Technical note

Nonlinear computed torque control for a high-speed planar parallel manipulator

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A B S T R A C T

A new computed torque (CT)-type controller termed nonlinear CT (NCT) controller is developed and applied to a high-speed planar parallel manipulator. The NCT controller is designed by replacing the linear PD in the conventional CT controller with the nonlinear PD (NPD) algorithm. The stability of the parallel manipulator system with the NCT controller is proven using the Lyapunov theorem, and the proposed controller is further proven to guarantee asymptotic convergence to zero of both tracking error and error rate. The superiority of the proposed NCT controller is verified through the trajectory tracking experiments of an actual high-speed planar parallel manipulator, and the experiment results are compared with the CT controller.

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1. Introduction

Comparing with the serial ones, the parallel manipulators have potential advantages in terms of high stiffness, accuracy, and speed [1]. Especially the high accuracy and speed performances make the parallel manipulators widely applied to the following fields, like the pick-and-place operation in food, medicine, electronic industry and so on. At present, the key issues are the ways to meet the demand of high accuracy in moving process under the condition of high speed. In order to realize the high speed and accuracy motion, it is very important to design efficient controllers for parallel manipulators.

In literatures, there are two basic control strategies for parallel manipulators [2]: kinematic control strategies and dynamic control strategies. In the kinematic control strategies, parallel manipulators are decoupled into a group of single axis control systems, so they can be controlled by a group of individual controllers. Proportional-derivative (PD) control [3,4], nonlinear PD (NPD) control [5,6], and fuzzy control [7] belong to this type of control strategies. These controllers do not always produce high control performance, and there is no guarantee of stability at the high speed. Unlike the kinematic control strategies, full dynamic model of parallel manipulators is taken into account in the dynamic control strategies. So the nonlinear dynamics of parallel manipulator can be compensated and higher performance can be achieved with the dynamic strategies. The traditional dynamic strategies are the augmented PD (APD) controller and the compute-torque (CT) controller [8–10]. In the APD controller, the control law is the combination of the PD control term and the feed-forward dynamic compensation term calculated with the desired velocity and acceleration signals. If the accurate dynamic model is established, the APD controller can achieve satisfied tracking performances. However, the feed-forward term cannot restrain the disturbance, thus the tracking accuracy will be affected when the disturbance exists.

In the traditional CT controller, the control law is the combination of the PD control term and the feedback dynamic compensation term calculated with the actual velocity and desired acceleration signals. That it is to say, the CT controller is a PD controller plus a feedback inner loop. Thus, the CT controller can get better trajectory tracking and disturbance rejection ability. However, the CT controller owns two main drawbacks. Firstly, the dynamic compensation is calculated based on the dynamic model with the fixed dynamic parameters, but the parameters are variable during the trajectory tracking. Thus the dynamic compensation in the CT controller cannot get good compensation performance. In order to realize better dynamic compensation, adaptive controller is designed for the parallel manipulators [11]. And the external disturbance is usually decreased by the adaptive robust controllers [12,13]. Secondly, in the CT controller, the linear PD with the proportional and derivative constant is used to eliminate the tracking error. For the presence of nonlinear factors such as modeling error and friction in the dynamic model of the parallel manipulators, sometimes the CT controllers will not get satisfied tracking accuracy. Thus, some new methods are used to tune the PD gains of the CT controller. In [14,15], the intelligent methods are used to optimize the PD gains in CT controller, and the trajectory tracking accuracy of the manipulators are improved. However, it is difficult to implement this type of controllers in practice for the complex structure and enormous calculation. Fortunately, the NPD control has good suppression of the uncertain factors, and its control performances are superior to the conventional PD.
control, also the NPD controllers have a simple structure [16–18]. In order to overcome the influences of modeling error and nonlinear friction, in this paper, the NPD controller is introduced to replace the PD control in the CT controller of parallel manipulator.

The main contribution of this paper is the development and application of the NPD controller proposed by Han [16] to an actual high-speed planar parallel manipulator. Compared with the work in [16], there are three novel results in our paper. First, the NPD controller is combined with the CT controller, and a new controller termed nonlinear CT (NCT) controller is developed. Second, the stability of the NCT controller containing the NPD is proved strictly. Third, the actual adjustment method of the parameters for the NPD controller is given in our paper. But, the stability proof and parameters adjustment of NPD controllers cannot be found in [5,6,16]. Based on the dynamic model in the task space of the high-speed planar parallel manipulator, the NCT controller is designed by replacing the linear PD in the CT controller with the NPD algorithm. The stability of the parallel manipulator system with the proposed NCT controller is proven using the Lyapunov stability theorem, and the NCT controller is further proven to guarantee asymptotic convergence to zero of both tracking error and error rate with LaSalle’s theorem. The trajectory tracking experiments of an actual high-speed planar parallel manipulator are carried out, and the experiment results of the proposed NCT controller are compared with the traditional CT controller. The detailed tuning procedures of the control parameters in the NCT controller are given in the experiments. Our experiment results indicate that, compared with the CT controller, the NCT controller can get better trajectory tracking accuracy of the end-effector.

The paper is organized as follows. In Section 2, a type of NPD controller is defined and its performances are analyzed. In Section 3, the dynamic model of a high-speed planar parallel manipulator is established. In Section 4, the NCT controller is designed based on the dynamic model using the NPD control, and the stability of the NCT controller is proved with the Lyapunov stability theorem and LaSalle’s theorem. In Section 5, the trajectory tracking experiments of an actual parallel manipulator are carried out, and the experiment results are compared with the conventional CT controller. Finally, several important remarks are concluded.

2. NPD controller

As well as we know, the linear PD controller takes the form

\[ u_i(t) = k_p e(t) + k_d \dot{e}(t), \quad (1) \]

where \( k_p \) and \( k_d \) are the proportional and derivative constants, respectively, and \( e(t) \) is the system error. The nonlinear PD (NPD) controller has a similar structure as the linear PD controller (1), the NPD controller may be any control structure of the form

\[ u_i(t) = k_p(\cdot) e(t) + k_d(\cdot) \dot{e}(t), \quad (2) \]

where \( k_p(\cdot) \) and \( k_d(\cdot) \) are the time-varying proportional and derivative gains, which may depend on system state, input or other variables.

Currently, several NPD controllers have been proposed for robotic application [17,19,20]. The NPD controller has superior trajectory tracking and disturbance rejection ability compared with the linear PD controllers for robot control. A NPD controller proposed by Han has a simple structure as [16]

\[ u_i(t) = k_p \text{fun}(e(t), \delta_1, \delta_2) + k_d \text{fun}(\dot{e}(t), \delta_2), \quad (3) \]

where the function \text{fun} can be defined as

\[ \text{fun}(x, \alpha, \delta) = \begin{cases} x^\gamma \text{sign}(x), & |x| > \delta, \\ x/\delta^{1-\gamma}, & |x| \leq \delta, \end{cases} \]

where \( \gamma \) refers to the nonlinearity, specially the NPD will degenerate into the linear PD when \( \gamma = 1 \); \( \delta \) refers to the threshold of the error (or error derivative), and it is at the same magnitude with the error (or error derivative). The NPD controller (3) can be rewritten as

\[
 k_p(e) = \begin{cases} k_p e^{\gamma_1-1}, & |e| > \delta_1, \\ k_p \delta_1^{\gamma_1-1}, & |e| \leq \delta_1. \end{cases}
 \]

Similarly, \( k_d(\cdot) \) can be expressed as

\[
 k_d(\cdot) = \begin{cases} k_d e^{\gamma_2-1}, & |\dot{e}| > \delta_2, \\ k_d \delta_2^{\gamma_2-1}, & |\dot{e}| \leq \delta_2. \end{cases}
 \]

In (5) and (6), \( \delta_1 \) and \( \delta_2 \) can be determined in the interval [0.5, 1.0] and [1.0, 1.5], respectively. This choice makes the nonlinear gains within the following characteristics [16]: on the one hand, large gain for small error and small gain for large error; on the other hand, large gain for large error rate and small gain for small error rate. Such variations of the gains result in a rapid transition of the systems with favorable damping. In addition, the NPD controller is robust against the changes of the system parameters and the nonlinear factors. Thus the NPD controller (3) is suitable to the trajectory tracking of the high-speed planar parallel manipulator.

3. Dynamic model

The experiment platform is a planar parallel manipulator. As shown in Fig. 1, a reference frame is established in the workspace of the parallel manipulator. The unit of the frame is meter. The parallel manipulator is actuated by three servo motors located at the base A1, A2, and A3, and the end-effector is mounted at the common joint O, where the three chains meet. Coordinates of the three bases are A1 (0, 0.25), A2 (0.433, 0), and A3 (0.433, 0.5), and all of the links have the same length 0.244 m. The definitions of the joint angels are shown in Fig. 1, \( q_{a1}, q_{a2}, q_{a3} \) refer to the active joint angles and \( q_{b1}, q_{b2}, q_{b3} \) refer to the passive joint angles.

Cutting the parallel manipulator at the common point O in Fig. 1, one can have an open-chain system including three independent planar 2-DOF serial manipulators, each of which contains an active joint and a passive joint. The dynamic model of the parallel manipulator equals to the model of the open-chain system plus the closed-loop constraints, thus the dynamics of the parallel manipulator can be formulated by combining the dynamics of the three serial manipulators under the constraints [9,21].

According to [9,21], the dynamic model in the task space of the parallel manipulator can be formulated as

\[
 M \ddot{q} + C \dot{q} + S^T f = S^T e, \quad (7)
\]

![Fig. 1. Coordinates of the high-speed planar parallel manipulator.](image)
where \( q_e = [x\ y]^T \) is the position coordinates of the end-effector; \( \tau_a \) is the actuator torque vector of the active joints; \( f_s \) is the friction torque vector of the active joints [21]; \( S \) is the Jacobian matrix between the velocity of the end-effector and the three active joints; \( M_\tau \) is the inertia matrix in the task space; and \( C_e \) is the Coriolis and centrifugal force matrix in the task space. The detailed definition of the above symbols can be found in [21].

4. Nonlinear computed torque control

An obvious drawback of the traditional CT controllers is the elimination of the tracking error by linear PD algorithm. However, the linear PD algorithm is not robust against the uncertain factors such as modeling error and nonlinear friction. To overcome this problem, the NPD algorithm can be combined with the conventional control strategy to improve the control accuracy. The NCT controller proposed in this paper is designed by replacing the linear PD in the CT controller with the NPD algorithm.

Let \( q_e(t) \) be the desired trajectory of the end-effector in the task space, the actual tracking error can be defined as

\[
e = q_e^d - q_e.
\]

According to the CT controller and the NPD algorithm [3], based on the dynamic model (7), the control law of the NCT controller can be written as

\[
\tau_e = M_\tau q_e^d + C q_e + S \dot{q}_e + M_e k_f (q_e) e + M_q (q_e) e.
\]  

The control law (9) can be divided into three items according to the different functions. The first item is the dynamics compensation defined by the desired acceleration and the actual velocity of the end-effector, which can be written as

\[
\tau_{e_1} = M_\tau q_e^d + C q_e.
\]  

The second item is the friction compensation, which can be written as

\[
\tau_{e_2} = S \dot{q}_e.
\]  

The third item is the error elimination, which can be written as

\[
\tau_{e_3} = M_e k_f (q_e) e + M_q (q_e) e.
\]

where \( k_f (q_e) \) and \( k_q (q_e) \) are symmetric, positive definite matrices of time-varying gains. From (5) and (6), \( k_f (q) \) and \( k_q (q) \) can be expressed as

\[
K_f (q_e) = \text{diag}(k_{f_1}(q_1)|x_1|^{\alpha_1-1}, k_{f_2}(q_2)|x_2|^{\alpha_2-1}),
\]

\[
K_q (q_e) = \text{diag}(k_{q_1}(q_1)|y_1|^{\alpha_1-1}, k_{q_2}(q_2)|y_2|^{\alpha_2-1}),
\]

Theorem 1. If the nonlinear gains \( k_f (\cdot) \) and \( k_q (\cdot) \) are defined by (11) and (12) respectively, the parallel manipulator system controlled by the NCT control law is asymptotically stable.

Proof. Choose the Lyapunov function candidate as

\[
V(e, \dot{e}) = \frac{1}{2} e^T e + \int_0^t e^T K_f (\xi) \dot{e} d\xi.
\]

where, \( \int_0^t e^T K_f (\xi) \dot{e} d\xi = \int_0^t |e|^2 k_f (\xi) d\xi \). Obviously, the first term in (13) is positive definite. In addition, the integral term can be interpreted as the potential energy induced by the position error-driven part of the controller. Next, we will prove that the second term in (13) is positive definite. Considering \( k_f (e) \) is defined as

\[
k_f (e) = \begin{cases} k_{f_1} |e|^{\alpha_1-1}, & |e| > \delta_1, \\ k_{f_0} \delta_1^{\alpha_1-1}, & |e| \leq \delta_1, 
\end{cases}
\]

and define class K functions \( \alpha_i (\cdot) \) as

\[
\alpha_i (|e_i|) = \begin{cases} e_i |e_i|^{\alpha_i}, & |e_i| > \delta_i, \\ e_i \delta_i^{\alpha_i-1}, & |e_i| \leq \delta_i, \quad \text{and} \quad k_f > e_i > 0. 
\end{cases}
\]

From (14) and (15), one knows \( |e_i| k_f (e_i) \geq \alpha_i (|e_i|) \). With the Lemma, one can get \( \int_0^t |e|^2 k_f (\xi) d\xi \to \infty \), and \( \int_0^t |e|^2 K_f (\xi) d\xi \to \infty \). So one can get the integral term in (13) is a radially unbounded positive definite function. Thus \( V(e, \dot{e}) \) is a positive definite function. Differentiating \( V(e, \dot{e}) \) with respect to time yields

\[
\dot{V} (e, \dot{e}) = e^T \dot{e} + e^T K_f (q_e) \dot{e}
\]

Combine the control law (9) and the dynamic model (7), considering \( S^T \tau_a = \tau_a \), the closed-loop system equation can be written as

\[
M_e (e + K_f (q_e) e + K_q (q_e) e) = 0.
\]

Since \( M_e \) is uniform positive definite, then \( M_e^{-1} \) exists and bounded, thus the closed-loop Eq. (17) can be written as

\[
\ddot{e} + k_f (q_e) e + k_q (q_e) e = 0.
\]

Multiplying both sides of the above equation by \( e^T \), and then substituting the resulting equation into (16) yields

\[
\dot{V} = -e^T k_q (q_e) e.
\]

As \( k_q (\cdot) \) is a symmetric, positive definite matrix, then \( \dot{V} \) is a semi-negative definite matrix, thus the closed-loop system is stable. Considering the closed-loop Eq. (18) is autonomous system, and defining the region \( \Omega \) as

\[
\Omega = \{ e \in \mathbb{R}^4 : \dot{V} (e, \dot{e}) = 0 \} = \{ e : e = 0 \} \in \mathbb{R}^4.
\]

Thus \( e = [0, 0, 0, 0]^T \) is the largest invariant set of \( \Omega = \{ e \in \mathbb{R}^4 : \dot{V} (e, \dot{e}) = 0 \} \) and constitutes an asymptotically stable equilibrium point. By using LaSalle’s theorem, we can get that the closed-loop system is asymptotically stable. □

5. Actual experiments

As shown in Fig. 2, the actual experiment platform is a high-speed planar parallel manipulator designed and produced by Gool Tech. Ltd. in Shenzhen, China. It is equipped with three permanent magnet synchronous servo motors with harmonic drive gears. The active joint angles are measured with absolute optical-electrical encoders. We programmed the NCT controller and the CT controller with the Visual C++, and the algorithms ran on a Pentium III CPU at 733 MHz. The sampling rate of the real time system used for the control is 2 ms.
In the trajectory tracking experiments, both the linear and circular trajectory in the workspace are selected as the desired trajectory. The desired trajectory and real trajectory are both shown in Fig. 3. From Fig. 3, one can see that the blue curves refer to the desired trajectory, and the red curves refer to the real trajectory. For the linear trajectory shown in Fig. 3a, the starting point is (0.22, 0.19) and the ending point is (0.35, 0.29), thus the motion distance is 0.164 m. The velocity profile of the linear trajectory is $(0.22, 0.19)$ and the ending point is $(0.35, 0.29)$, thus the motion distance is 0.164 m. The velocity profile of the linear trajectory is a S-type curve $[9]$, the max velocity is 0.5 m/s, the max acceleration is $0.164 m$. The velocity profile of the circular trajectory in the workspace is selected as the desired trajectory, and the red curves refer to the real trajectory. For the circular trajectory with the constant speed of 0.5 m/s, the center is $(0.29, 0.25)$ and the radius is 0.06 m.

In order to implement the controller (9), the dynamic parameters in (10a) and the friction parameters in (10b) must be known. In the experiment, the nominal values are selected as the values of the actual dynamic parameters, which can be found in [21]. Then, with the known dynamic parameters, the friction parameters can be identified by the Least Squares method [21]. And the values of the control parameters in (10c) are tuned and determined by the actual experiments.

The procedures of tuning parameters in (10c) are as follows:

1. Assume $k_{p1} = k_{p2} = k_p$, $k_{d1} = k_{d2} = k_d$. Let $k_d = 0$, $x_1 = 1$, $x_2 = 1$, and increase the value of $k_p$ from zero until the system show a little oscillation to some extent.
2. Keep the value of $k_p$ tuned well in the first stage, and increase the value of $k_d$ to improve the dynamic performance further.
3. Regulate finely the above two values and make tradeoffs between $k_p$ and $k_d$.
4. Find the maximum error and error rate of the end-effector under the tuned value of $k_p$ and $k_d$.
5. In the NPD algorithm, $\delta_1$ and $\delta_2$ are the threshold of the error and the error rate. If $\delta_1$ is tuned bigger than the maximum error, then the proportional gain $k_p(e_i)$ will always equals to $k_p\delta_1^{-1}$; and $\delta_2$ is tuned close to 0, then $k_d(e_i)$ will always equal to $k_d\delta_2^{-1}$. So, $\delta_1$ should be made a trade-off between the maximum error and 0 error. Similar method can be used to tune parameter $\delta_2$. From our actual experiences, the value of $\delta_1$ is tuned to the half value of the maximum error, and the value of $\delta_2$ is tuned to the half of the maximum error rate. This choice has good control performance and it is easy to implement.
6. For the parameters $x_1 = 1$ and $x_2 = 1$, the proportional gain $k_p(e_i)$ is a constant of $k_p$, and the derivative gains $k_d(e_i)$ is a constant of $k_d$. Thus the NPD algorithm can be considered as the linear PD algorithm. So decrease the value of $x_1(0.5 < x_1 < 1)$, and decrease the value of $k_p$ at the same time to improve the error curve further, and make tradeoffs between the two values. Using this step, one can get the nonlinear proportional gain of the NCT controller.
7. Increase the value of $x_2(1 < x_2 < 1.5)$, and decrease the value of $k_p$ at the same time to improve the error rate curve further, then make tradeoffs between the two values.

Using the above procedures, the NCT controller parameters are tuned as follows: $k_p = 2400$, $k_d = 240$, $\delta_1 = 3 \times 10^{-4}$, $\delta_2 = 3 \times 10^{-3}$, $x_1 = 0.7$, $x_2 = 1.1$, and the above values of the parameters are called parameters 1.

Moreover, to demonstrate that the NCT controller can improve the tracking accuracy of the end-effector, experiments using the CT controller [9] are carried out as comparison. The CT controller is chosen because it has nonlinear dynamics compensation and friction compensation. In the CT controller, the control input vector of the three active joints can be calculated as

$$r_a = (S^T)^{-1}(M_a\dddot{q}_a + C_a\dot{q}_a + M_a(K_p\dot{e} + K_d\dot{e})) + f_a,$$  \hspace{1cm} (21)

where $K_p$ and $K_d$ are both symmetric, positive definite matrices of constant gains; In (21), $(S^T)^{-1}$ is the pseudo-inverse of $S$, satisfying $S(S^T)^{-1}S = I$ [9].

In the CT controller (21), the dynamic parameters in $M_a$ and $C_a$ and the friction parameters in $f_a$ are the same with those of the NCT controller. The procedures of tuning parameters of $K_p$ and $K_d$ in the CT controller are similar to the procedures of tuning parame-
ters of $k_p$ and $k_d$ in the NCT controller. Thus, the tuning procedures (1)–(3) can be used to tune the parameters of $K_{lp}$ and $K_{ld}$. Using the above methods, the CT controller parameters are tuned as follows: $K_{lp} = \text{diag}(20000, 20000)$, $K_{ld} = \text{diag}(150, 150)$.

Considering the NCT controller has the constant proportional and derivative gains when the error and error rate close to zero, then the NCT controller has the same behavior with the CT controller. In order to compare the performances of the two controllers further, the local gains close to zero error of the NCT controller are tuned to the same with the gains of the CT controller. The non-linear proportional and derivative gain are selected as follows: $k_p(\varepsilon_l) = k_p(\varepsilon_l)^{1/2} = 20000$, when the error is small; and $k_d(\dot{\varepsilon}_l) =$

![Fig. 4. Linear trajectory tracking errors of the end-effector: (a) X-direction and (b) Y-direction.](image)

![Fig. 5. Circular trajectory tracking errors of the end-effector: (a) X-direction and (b) Y-direction.](image)

![Fig. 6. The control inputs of the three active joints of the circular trajectory: (a) active joint 1; (b) active joint 2; and (c) active joint 3.](image)
$k_b \delta_1^{2-1} = 150$ when the error rate is small. Then, the local behavior close to the small errors of the two controllers is the same, and the difference in the behavior for large errors can be analyzed. Then the value of the NCT controller parameters are solved as follows: $k_p = 1755$, $k_u = 268$, and other controller parameters are selected same as the parameters 1. $\delta_1 = 3 \times 10^{-4}$, $\delta_2 = 3 \times 10^{-3}$, $\gamma_1 = 0.7$, $\gamma_2 = 1.1$. And the above value of the controller parameters is called parameters 2.

The tracking error curves of the end-effector controlled by the CT and NCT controller are shown in Figs. 4 and 5. Fig. 4 presents the linear trajectory tracking errors of the end-effector on the X-direction and Y-direction. From the experiment curves, one can see that the tracking errors of the NCT controller with the parameters 2 are smaller than the tracking errors of the CT controller. Considering the NCT controller has the same behavior with the CT controller when the error and error rate are small, thus the tracking ability of the NCT controller for large errors is better than the CT controller. Especially, the tracking accuracy of the NCT controller with the parameters 1 is better than the tracking accuracy with the parameters 2. This experiment result validates that the tuning procedure of the NCT controller is satisfying. Fig. 5 shows the circular trajectory tracking errors of the end-effector on the X-direction and Y-direction. From the curves, one can see that the tracking accuracy is improved obviously using the NCT controller with the parameters 1 or parameters 2, compared with the CT controller. Also, one can see that the tracking accuracy of the NCT controller with the parameters 1 is better than the result with the parameters 2.

In order to further study the control performances of the NCT controller, the control inputs of the three active joints with the NCT and CT controllers are provided. The control inputs of the three active joints of the circular trajectory are shown in Fig. 6. From the control inputs curves, one can see that the control input of the NCT controller changes more quickly. These changes will help decreasing the influences of the modeling error and the nonlinear friction.

6. Conclusion

A new CT-type controller termed NCT controller is developed and applied to a high-speed planar parallel manipulator. The NCT controller is designed by replacing the linear PD in the CT controller with the NPD algorithm. So the NCT controller inherits merits from the CT controller, such as simple structure and clear physical meaning of each control parameter, also it owns the good performances of the NPD algorithm in elimination of the nonlinear factors such as the modeling error and the nonlinear friction. According to the best knowledge of the authors, this is the first time that the NCT controller was proposed, especially was used for an actual parallel manipulator. Also the NCT controller can be used to other manipulators, such as serial ones, or parallel manipulators without redundant actuation to realize high speed and high accuracy motion.

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