

**Surgical Robotics**  
**2<sup>nd</sup> Summer European University**  
**Montepellier , France**  
**September 7-14 , 2005**

**“A Mathematical Model of Hysteresis”**

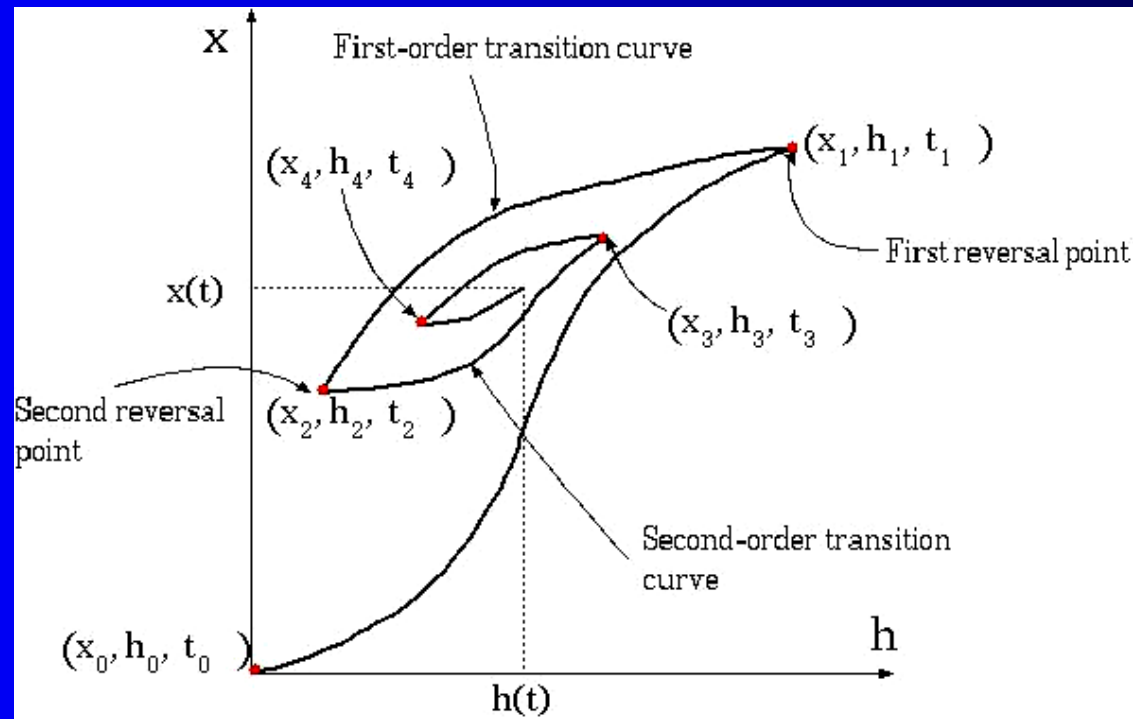
**Daniele Carnevale, Salvatore Nicosia, Luca Zaccarian**  
*University of Roma Tor Vergata, Italy*  
*Dipartimento di Informatica Sistemi e Produzione*

# Outline

- **Hysteresis?...Where?...Why?**
- **Previous Models**
- **The Proposed Model**
- **Conclusions**
- **Future Works**

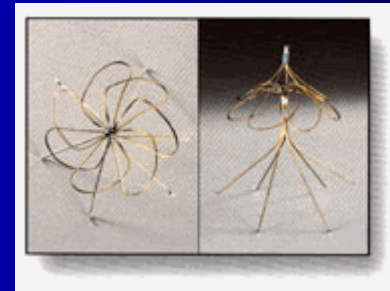
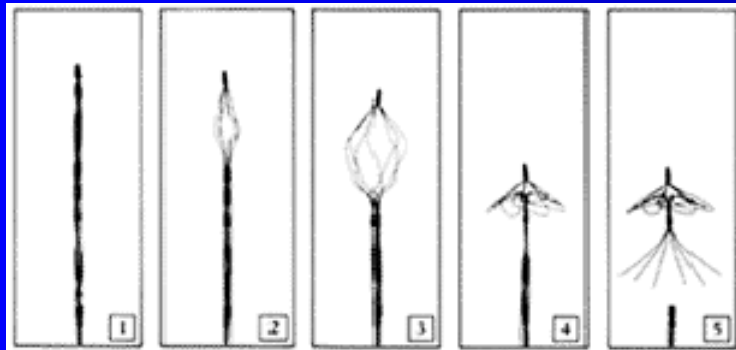
## Hysteresis?...

Hysteresis is a strongly nonlinear phenomenon. The state/output of the hysteretic system depends by past input history (memory effect). The free energy function has many local minima and saddles points.



## ...Where?

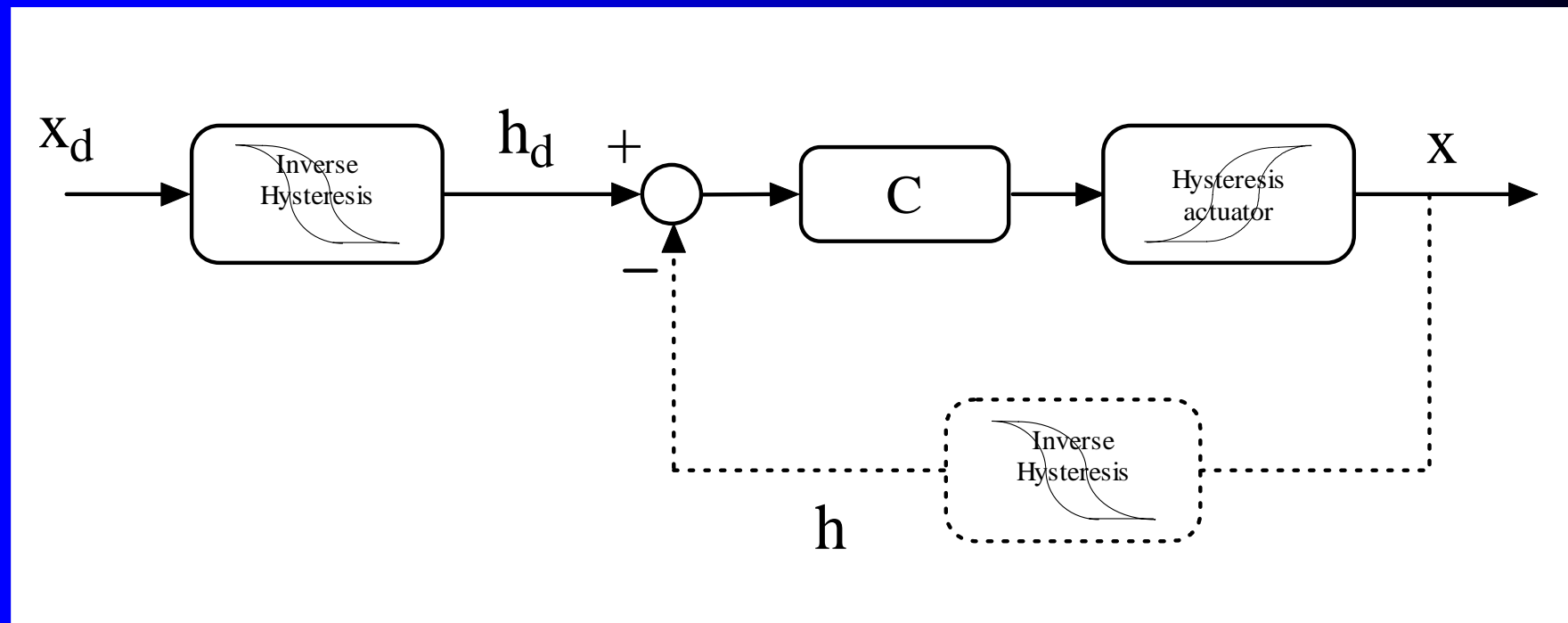
- Shape Memory Alloy (SMA). NiTiNol: muscle wire, aortic stents, Simon filter, knotting wire...
- Piezoelectric/Piezoceramics Actuators: MIS devices...
- Mechanic Friction: robot motor and joints, tissue modeling...
- Magnetic Materials
- Semiconductors, Biology, Laser Beam, Nanotechnology...



Simon Filter

...Why?

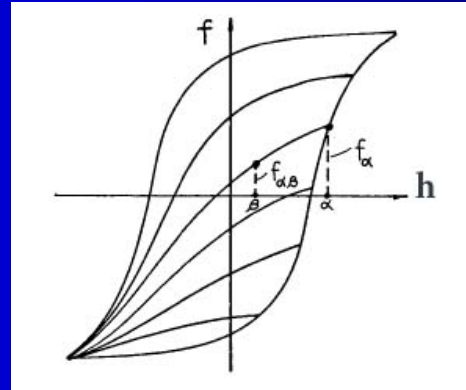
If we came up with the inverted model of hysteresis the fdt between  $x_d$  and  $x$  their cascade becomes the identity...



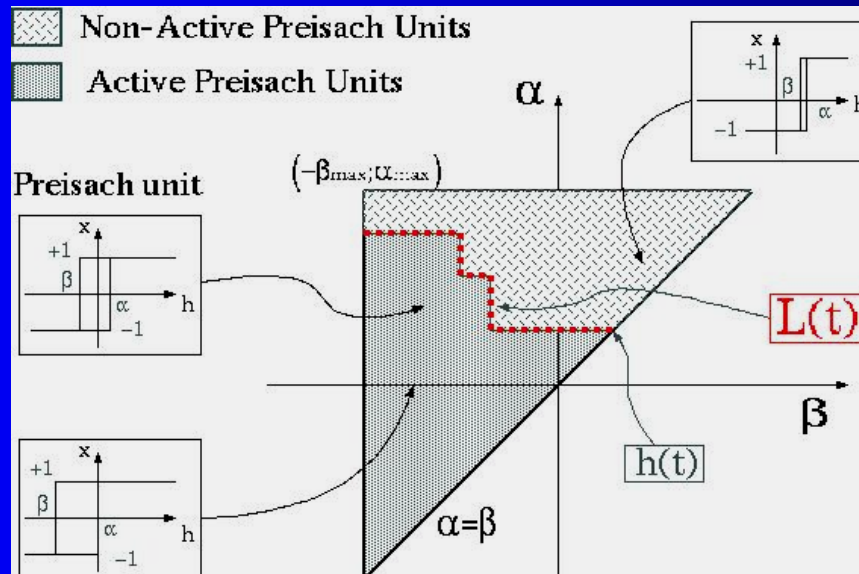
# Previous Models

**Preisach model** has been introduced in 1935. The system has been seen as a collection of bistable units (relays).

- Identification phase



- Run time

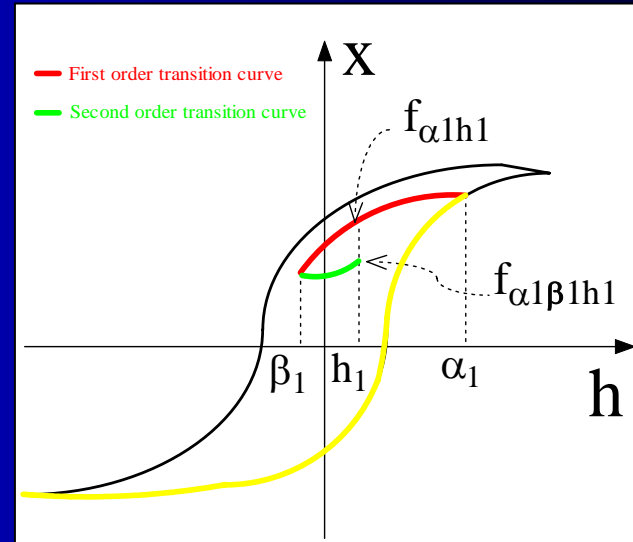


$$x(t) = \iint_{P_{\alpha\beta}} \mu(\alpha, \beta) h(t) d\alpha d\beta$$

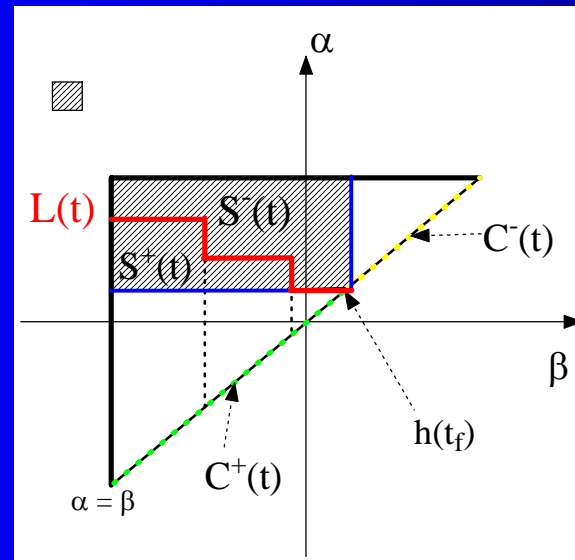
# Previous Models

Generalized model has been introduced by I.D. Mayergoyz.

- Identification phase



- Run time

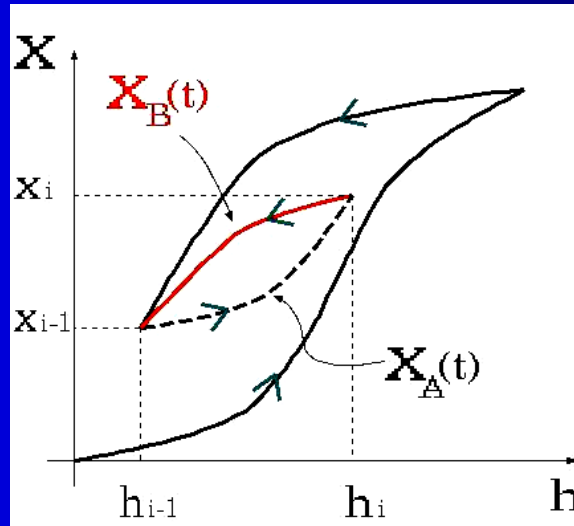


$$x(t) = \iint_{P_{\alpha\beta u}} \mu(\alpha, \beta, h) h(t) d\alpha d\beta$$

## Proposed Model

A time varying function  $\mathbf{W}$  is used in order to maintain the system “memory”.

- Identification phase



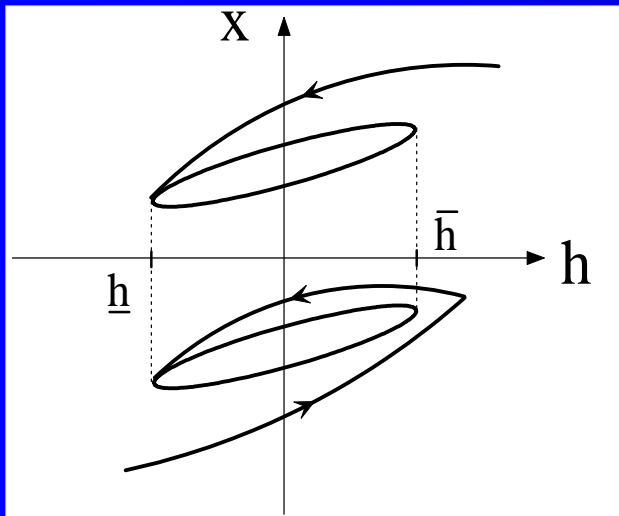
- Run time : 
$$x(t) = x(t_{k-1}) + \int_{h(t_{k-1})}^{h(t)} W_h(\zeta) d\zeta, \quad \forall t \in \varepsilon_k.$$

$$W_h(k, \zeta) = w_{h0}(\zeta) + \sum_{i \in \mathcal{A}_k} w_h(h_{i-1}, x_i, h_i)(\zeta)$$

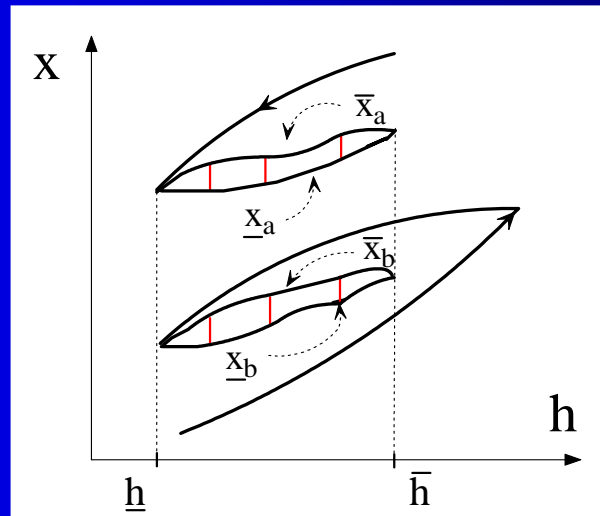


## Conclusions

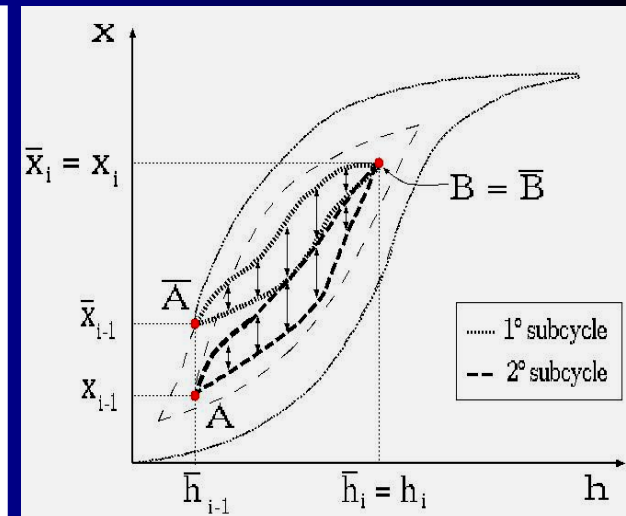
- Preisach Model  $\Leftrightarrow$  Wiping-out & Congruency properties, 1° rc.
- Generalized Model  $\Leftrightarrow$  Wiping-out & Equal chords properties, 2° rc.
- Proposed Model  $\Leftrightarrow$  Wiping-out & Same Vertex Equal Chords, n° rc.



Congruency

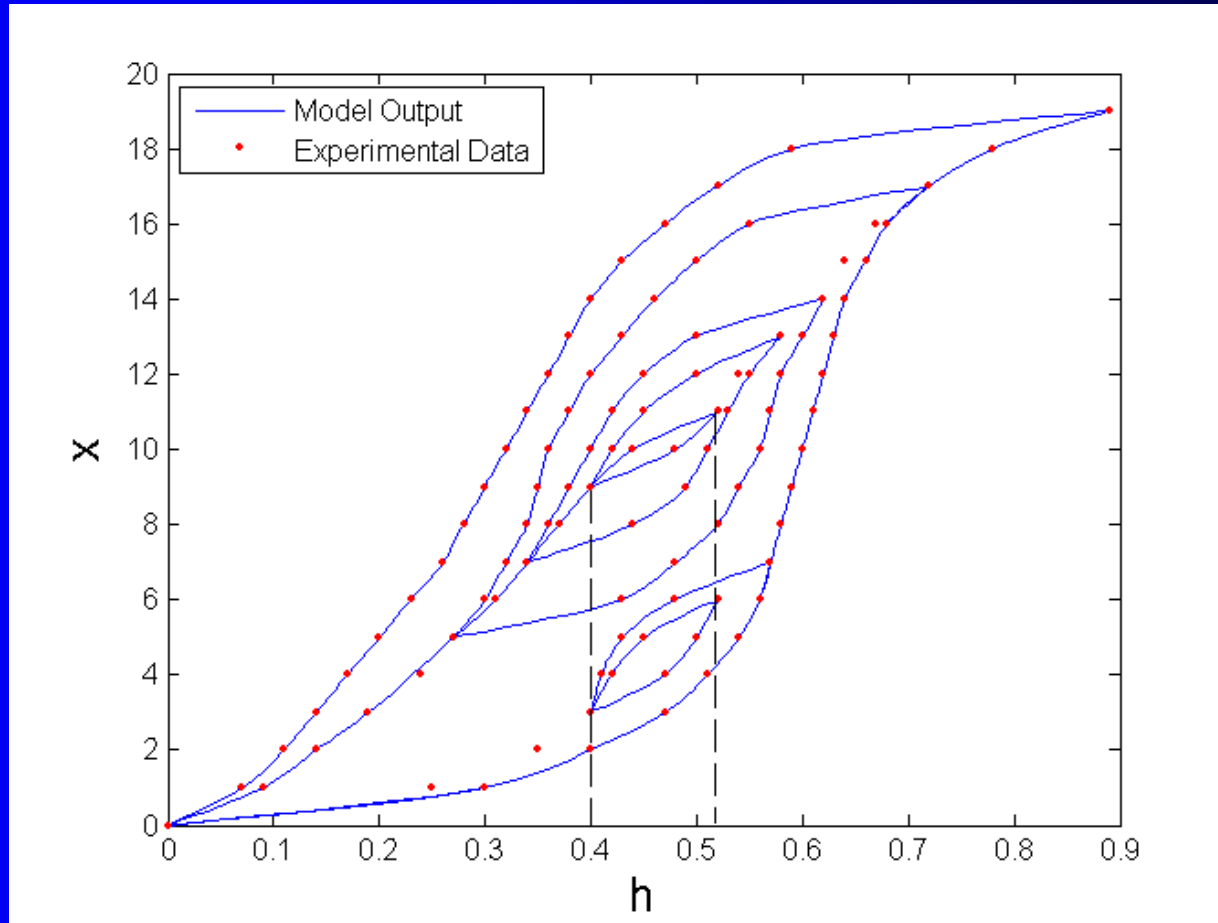


Equal chords



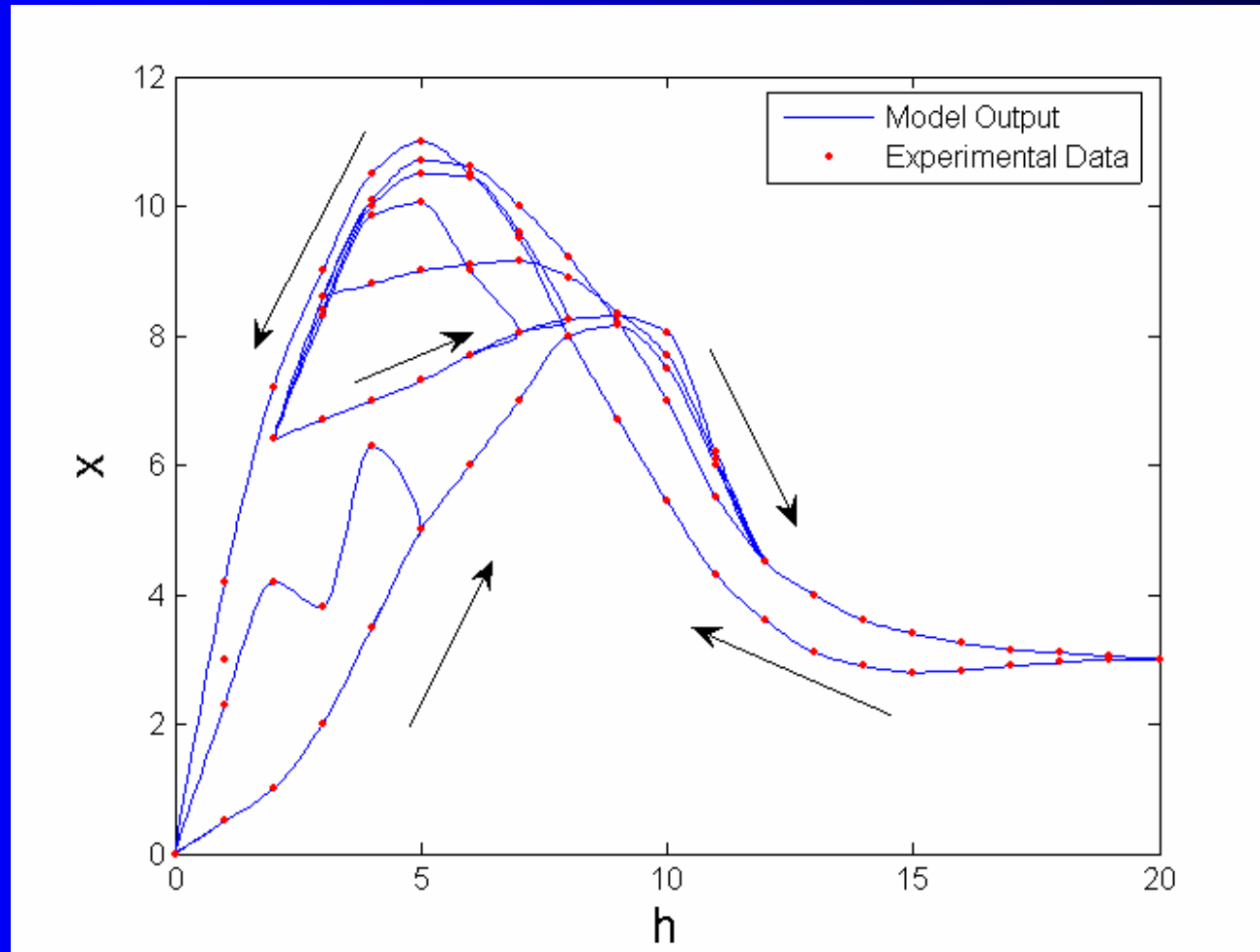
SVEC

## ...Conclusions



Example of minor loops and 6<sup>o</sup> order reversal curves reconstruction.

## ...Conclusions



Example of Resistivity VS Temperature in NiTiNol actuator wire.

## Future Works

- Develop a dynamic version of the proposed model to take into account *rate dependence* hystereses and *accommodation* effect;
- Setup an hybrid framework for the model analysis when used in control loop;
- Generate an algorithm for the adaptive version;
- Vector version of the model.