NAFIT: New Algorithmic Forms of Information Theory

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LIRMM – Université de Montpellier II – CNRS

Rencontres du numérique, April 17th 2013

NAFIT project

DEFIS 2008 ANR-08-EMER-008-01

Title Nouvelles formes algorithmiques de la théorie de l'information

Janvier 2009 – Décembre 2012
extension: Juin 2013

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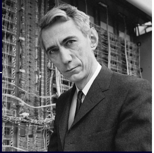
Nikolai Vereshchagin
Laboratoire Poncelet, Moscou

Algorithmic information theory

Information theory: measures the amount of information…

Algorithmic information theory



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Shannon's

…in random variables

(Shannon entropy)

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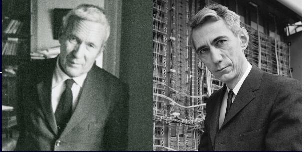

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Local rules and global structure

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Mathematical model: configuration is an infinite cell paper where each cell carries finite amount of information

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Local rule says which combinations of neighbor cells are allowed

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Global structure: the properties of configurations that satisfy local rules

Classical: local rules imply global order

An example: two types of cells: black and white

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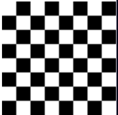

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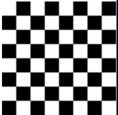

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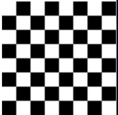

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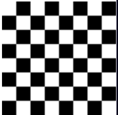

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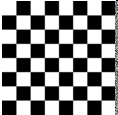
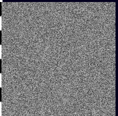
Another trivial case:

Local rule: allow everything

Global configuration: all allowed, both simple and very complex





A paradox: local rules imply global disorder

THM

DLS 2001 A set of local rules exists such that it is consistent (∃ configurations that satisfy all the local rules) and all configurations that satisfy these rules have high complexity.

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DEF

Kolmogorov complexity of a finite object: minimal bit length of a program that produces this object

New technique for constructing local rules

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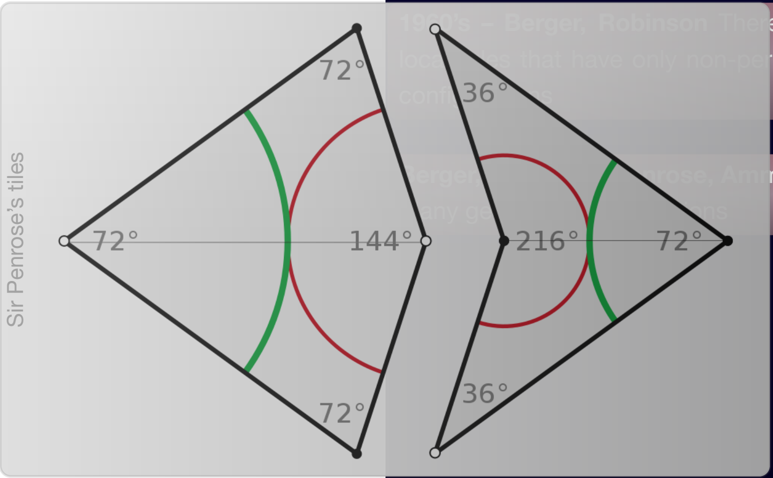

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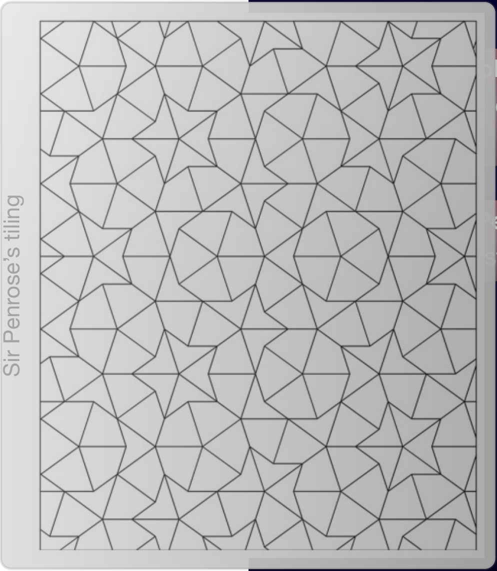

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See `www.lirmm.fr/~ashen` for more information and references

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