSGQ)

In this and the following sections, we only discuss the elements of the system relevant for the linguistic purposes of this paper. For the full system, see [3].

Polymorhic interpretation of quantifiers. Standard Montague-style semantics is single-sorted in the sense that there is a single type e of all entities. On the Montague-style analysis, quantifiers are interpreted over the universe of all entities E. Our semantics is many-sorted in the sense that there are many types and we have a polymorphic interpretation of quantifiers. On the Montague-style analysis, quantifier phrases, e.g. *some woman*, are interpreted as sets of subsets of E

$$\|\exists x : woman\ x\| = \{X \subseteq E : \|W\| \cap X \neq \emptyset\}.$$

On our analysis, a generalized quantifier associates to every set Z a subset of the power set of Z: $\|Q\|(Z) \subseteq \mathcal{P}(Z)$; quantifier phrases, e.g. *some woman*, are interpreted as follows

$$\|\exists_{w:Woman}\| = \{X \subseteq \|W\| :\ X \neq \emptyset\}.$$

As a consequence of our many-typed analysis, predicates are also defined polymorphically, i.e. predicates are interpreted over many types (and not over the universe of all entities).

Combining quantifier phrases. To handle multi-quantifier sentences, the interpretation of quantifier phrases is further extended into the interpretation of (generalized) quantifier prefixes. (Generalized) quantifier prefixes can be built from quantifier phrases using the sequential composition ?|? constructor. The corresponding semantical operation is known as iteration (see [3]). To illustrate with an example: *Every linguist has studied some problem* can be understood to mean that each of the linguists has studied a potentially different problem. To capture this reading:

- a sequential composition constructor ?|? is used to produce a multi-quantifier prefix: $\forall_{l:L}|\exists_{p:P}$;
- the corresponding semantical operation of iteration is defined as follows

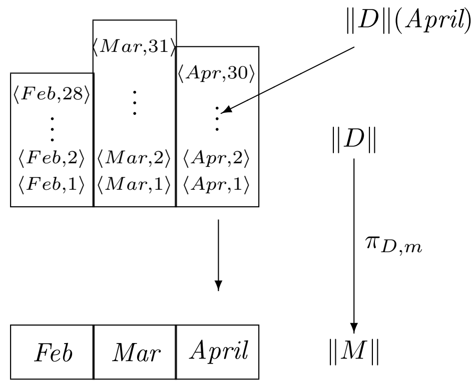
  $$\|\forall_{l:L}|\exists_{p:P}\| =$$

  $$\{R \subseteq \|L\| \times \|P\| : \{a \in \|L\| : \{b \in \|P\| : \langle a, b\rangle \in R\} \in \|\exists_{p:P}\|\} \in \|\forall_{l:L}\|\}.$$

The multi-quantifier prefix $\forall_{l:L}|\exists_{p:P}$ denotes a set of relations such that the set of linguists such that each linguist is in this relation to **at least one problem** is the set of all linguists. Obviously, the iteration rule gives the same result as the standard nesting of quantifiers in first-order logic.

Dependent types. Crucially, in a system with many types we can also have dependent types. One example of such a dependence of types is that if m is a variable of the type of months M, there is a type $D(m)$ of the days in that month

$$m : M, d : D(m)$$



If we interpret type M as a set $\|M\|$ of months, then we can interpret type D as a set of the days of the months in $\|M\|$, i.e. as a set of pairs

$$\|D\| = \{\langle a, k\rangle : k \text{ is (the number of) a day in month } a\},$$

equipped with the projection $\pi_{D,m} : \|D\| \to \|M\|$. The particular sets $\|D\|(a)$ of the days of the month a can be recovered as the fibers of this projection (the preimages of $\{a\}$ under $\pi_{D,m}$)

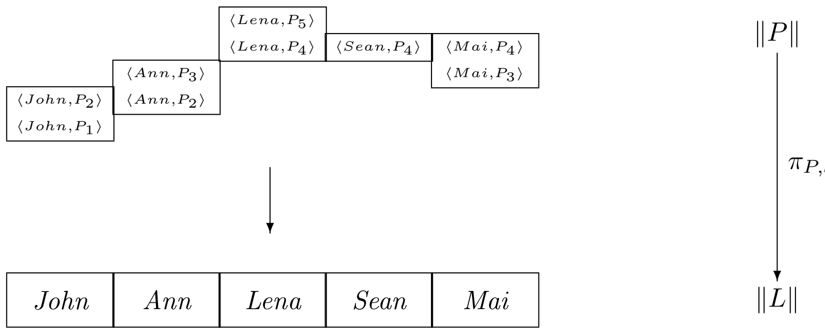
$$\|D\|(a) = \{d \in \|D\| : \pi(d) = a\}.$$

Generalized quantifiers on dependent types. Generalized quantifiers are extended to dependent types in our system

$$\|\forall_{l:L}|\exists_{p:P(l)}\| =$$

$$\{R \subseteq \|P\| : \{a \in \|L\| : \{b \in \|P\|(a) : \langle a, b\rangle \in R\} \in \|\exists_{p:P(l)}\|(\|P\|(a))\} \in \|\forall_{l:L}\|\}.$$

The multi-quantifier prefix $\forall_{l:L}|\exists_{p:P(l)}$ denotes a set of relations such that the set of linguists such that each linguist is in this relation to **at least one problem in the corresponding fiber of problems** is the set of all linguists. By extending the interpretation of generalized quantifiers to dependent types, our semantics introduces quantification over fibers, e.g. quantification over the fiber of the problems of John - $\|P\|(John)$



2.2 DTSGQ Analysis of Sentence (1)

Alphabet. The alphabet of the system consists of:
type variables: $X, Y, Z, \ldots$;
type constants: $Linguist, Problem, Solution, \ldots$;
type constructor: $\mathbb{T}$;
individual variables: $x, y, z, \ldots$;
predicates: $P^n, P^n_1, \ldots$;
quantifier symbols: $\exists, \forall, \ldots$;
prefix constructors: $?|?, \ldots$.

English-to-formal language translation. Consider now a sentence in (1):

(1) Every linguist has studied every solution that some problem might have.

Our English-to-formal language translation process consists of two steps (i) *representation* and (ii) *disambiguation*. The syntax of the representation language - for the English fragment considered in this paper - is as follows

$S \to Prd^n(QP_1, \ldots, QP_n)$;
$MCN \to Prd^n(QP_1, \ldots, \ CN \ , \ldots, QP_n)$;
$MCN \to CN$;
$QP \to Det\ MCN$;
$Det \to every, some, \ldots$;
$CN \to linguist, problem, \ldots$;
$Prd^n \to study, have, \ldots$

Sentence (1) is accordingly represented as

Study^2 (*every linguist*, *every solution that some problem might have*).

Multi-quantifier sentences of English, contrary to sentences of our formal language, are often ambiguous. Hence one sentence representation can be associated with more than one sentence in our formal language. The second step thus involves disambiguation. We take quantifier phrases out of a given representation and organize them into possible prefixes of quantifiers. In the case of our example, the sentence translates as

$$\forall_{l:Linguist}|\forall_{t_s:\mathbb{T}_{Solution\ to\ some\ problem}} Study^2(l, t_s).$$

Interpretation. In the Montague-style semantics, common nouns are interpreted as predicates (expressions of type $e \rightarrow t$). In our type-theoretic setting, common nouns (CN), e.g. *linguist*, are interpreted as types; modified common nouns (MCN), e.g. *solution that some problem might have*, are treated as $*$-*sentences* (= *Have*^{2} (*some problem*, *solution*) determining a system of (possibly dependent) types, and the types so determined are interpreted using an interpretational algorithm defined in [3]. In the case of our example, the type *Linguist* is interpreted as a set of linguists (indicated in the context) $\|Linguist\|$, and the type $\mathbb{T}_{Solution\ to\ some\ problem}$ is interpreted as

$$\|\mathbb{T}_{Solution\ to\ some\ problem}\| =$$

$$\{c \in \|Solution\| : \{b \in \|Problem\| \ : \langle b, c\rangle \in \|Have\|\} \in \|\exists_{p:Problem}\|\}.$$

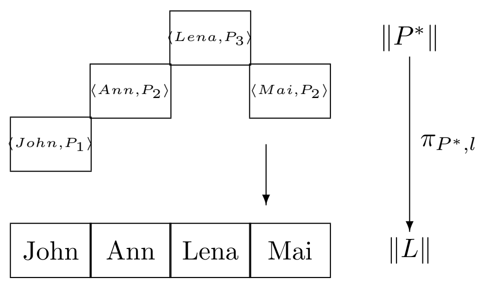
Thus, as can be seen from this analysis, DTSGQ can only yield a narrow scope reading for the indefinite *some problem* in (1): every a in $\|Linguist\|$ has studied every c in $\|\mathbb{T}_{Solution\ to\ some\ problem}\|$.

2.3 DTSGQ Solution to the Puzzle about Long-Distance Indefinites

Indefinites. Unlike standard quantifier expresions, indefinites have been claimed to be ambiguous between a quantificational and a referential reading. Our proposal ties this ambiguity to the variability in type assignment. A quantificational indefinite *a/some problem* combines a determiner *a/some* and the variable of the type *Problem*, interpreted as the set of all problems (given in the context) - $\|Problem\|$. A referential indefinite *a/some (certain) problem* combines the determiner with the variable of the referential type *Problem**, interpreted as a certain singleton set consisting of a problem that the speaker has in mind - $\|Problem^*\|$. Correspondingly to referential types, we can also have dependent referential types in our semantics.

Dependent and functional readings. Our proposal distinguishes dependent referential and functional readings. If a sentence like (1) involves a hidden dependent referential indefinite, e.g. *a (certain) problem (that intrigues him/her)*, then it quantifies over the dependent referential type:

$$l : L; p : P^*(l)$$



yielding the dependent referential reading saying that every linguist a in $\|L\|$ has studied every solution that a certain (one) problem b in $\|P^*\|(a)$ might have. That is, DTSGQ gives a reading: every a in $\|L\|$ has studied every c in $\|\mathbb{T}_{\textit{Solution to a (certain) problem (that intrigues him/her)}}\| =$

$$\{c \in \|S\| : \{b \in \|P^*\|(a)\ : \langle b, c\rangle \in \|Have\|\} \in \|\exists_{p:P^*(l)}\|(\|P^*\|(a))\}.$$

If a sentence like (1) involves a hidden function inducing element (e.g. the *most intriguing problem* function), then we get a functional reading saying that every linguist a has studied every solution to $f(a)$.

If sentence (1) involves a quantificational indefinite, DTSGQ does not give a long-distance intermediate (Chierchia's) reading (as explained above, DTSGQ can only yield a narrow scope reading for the indefinite). Chierchia's reading, however, can be explained away. As observed by Chierchia in [2], special context is needed to get a long-distance intermediate reading for (1) (in the absence of factors inducing dependent referential or functional readings), e.g. 'You know, linguists are really systematic: Lee studied every solution to the problem of weak crossover, Kim every solution to the problem of donkey sentences, etc.' We propose that people posit certain dependencies (given some such context), e.g.: that the type of problems depends on the type of linguists and the type of solutions depend on the type of problems $l : L; p : P(l); s : S(p)$ - by quantifying over the so posited dependent types, we get the apparent dependent ($\simeq$ Chierchia's) reading

$$\forall_{l:L}|\exists_{p:P(l)}|\forall_{s:S(p)}Study^2(l, s)$$

(for every linguist a in $\|L\|$ there is a problem b in $\|P\|(a)$ such that a has studied every solution c in $\|S\|(b)$).

The negative sentence (2) would then claim that for some a in $\|L\|$ there is no problem b in $\|P\|(a)$ such that a has studied every solution c in $\|S\|(b)$.

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