Factivity and Presupposition in Dependent Type Semantics*

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1 Introduction

Dependent Type Semantics (DTS, Bekki [3]) is a framework of natural language semantics based on dependent type theory (Martin-Löf [19]). In contrast to traditional model-theoretic semantics, DTS is a proof-theoretic semantics, where entailment relations are characterized as provability relations between semantic representations. One of the distinctive features of DTS, as compared to other type-theoretical frameworks, is that it is augmented with underspecified terms so as to provide a unified analysis of entailment, anaphora and presupposition from an inferential/computational perspective. In contrast to previous work on anaphora in dependent type theory (cf. Ranta [22]), DTS gives a fully compositional account of inferences involving anaphora (Bekki [3]). It is also extended to the analysis of modal subordination (Tanaka et al. [25]).

In this paper, we provide the framework of DTS with a mechanism to handle entailment and presupposition associated with factive verbs such as *know*. Although there are numerous studies on factive verbs in natural language semantics, they are usually based on model-theoretic approaches; it seems fair to say that there has been little attempt to formalize inferences with factivity from a computational and proof-theoretical perspective. On the other hand, various proof systems for knowledge and belief have been studied in the context of epistemic logic (cf. Meyer and van der Hoek [20]). However, such systems are mainly concerned with knowledge and belief themselves, not with how they are expressed in natural languages, nor with linguistic phenomena such as factivity presuppositions. Our study aims to fill this gap by providing a framework that explains entailments and presuppositions with factive verbs in dependent type theory.

2 Factive verbs and presupposition

We briefly summarize entailments and presuppositions triggered by factive verbs. Factive verbs like *know*, in contrast to non-factive verbs like *believe*, presuppose that the complement is true. There are two characteristic properties of presuppositions. First, a presupposition *projects* out of embedded contexts such as negation, question, and the antecedent of a conditional. Thus, not only (1) but also (3a-c) imply (2).

(1) John knows that he is successful.

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(2) John is successful.

(3) a. John does not know that he is successful. NEGATION

b. Does John know that he is successful? QUESTION

c. If John knows that he is successful, ... CONDITIONAL

(4) a. If John is successful, he knows that he is.

b. John is successful, and he knows that he is.

Second, a presupposition is *filtered* when it occurs in contexts such as (4a, b). In general, if S' entails the presuppositions of S, constructions like S' and S and If S' then S do not inherit the presuppositions of S.

Unlike non-factive verbs, factive verbs can take interrogative complements as in (5), and license inferences as shown in (6) and (7) (Groenendijk and Stokhof [9]).[3]

(5) a. John $\{$knows, *believes$\}$ whether Ann or Bob came.

b. John $\{$knows, *believes$\}$ who came.

(6) John knows whether Ann or Bob came.
Ann came.
―――――――――――
John knows that Ann came.

(7) John knows who came.
Ann came.
―――――――――――
John knows that Ann came.

Interrogative complements themselves have presuppositions (Hintikka [12]; Karttunen [14]). For instance, it is natural to take *whether Ann or Bob came* as presupposing "Ann or Bob (but not both) came" and *who came* as presupposing "someone came". Figure 1 summarizes basic inference patterns for factive verb *know*.

E1	x knows whether A or B, $A \Rightarrow x$ knows that A
E2	x knows whether A or B, $B \Rightarrow x$ knows that B
E3	x knows who F, $F(a) \Rightarrow x$ knows that $F(a)$
P1	x knows that $P \triangleright P$
P2	x knows who $F \triangleright$ someone F
P3	x knows whether A or $B \triangleright A$ or B (but not both)

Fig. 1. Entailments ($\Rightarrow$) and presuppositions ($\triangleright$) associated with factive verbs.

3 Dependent Type Semantics

DTS (Bekki [3]) is a natural language semantics based on dependent type theory (Martin-Löf [19]). Since the work of Sundholm [24] and Ranta [22], dependent type theory has been applied to the analysis of various dynamic discourse phenomena, providing a type-theoretic alternative to model-theoretic frameworks such as DRT and Dynamic

[3] Here and henceforth, we take *whether A or B* as expressing an alternative question.

Semantics. Dependent type theory has also been applied to the study of natural language inferences in computational semantics with the help of modern proof assistants (Chatzikyriakidis and Luo [6]).

In dependent type theory, two type constructors Σ and Π play a crucial role in forming the semantic representations for natural language sentences. The type constructor Σ is a generalized form of the product type and behaves as an existential quantifier. An object of type $(\Sigma x : A)B(x)$ is a pair (m, n) such that m is of type A and n is of type $B(m)$. Conjunction $A \wedge B$ is a degenerate form of $(\Sigma x : A)B$ if x does not occur free in B. Σ-types are associated with projection functions π_1 and π_2 that are computed with the rules $\pi_1(m, n) = m$ and $\pi_2(m, n) = n$, respectively. The type constructor Π is a generalized form of the functional type and behaves as a universal quantifier. An object of type $(\Pi x : A)B(x)$ is a function f such that for any object a of type A, fa is an object of type $B(a)$. Implication $A \to B$ is a degenerate form of $(\Pi x : A)B$ if x does not occur free in B. See e.g., Martin-Löf [19] and Ranta [22] for more details.

Common nouns: types or predicates? There are two possible approaches to representing basic sentences like *A man entered* in dependent type theory. One is the approach proposed in Ranta [22] and Luo [17, 18], according to which common nouns like *man* are interpreted as *types* so that the sentence is represented as $(\Sigma x : \mathbf{man})\,\mathbf{enter}(x)$. A problem with this approach is that it is not straightforward to analyze sentences containing *predicate nominals*, such as (8a, b).

(8) a. John is *a man*.
  b. Bob considers Mary *a genius*.

One might analyze (8a) as a judgement **john** : **man**; but then it is not clear how to account for the fact that such a sentence can be negated (*John is not a man*) or appear in the antecedent of a conditional (*If John is a man, ...*). One possible solution is to construe *be*-verbs as the so-called "*is*-of identity" along the Russell-Montague lines. Thus, (8a) is represented as $(\Sigma x : \mathbf{man})\,\mathbf{john} =_{\mathbf{man}} x$. This predicts that the predicate nominal *a man* introduces a discourse referent (in terms of Σ-types). However, contrary to this prediction, predicate nominals cannot serve as an antecedent of an anaphoric pronoun like *he* or *she* (Mikkelsen [21]); hence they do not introduce a standard discourse referent.[4]

As an alternative approach, we interpret a common noun as a predicate; thus *A man entered* is represented as $(\Sigma u : (\Sigma x : \mathbf{entity})\,\mathbf{man}\,x)\,\mathbf{enter}(\pi_1 u)$. This approach is in line with the traditional analysis of common nouns, so we can integrate standard assumptions in formal semantics into our framework. Note that although we do not take common nouns to be types, it is possible to refine type **entity** by introducing more fined-grained types such as ones representing animate/inanimate objects, physical/abstract objects, events/states, and so on (Asher [1]; Asher and Luo [2]; Bekki and Asher [4]; Retoré [23]). Such richer type structures will be needed to provide a proper treatment of lexical phenomena such as polysemy, coercion and, selection restriction

[4] See Fara [7] and Heim [11] for more discussion on the problems of the Russell-Montague analysis of predicate nominals.

	Π-types	Σ-types
Standard notation	$(\Pi x : A)B(x)$	$(\Sigma x : A)B(x)$
Notation in DTS	$(x{:}A) \rightarrow B(x)$	$\begin{bmatrix} x{:}\, A \\ B(x) \end{bmatrix}$
When $x \notin fv(B)$	$A \rightarrow B$	$\begin{bmatrix} A \\ B \end{bmatrix}$

Fig. 2. Notation for Π-types and Σ-types in DTS. $fv(B)$ means the set of free variables in B.

within the framework of dependent type theory. Our framework is consistent with such an extended type system.

In what follows, we will make use of the notation in DTS for Π-types and Σ-types as shown in Figure 2.

Presupposition in DTS. DTS is based on the paradigm of the Curry-Howard correspondence, according to which propositions are identified with *types*; the truth of a proposition is then defined as the existence of a proof (i.e., proof-term) of the proposition. In order to handle anaphora and presupposition in a compositional setting (see Bekki [3] for detail), DTS distinguishes two kinds of propositions, *static* and *dynamic* propositions. For any static proposition P, we say that P is true if P is *inhabited*, that is, there exists a proof-term t such that $t : P$. A *dynamic* proposition in DTS is a function mapping a proof c of a static proposition γ, a proposition representing the preceding discourse, to a static proposition. Such a proof term c is called a *local context*.

Underspecified term $@_i$ (where i is a natural number) is used to represent presupposition triggers. For instance, (9a) is represented as (9b), where definite article *the* introduces the term $@_1$ in the semantic representation.

(9) a. The apple is red.

b. $\lambda c.\, \mathbf{red}(\pi_1(@_1 c : \begin{bmatrix} x{:}\, \mathbf{entity} \\ \mathbf{apple}(x) \end{bmatrix}))$

The underspecified term $@_1$ is a function that takes a local context c as argument. A term of the form $@_i c : \Lambda$ is called *type annotation* and specifies that the term $@_i c$ has type Λ. In the case of (9b), the term $@_1 c$ is annotated with a type corresponding to the proposition *there is an apple* represented as a Σ-type. This means that the underspecified term $@_1$ takes a local context c as argument and returns a proof of that proposition. In this way, the annotated type represents the existential presupposition triggered by the definite description *the apple*.[5]

The type of an underspecified term $@_i$ can be specified by a type-checking algorithm (Bekki and Satoh [5]). Based on the inferred type, a proof search is carried out to

[5] Here we take it that the uniqueness presupposition is not part of the conventional meaning of a definite description but can be derived on pragmatic considerations along the lines of Heim [10]. Although it is possible to include the uniqueness presupposition in the type annotation Λ associated with *the*, the proof-search procedure to find the antecedent of an underspecified term would then become complicated.

construct a term of that type; then the underspecified term $@_i$ is replaced by the obtained term. This whole process corresponds to the process of presupposition resolution.

In the case of (9b), if a proof term for the existential proposition *there is an apple* is constructed given a local context c, it can substitute $@_1$. Such a proof construction is possible when (9a) appears in contexts such as *There is an apple and the apple is ...* and *If there is an apple, then the apple is ...*. These cases correspond to the case of *presupposition filtering*. Note that given its type, $@_1 c$ is a pair of an entity and a proof of the entity being an apple. Accordingly, in (9b), the projection function π_1 is applied and returns the first element of the pair, i.e., an entity corresponding to the apple in question; then the predicate **red** takes this entity as argument.

When one cannot construct a proof required by $@_i$ from a given local context c, the existence of a term having the intended type can be assumed by means of the process of *accommodation*. The whole process of resolving an underspecified term $@_i$ is the same if the presupposition trigger is embedded under such a context as the scope of negation or the antecedent of a conditional. In this way, we can explain basic projection patterns of presuppositions.

4 Analyzing factivity in DTS

We will provide semantic representations for factive verbs by using underspecified term $@_i$ in DTS. For reasons of space, we will concentrate on the case of declarative complements and leave the analysis of interrogative complements for another occasion. We take the factive verb *know* as a representative case.

Declarative complements. We represent a sentence of the form a *knows that* P as $\lambda c.\, \mathbf{kn}(a, @_i c : Pc)$. The underspecified term $@_i$ here takes a local context c as argument and requires one to construct a proof term of type Pc, i.e., to find *evidence* of the (static) proposition Pc being true given the context c. Here P is a dynamic proposition expressed by the declarative complement of *know*. If such a proof term is constructed, it fills the second argument position of **kn**. Here predicate $\mathbf{kn}(x, t)$ can be read as *agent* x *obtains evidence* t. In sum, given a context c, the sentence x *knows that* P presupposes that there is a proof (evidence) of Pc and asserts that the agent x obtains it, i.e., x has a proof (evidence) of the proposition Pc. In the same way as the case of definite descriptions the analysis of presuppositional contents in terms of $@_i$ terms accounts for the projection and filtering properties of *know* as shown in (3) and (4).

The standard analysis of *know* in formal semantics follows Hintikka's [13] possible world semantics, which fails to capture the notion of evidence or justification that has been traditionally associated with the concept of knowledge. An advantage of dependent type theory is that it is equipped with proofs as first-class objects and thus enables us to analyze the factive verb *know* as a predicate over a proof (evidence) of a proposition.[6]

[6] The idea that a proof term serves as an antecedent of anaphor can be traced back to Ranta [22], where under the the assumption that proofs are identified with *events* it is claimed that aspectual verbs like *stop* presuppose the existence of a proof. See also Krahmer and Piwek [15],

In contrast to *know*, a verb like *believe* does not have factivity presupposition. Accordingly, we analyze non-factive attitude verbs like *believe* as a predicate over a proposition (cf. Tanaka et al. [25]). Thus, we treat factive and non-factive verbs as predicates having a different semantic type. This treatment of factive and non-factive verbs is consistent with Zeno Vendler's view that *know* and *believe* select different semantic objects, i.e., *know* selects a fact, while *believe* selects a proposition (Vendler [27]; Ginzburg [8]). Note that in our approach, the notion of facts is not taken as primitive but analyzed in terms of the notion of *evidence* of a proposition.

One advantage of this analysis is that it is consistent with, and directly applicable to, a language like Japanese in which factive and non-factive verbs require a different complementizer. In Japanese, there are two types of complementizer, *koto* and *to*. As Kuno [16] observed, factive verbs usually take a clause ending with *koto*, while non-factive verbs take a clause ending with *to*:

(10) John-wa Mary-ga kita *koto*-o sitteiru.
John-TOP Mary-NOM came COMP-ACC know.
'John knows (the fact) that Mary came.'

(11) John-wa Mary-ga kita *to* sinziteiru.
John-TOP Mary-NOM came COMP believe.
'John believes that Mary came.'

In general, *koto*-clauses trigger factive presupposition, while *to*-clauses do not. This contrast can be captured by assuming that a factive verb like *sitteiru* takes as its object a proof (evidence) of the proposition expressed by a *koto*-clause, while a non-factive verb selects a proposition denoted by a *to*-clause.

NP-complements. Our analysis can be naturally extended to factive verbs taking NP-complements. The factive verb *know* taking an NP-complement of the form *the N that A* shows different entailment patterns from non-factive verbs like *believe* and interrogative verbs like *ask* (Vendler [27]; Ginzburg [8]; Uegaki [26]): *know* does not license the entailment from x *Vs the rumor that* P to x *Vs that* P, nor that from x *Vs the question whether* A *or* B to x *Vs whether* A *or* B.

(12) a. John believes the rumor that Mary came. $\Rightarrow$ John believes that Mary came.
b. John knows the rumor that Mary came. $\not\Rightarrow$ John knows that Mary came.

(13) a. John asks the question whether Mary or Bob came. $\Rightarrow$ John asks whether Mary or Bob came.
b. John knows the question whether Ann or Bob came. $\not\Rightarrow$ John knows whether Mary or Bob came.

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where it is briefly mentioned that the presuppositions triggered by noun phrases like *the fact that P* can be treated in a similar way. Although space limitations preclude us from examining these analyses in detail, our claim is that the idea that proofs act as antecedents of anaphora can best be applied to the presuppositions of factive verbs in general.

We take it that *know* is ambiguous between the *evidence*-taking reading (**kn**) and the so-called *acquaintance* reading. The latter is denoted as $\mathbf{kn}_{np}$. For example, x *knows the man* is represented as (14).

(14) $\mathbf{kn}_{np}(x, \pi_1(@_i c : \begin{bmatrix} x\colon \mathbf{entity} \\ \mathbf{man}(x) \end{bmatrix}))$

(15) $\mathbf{kn}_{np}(x, \pi_1(@_i c : \begin{bmatrix} p\colon \mathsf{type} \\ \begin{bmatrix} p = P \\ \mathbf{rumor}(P) \end{bmatrix} \end{bmatrix})$

Using the predicate $\mathbf{kn}_{np}$, we represent x *knows the rumor that* P as (15), which presupposes that there is a rumor whose content is identified with type P. When this presupposition is satisfied, (15) is provably equivalent to $\mathbf{kn}_{np}(x, P)$, which is clearly distinguished from the reading that x *has evidence of* P, hence, the non-entailment in (12b) follows. (12a), which contains *believe*, is represented in the same way as (15); thus it is equivalent to $\mathbf{believe}(x, P)$, hence we can derive the entailment in (12a). The asymmetry between *ask* and *know* in (13) can be explained in a same manner.

5 Conclusion

This paper analyzed entailments and presuppositions associated with factive verbs in the framework of DTS. We analyzed factive verbs as predicates taking a proof-object as argument, and non-factive verbs as predicates taking a proposition in the sense of dependent type theory. A fully compositional analysis of factive verbs in English and Japanese, including those with NP-complements, as well as an analysis of *wh*-complements, is left for another occasion.

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