Performance comparisons of model-free control strategies for hybrid magnetic levitation system

R.-J. Wai and J.-D. Lee

Abstract: The study mainly focuses on the development of three model-free control strategies including a simple proportional-integral-differential (PID) scheme, a fuzzy-neural-network (FNN) control and an adaptive control for the positioning of a hybrid magnetic levitation (maglev) system. In general, the lumped-parameters dynamic model of a hybrid maglev system can be derived from the energy balance. In practice, the mathematical model can not be established precisely because this hybrid maglev system is inherently unstable in the levitated direction, and the relationships between current and electromagnetic force are highly nonlinear. To cope with the unavailable dynamics, model-free control design is used to handle the system behaviour. In this study the experimental results of PID, FNN and adaptive control schemes for the hybrid maglev system are reported. As can be seen from performance comparisons, the adaptive control system yields favorable control performance superior to that of PID and FNN control systems. Moreover, it not only has a learning ability similar to that of FNN control but also the simple control structure of PID control.

1 Introduction

In recent years, magnetic levitation (maglev) techniques have been respected for eliminating friction due to mechanical contact, decreasing maintaining cost, and achieving high-precision positioning. They are therefore widely used in various fields, such as high-speed trains [1–5], magnetic bearings [6, 7], vibration isolation systems [8], wind tunnel levitation [9] and photolithography steppers [10]. In general, maglev techniques can be classified into two categories: electrodynamic suspension (EDS) and electromagnetic suspension (EMS). EDS systems are commonly known as “repulsive levitation”, and the corresponding levitation sources are from superconducting magnets [11] or permanent magnets [12]. However, the repulsive magnetic poles of superconducting magnets cannot be activated at low speed so that they are only suitable for long-distance and high-speed train systems. Basically the magnetic levitation force of EDS is partially stable and allows a large clearance. Nevertheless, the productive process of magnetic materials is more complex and expensive. On the other hand, EMS systems are commonly known as “attractive levitation”, and the magnetic levitation force is inherently unstable so that the control problem becomes more difficult. Generally speaking the manufacturing process and cost of EMS are lower than that of EDS, but extra electric power is required to maintain levitation height.

To merge the merits of these two kinds of levitation systems, in this study a hybrid maglev system adopted is combined with an electromagnet and a permanent magnet. The magnetic force generated by the additional permanent magnet is used to alleviate the power consumption for levitation.

Because the EMS system has unstable and nonlinear behaviour, it is difficult to build a precise dynamic model. Some research has derived various mathematical models for many kinds of maglev systems in numerical simulation [13, 14], but there still exist unpredictable uncertainties in practical applications. In general, linearised control strategies based on a Taylor-series expansion of the actual nonlinear dynamic model and force distribution at nominal operating points are often employed. Nevertheless, the tracking performance of the linearised control strategies [15–18] deteriorate rapidly with increasing deviation from nominal operating points. Many approaches introduced to solve this problem for ensuring consistent performances independent of operating points have been reported in previous literature. Backstepping methods were reported in [12, 19] due to the systematic design procedure. Huang et al. [12] addressed an adaptive backstepping controller to achieve a desired stiffness for a repulsive maglev suspension system. Queiroz and Dawson [19] utilised a nonlinear model of an active magnetic bearing system for developing a nonlinear backstepping controller. Unfortunately some constraining conditions should be satisfied for precision positioning. Moreover, the approach of gain scheduling [20, 21] can linearise the nonlinear relationships of the magnetic suspension at various operating points with a suitable controller designed for each of these operating points. To achieve better control performance over the entire operational range, it needs to subdivide the operating range into appropriate intervals. In this way favourable control gains collected in the lookup table will occupy a large memory to bring about the heavy computation burden. In addition, Sinha and Pechev [22] presented an adaptive controller to compensate for payload variations and external force disturbance using the criterion of stable maximum descent. Overall, the detailed or partial mathematical models of complicated modelling processes are usually required to design a suitable control law for
achieving positioning demand. The aim of this study is to introduce model-free control strategies for a hybrid maglev system and to compare their superiority or defect via experimental results.

Conventional proportional-integral-derivative (PID) controllers have been widely used in industry due to their simple control structure, ease of design and inexpensive cost. However, this model-free PID-type controller cannot provide perfect control performance if the controlled plant is highly nonlinear and uncertain. On the other hand, intelligent control techniques (fuzzy control or neural network control) have been adopted in the control field for their powerful learning ability and unnecessary prior knowledge of the controlled plant in the design process [23–28]. Lepeti et al. [25] introduced a predictive functional controller based on a Takagi-Sugeno fuzzy model for a magnetic suspension system. Hong and Langari [26] represented a nonlinear magnetic bearing system by a Takagi-Sugeno-Kang fuzzy model, where a nonlinear global model is approximated by a set of linear local models. In the application of neural networks, Cole et al. [27] utilised a neural network to identify faults associated with the system position transducer measurements so that the output from the neural network can be used as the decision tool for reconfiguration control. Jeng and Lee [28] proposed a Chebyshev-polynomial-based unified-model (CPBUM) neural network with faster learning speed than conventional feedforward/recurrent neural networks for the position control of a magnetic bearing system. In recent years, the concept of incorporating fuzzy logic into a neural network has grown into a popular research topic [29–31].

Unfortunately the stability of these control strategies cannot be guaranteed, or their control structures are more complex than that of the conventional PID controller. To simplify the control framework and ensure system stability some literature is mentioned [4, 32]. Kaloust et al. [4] presented a nonlinear robust control design for the levitation and propulsion of a maglev system, where a recursive controller was designed via the nonlinear state transformation and Lyapunov direct method to guarantee global stability for the nonlinear maglev system. Wai [32] developed a robust control system based on a hypothetical dynamic model to achieve high-precision position control for a linear piezoelectric ceramic motor.

Due to inherent instability and high nonlinearity associated with the electromechanical dynamics in the hybrid maglev system, the control problem is usually quite challenging to control engineers, especially in model-free control design. In this study, three model-free control strategies including PID, FNN and adaptive control strategies are implemented for the levitation positioning of a hybrid maglev system, and the corresponding experimental results are provided to compare individual diversities.

2 Hybrid maglev system

The configuration of a hybrid maglev system is depicted in Fig. 1a, which consists of a hybrid electromagnet, a ferrous plate, a load carrier and a gap sensor. Among these, the hybrid electromagnet is composed of a permanent magnet and an electromagnet. It forms two flux-loops in the E-type hybrid electromagnet, and the flux passes through a permanent magnet, a ferrous plate, an air gap and a core in each loop. The magnetic equivalent circuit can be represented as Fig. 1b. The magnetomotive force (MMF) of this hybrid electromagnet is the summation of the permanent magnet MMF $F_p$ and the electromagnet MMF $F_{el}$, where $N_m$ is the coil turns and $i$ is the coil current. Moreover, the total reluctance of the magnetic path is

$$\frac{1}{R} = \frac{1}{R_p} + \frac{1}{R_F} + \frac{1}{R_s} + \frac{1}{R_c} + \frac{1}{R_p}$$

(1)

where $R_p$, $R_F$, $R_s$ and $R_c$ are the reluctances of the permanent magnet, ferrous plate, air gap and core in the magnetic path, respectively. In addition, the flux $\Phi$ produced against the magnetic reluctance $R$ by this hybrid electromagnetic MMF can be denoted as

$$\Phi = \frac{F_p + N_m i}{R}$$

(2)

The energy in this magnetic field is

$$W_f = \frac{1}{2} L i^2 = \frac{1}{2} N_m (F_p + N_m i) i P$$

(3)

where $P = 1/R$ is the permeance of the magnetic path; $L$ is the inductance of the hybrid electromagnet and is given by neglecting magnetic saturation as

$$L = \frac{i \lambda}{i} = \frac{N_m \Phi}{i} = \frac{N_m}{R} (F_p + N_m i)$$

(4)

in which $\lambda$ means the flux linkage. Assume that there is no loss in energy transmission, the power produced by the magnetic field can be represented via the principle of the conservation of energy as

$$\frac{dW_f}{dt} = i \frac{d\lambda}{dt} - F \frac{dx}{dt}$$

(5)

where $F$ is the produced mechanical force, and $x$ is the air-gap length. Multiply $dt$ on both sides of (5), then

$$dW_f = i d\lambda - F dx$$

(6)
Since the magnetic energy is a function of \( \lambda \) and \( x \), one can obtain
\[
dW_f(\lambda, x) = \frac{\partial W_f}{\partial \lambda} d\lambda + \frac{\partial W_f}{\partial x} dx
\] 
(7)

To compare (6) with (7), the mechanical force \( F(x, i) \) can be expressed via (3) as
\[
F(x, i) = -\frac{\partial W_f(\lambda, x)}{\partial x} = -\frac{1}{2} N_m (F_p + N_m i) l \frac{\partial P}{\partial x} \equiv -G_f i
\] 
(8)

where the term \( G_f \) is related to the total magnetomotive force, coil turns and the permeance in the magnetic path. According to Newton’s law, the dynamic behaviour of the hybrid maglev system is governed by the following equation:
\[
\ddot{x}(t) = \frac{G_f}{m} i + g + \frac{f_d}{m} = \frac{G_f G_t}{m} U + g + \frac{f_d}{m}
\]
\[
\equiv G(x, t) U(t) + M(x, t)
\] 
(9)

where \( m \) is the mass of total suspension object, \( g \) is the acceleration of gravity, \( f_d \) is the external disturbance force, \( G_t \) represents the transfer function of a power amplifier, and \( U \) is the control effort. Moreover, \( G(x, t) = G_t G_t / m \) expresses the control gain, and \( M(x, t) = g + f_d / m \). Due to the nonlinear and time-varying characteristics of the hybrid maglev system, accurate model dynamics \( G(x, t) \) and \( M(x, t) \) are assumed to be unknown in this study. Without loss of generality it is assumed that \( G(x, t) \) is finite and bounded away from zero for all \( x \).

3 Control systems design

3.1 PID control system

In industrial application the PID-type control is usually used due to its simple scheme. The PID control system in this study is depicted in Fig. 2, where \( x \) is the Laplace operator and the tracking error is defined as
\[
e = x_m - x
\] 
(10)

In which \( x_m \) represents the reference air-gap length. The PID control law can be represented as
\[
U = U_p + U_i + U_D = K_p e + K_I \int e + K_D \frac{de}{dt}
\] 
(11)

where \( U_p \) is a proportional controller; \( U_i \) is an integral controller; \( U_D \) is a differential controller; \( K_p, K_I \) and \( K_D \) are the corresponding control gains. Selection of the values for the gains in the PID control system has a significant effect on the control performance. In general they are determined according to desirable system responses, e.g. rise time, settling time, etc. With proportional control, a corrective force is proportional to the tracking error for the hybrid maglev system, and the coefficient \( K_p \) is designed according to the amount of initial tracking error. Moreover, the integral controller \( U_I \) can alleviate the steady-state error due to the load force, and the gain \( K_I \) is selected based on the amount of steady-state error. In addition, the differential controller \( U_D \) is good for the faster system response and the reduction of the tendency to overshoot, and the parameter \( K_D \) is chosen as small as possible to prevent enlarging the noise effect.

3.2 FNN control system

A FNN control system is depicted in Fig. 3, where a four-layer network structure with the input (i layer), membership (j layer), rule (k layer) and output (o layer) layers is adopted [30, 31]. The membership layer acts as the membership functions. Moreover, all the nodes in the rule layer form a fuzzy rule base. The signal propagation and the basic function in each layer of the FNN are introduced in the following paragraph.

For every node \( i \) in the input layer transmits the input variables \( x_i (i = 1, \ldots, n) \) to the next layer directly, and \( n \) is the total number of the input nodes. Moreover, each node in the membership layer performs a membership function. In this study the membership layer represents the input values with the following Gaussian membership functions:
\[
net_j(x_i) = -\frac{(x_i - m_j^i)^2}{(\sigma_j^i)^2},
\]
\[
\mu_j^i (net_j(x_i)) = \exp (net_j(x_i))
\] 
(12)

where \( m_j^i \) and \( \sigma_j^i (i = 1, \ldots, n; j = 1, \ldots, n_p) \) are, respectively, the mean and the standard deviation of the Gaussian
function in the \( j \)th term of the \( i \)th input variable \( x_i \) to the node of this layer, and \( n_{yi} \) is the total number of the linguistic variables with respect to the input nodes. In addition, each node \( k \) in the rule layer is denoted by \( \Pi_k \), which multiplies the input signals and outputs the result of the product. The output of this layer is given as

\[
\phi_k = \prod_{i=1}^{n} w_{ki}^j \eta_i(\text{net}_i(x_i))
\]

(13)

where \( \phi_k \ (k = 1, \ldots, n_x) \) represents the \( k \)th output of the rule layer; \( w_{ki}^j \), the weights between the membership layer and the rule layer, are assumed to be unity; \( n_i \) is the total number of rules. Furthermore, the node \( o \) in the output layer is labelled with \( \Sigma \); each node \( y_o \ \ (o = 1, \ldots, n_o) \) computes the overall output as the summation of all input signals, and \( n_o \) is the total number of output nodes

\[
y_o = \sum_{k=1}^{n} w_{ko}^j \phi_k
\]

(14)

where the connecting weight \( w_{ko}^j \) is the output action strength of the \( o \)th output associated with the \( k \)th rule. In this study the inputs of the FNN control system are the tracking error \( (x_i = e) \) and its derivative \( (x_{2i} = e_x) \), and the single output is the control effort for the hybrid maglev system, i.e. \( \dot{U} = y_o \). The FNN controller is similar to a nonlinear PD controller as it is based on \( e \) and \( e_x \).

To describe the online learning algorithm of this FNN control system via supervised gradient decent method, first the energy function \( E \) is defined as

\[
E = (x_n - \dot{x})^2/2 = e^2/2
\]

(15)

In the output layer the error term to be propagated is given by

\[
d_o = -\frac{\partial E}{\partial y_o} = -\frac{\partial E}{\partial e} \frac{\partial e}{\partial y_o} = \frac{\partial E}{\partial e} \frac{\partial e}{\partial x} \frac{\partial x}{\partial y_o}
\]

(16)

and the weight is updated by the amount

\[
\Delta w_{ko}^j = -\eta_e \frac{\partial E}{\partial w_{ko}^j} = \eta_e \frac{\partial E}{\partial y_o} \left( \frac{\partial y_o}{\partial w_{ko}^j} \right) = \eta_e \delta_o \phi_k
\]

(17)

where \( \eta_e \) is the learning-rate parameter of the connecting weights. The weights of the output layer are updated according to the following equation:

\[
w_{ko}^j(N + 1) = w_{ko}^j(N) + \Delta w_{ko}^j
\]

(18)

where \( N \) denotes the number of iterations. Since the weights in the rule layer are unified, only the error term to be calculated and propagated.

\[
\delta_k = \frac{\partial E}{\partial \phi_k} = \delta_o w_{ko}^j
\]

(19)

In the membership layer, the error term is computed as follows:

\[
\delta_i = -\frac{\partial E}{\partial \text{net}_i} = \sum_k \delta_k \phi_k
\]

(20)

The update laws of \( m_i^j \) and \( \sigma_i^j \) can be denoted as

\[
m_i^j(N + 1) = m_i^j(N) + \Delta m_i^j
\]

(21)

with \( \Delta m_i^j = -\eta_m \frac{\partial E}{\partial m_i^j} = \eta_m \delta_i \frac{2(x_i - m_i^j)}{\sigma_i^j} \)

\[
\sigma_i^j(N + 1) = \sigma_i^j(N) + \Delta \sigma_i^j
\]

(22)

with \( \Delta \sigma_i^j = -\eta_e \frac{\partial E}{\partial \sigma_i^j} = \eta_e \delta_i \frac{2(x_i - m_i^j)}{\sigma_i^j} \)

where \( \eta_m \) and \( \eta_e \) are the learning-rate parameters of the mean and the standard deviation of the Gaussian function. The exact calculation of the jacobian of the actual plant, \( \partial \dot{x}_i/\partial y_o \) in (16), cannot be determined due to the unknown system dynamics. Similar to [30, 31], the delta adaptation law \( \delta_i = e + e_x \) is used for approximating (16) in this study. Moreover, varied learning rates in [31], which are derived based on the analyses of a discrete-type Lyapunov function, are adopted to guarantee convergence of the tracking error.

### 3.3 Adaptive control system

To control the air-gap length of the hybrid maglev system more effectively, an adaptive control system shown in Fig. 4 is implemented [32-34], and the state variables are defined as follows:

\[
X_1 = x
\]

(23)

\[
\dot{X}_1 = v = X_2
\]

(24)

where \( v \) represents the derivative of \( x \). Rewriting (9) the hybrid maglev system can be represented in the following state space form:

\[
\begin{bmatrix}
\dot{X}_1 \\
\dot{X}_2
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} + 
\begin{bmatrix}
0 \\
G(x, t)
\end{bmatrix} U + 
\begin{bmatrix}
0 \\
M(x, t)
\end{bmatrix}
\]

(25)

The equation can be expressed as

\[
\dot{X}_P = AX_p + BU - D
\]

\[
= AX_p + (B + \Delta B)U - (D + \Delta D)
\]

\[
= AX_p + BU - D + L
\]

(26)

where \( X_P = [X_1 \ X_2 \ X_3 \ X_4] \); \( A = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} ; B = \begin{bmatrix} 0 & G(x, t) \end{bmatrix} \); \( D = \begin{bmatrix} 0 & -M(x, t) \end{bmatrix} \); \( B \) and \( D \) are the nominal parametric matrixes of \( B \) and \( D \); \( \Delta B \) and \( \Delta D \) denote the uncertainties.
introduced by parameter variation and external disturbances \( \mathbf{L} = \mathbf{A} \mathbf{B} \mathbf{U} - \mathbf{A} \mathbf{D} \) is the lumped uncertainty. 

In the adaptive control system design the desired behaviour of the hybrid maglev system is expressed through the use of a reference model driven by a reference input. Typically a linear model is used. A reference model of the hybrid maglev system.

Typically a linear model is used. A reference model of the hybrid maglev system.

The control problem is to find a control law so that the control error vector be

\[
\mathbf{E}_C = \mathbf{X}_M - \mathbf{X}_P = [x_m - x, v_m - v]^T = [e, e_s]^T
\]

To make the control error vector tend to zero with time, the control error equation governing the closed-loop system can be obtained from (25) through (29) as follows:

\[
\dot{\mathbf{E}}_C = \mathbf{A}_M \mathbf{E}_C + (\mathbf{A}_M - \mathbf{A} - \mathbf{B} \mathbf{\Theta}) \mathbf{X}_P + (\mathbf{B}_M - \mathbf{B} \mathbf{K}) \mathbf{R} + (\mathbf{D} - \mathbf{B} \mathbf{\xi} - \mathbf{L})
\]

If the precise model dynamics and the uncertainties in practical applications are available, there exist ideal control gains \( \mathbf{\Theta}^*, \mathbf{K}^* \) and \( \mathbf{\xi}^* \) in the following equations such that the control error vector tend to zero with time:

\[
\mathbf{\Theta}^* = \mathbf{B}^+ (\mathbf{A}_M - \mathbf{A})
\]

\[
\mathbf{K}^* = \mathbf{B}^+ \mathbf{B}_M
\]

\[
\mathbf{\xi}^* = \mathbf{B}^+ (\mathbf{D} - \mathbf{L})
\]

where \( \mathbf{B}^* \) is the left penrose pseudo inverse of \( \mathbf{B} \), i.e. \( \mathbf{B}^* = (\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \). Since the dynamic model and the uncertainties of the controlled system may be unknown or perturbed, the ideal control gains shown in (31)–(33) cannot be implemented in practice. Reformulate (30), then

\[
\dot{\mathbf{E}}_C = \mathbf{A}_M \mathbf{E}_C + \mathbf{B}(\mathbf{E}_0 \mathbf{X}_P + \mathbf{E}_k \mathbf{R} + \mathbf{E}_\xi) \]

in which the control parameter errors \( \mathbf{E}_0, \mathbf{E}_k \) and \( \mathbf{E}_\xi \) are defined as

\[
\mathbf{E}_0 = \mathbf{\Theta}^* - \mathbf{\Theta}
\]

\[
\mathbf{E}_k = \mathbf{K}^* - \mathbf{K}
\]

\[
\mathbf{E}_\xi = \mathbf{\xi}^* - \mathbf{\xi}
\]

The ideal control gains are observed by simple adaptation laws and are assumed to be constant during the observation. The assumption is valid in practical digital processing of the adaptation laws because the sampling periods of the adaptation laws are short enough compared with the variation of ideal control gains.

**Theorem 1:** Consider the hybrid maglev system represented by (25); if the adaptive control law is designed as (29) in which the adaptation laws of the control gains are designed as (38)–(40), then the stability of the adaptive control system can be guaranteed

\[
\mathbf{\Theta} = \gamma_1 \mathbf{E}_0^T \mathbf{B} \mathbf{X}_P
\]

\[
\mathbf{K} = \gamma_2 \mathbf{B}^T \mathbf{E}_C \mathbf{R}
\]

\[
\mathbf{\xi} = \gamma_3 \mathbf{B}^T \mathbf{E}_C
\]

where \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) are positive constants.

**Proof:** The proof of the theorem is similar to [32–34] and is omitted here.

From theorem 1 it follows that the tracking error will tend to zero under the level of slowly varying uncertainties. However, the control gains will not necessarily converge to their ideal values in (31)–(33); it is shown only that they are bounded. Moreover, according to the unavailable system parameters, the nominal value of control gain \( G(x, t) \) in the tuning algorithms is reorganised as \( G(x, t) \) in (40) can be reorganised as follows:

\[
\dot{\mathbf{\Theta}} = \beta_1 e \mathbf{X}_P^T \mathbf{sgn}(G(x, t))
\]

\[
\dot{\mathbf{K}} = \beta_2 e \mathbf{R} \mathbf{sgn}(G(x, t))
\]

\[
\dot{\mathbf{\xi}} = \beta_3 e \mathbf{X}_P \mathbf{sgn}(G(x, t))
\]

where \( \mathbf{sgn(\cdot)} \) is a sign function; the terms \( \gamma_1 |G(x, t)|, \gamma_2 |G(x, t)| \) and \( \gamma_3 |G(x, t)| \) are absorbed by the tuning gain, \( \beta_1, \beta_2 \) and \( \beta_3 \) individually. Consequently only the sign of \( G(x, t) \) is required in the design procedure, and it can be easily obtained from the physical characteristic of the hybrid maglev system.

4 Experimental results

The block diagram of a computer-based control system for the hybrid maglev system is depicted in Fig. 5. In the hybrid maglev system, it can be divided into two parts: A ferrous frame and a levitation table. A hybrid electromagnet is fixed on the levitation table, and the attracting levitation force is produced by the magnetisation of the electromagnetic coil.

**Fig. 5** Computer-based control system
A servo control card is installed in the control computer, which includes multichannels of D/A, A/D, PIO and encoder interface circuits. The moving displacement of the levitation table $x_{\text{table}}$ is fed back using a gap sensor, and the state $x$ in (10) can be calculated by $x = x_t - x_{\text{table}}$, where $x_t$ is the total air-gap length. In this study the value of $x_t$ is equal to 12.5 mm. Moreover, the control systems are realised on a Pentium PC via Turbo C language to manipulate the coil current $i$ in the electromagnetic coil by way of voltage control $U$ passing through a linear power amplifier, and the sampling time is 6 ms. In addition, the output of the reference model is designed as a step command in the following experimentation. Furthermore, the mean-square-error (MSE) value of the moving displacement is defined as

$$MSE = \frac{1}{T} \sum_{z=1}^{T} [x_{\text{ref,table}}(z) - x_{\text{table}}(z)]^2$$  (44)

where $T$ is total sampling instants. According to (44) the normalised MSE value of the moving displacement using per-unit values with a 1 mm base is used for examining the control performance in this study.

Some experimental results are provided here to demonstrate the effectiveness of the PID, FNN and adaptive control systems. In the experimentation the initial condition of this hybrid maglev system is loaded by two pieces of iron disk with 3.7 kg weight. The experimental results of the PID control system due to reference table position $x_{\text{ref,table}}$ are depicted in Fig. 6. In Fig. 6a a 1 mm-step command is set, and the gains of the PID control system are given via trial and error as

$$K_P = 32, K_I = 10, K_D = 0.6$$  (45)

One iron disc is unloaded at 6 s and reloaded at 12 s and it is obvious that the position drift of the levitation is almost 1 mm when unloading. Because the gains in (45) are selected under the loading condition these control gains may not keep the levitation height at 1 mm during the unloading duration. The normalised MSE value is $6.041 \times 10^{-3}$ mm. Moreover, the gains of the PID control system for a 2 mm-step command are designed via trial and error as

$$K_P = 25, K_I = 10, K_D = 0.5$$  (46)

In Fig. 6b there are similar results as Fig. 6a, and the normalised MSE value is $6.902 \times 10^{-3}$ mm. As seen from Fig. 6 there are apparent static errors after unloading. Though this problem could be improved by increasing the integral gain $K_i$ it will result in a higher inrush control effort. In conclusion, it is time-consuming and laborious because the control coefficients of the PID control system should be redesigned for various demands to satisfy the desirable dynamic behaviour.

For comparison the FNN control system in Section 3.2 is also applied to control the hybrid maglev system. To show the effectiveness of the FNN with a small rule set, the FNN has two, six, nine and one neuron at the input, membership, rule and output layer, respectively. It can be regarded that
the associated fuzzy sets with a Gaussian function for each input signal are divided into N (negative), Z (zero) and P (positive), and the number of rules with complete rule connection is nine. Moreover, some heuristics can be used to roughly initialise the parameters of the FNN for practical applications, e.g. the means and the standard deviations of the Gaussian functions can be determined according to the maximum variation of $e$ and $e_s$. The effect due to the inaccurate selection of the initialised parameters can be constantly adjusted by the online learning methodology. Therefore, for simplicity, the means of the Gaussian functions are set at $\frac{1}{\sqrt{C_1}}, 0, \frac{1}{\sqrt{C_2}}$ for the N, Z, P neurons, respectively, and the standard deviations of the Gaussian functions are set at one. In addition, to test the learning ability of the FNN control system, all the initial connecting weight between the output layer and the rule layer are set to zero in the experimentation.

The responses of the table position, tracking error and coil current using the FNN control system due to 1 and 2 mm-step commands are depicted in Fig. 7a and 7b, where respective normalised MSE values are 1.108 and 1.492 $\times 10^{-3}$ mm. From the experimental results the overshoot responses at the transient state are caused by the rough initialisation of the network parameters. After this, the tracking errors reduce to zero quickly even under the load variations. Although favourable tracking performance can be obtained, this control scheme seems to be too complex in practical applications.

In the end the experimentation of the adaptive control system is implemented based on the scheme shown in Fig. 4. The gains of the adaptive control system are given as

$$\beta_1 = 1000, \beta_2 = 65, \beta_3 = 65$$ (47)

The selection of the positive tuning gains $\beta_1, \beta_2$ and $\beta_3$ is concerned with the tracking speed. The tracking response converges slowly with small tuning gains, and the tracking speed increases with large tuning gains. Due to the unavailable system dynamics, they are chosen via a trial and error process to achieve the superior transient response in the experimentation considering the requirement of stability, the limitation of control effort and the possible operating conditions. The table position, tracking error, and coil current at 1 and 2 mm-step commands are depicted in Fig. 8a and 8b, where respective normalised MSE values are 6.150 and 8.061 $\times 10^{-3}$ mm. From the experimental results, good tracking responses can be obtained. With the variation of load during 6–12 s the table position can be returned to the command position quickly.

To exhibit the flexible control performance of the adaptive control system, the experimental results due to step-command changing from 1 to 2 mm are given in Fig. 9. In Fig. 9a, there is no load variation. On the contrary, it unloads one iron disc at 2 and 12 s, and reloads at 4 and 14 s in Fig. 9b. The respective normalised MSE values are 4.772 $\times 10^{-4}$ and 1.040 $\times 10^{-3}$ mm. From the experimental results the tracking errors converge quickly and the robust
control characteristics under the occurrence of different load variations and step commands can be clearly observed. Comparing Figs. 8 and 9 with Figs. 6 and 7, it is obvious that the adaptive control system with simple framework yields superior control performance than the PID and FNN control systems. In the whole design process no prior knowledge of the controlled plant was required and the stability of the control system can be guaranteed. It not only has the learning ability similar to intelligent control, but also its control framework is simple like PID control.

5 Conclusions

This study has successfully implemented the PID, FNN and adaptive control systems for the levitated positioning of a hybrid maglev system to demonstrate their individual control diversities. Performance comparisons of the PID, FNN and adaptive control systems are summarised in Table 1. The PID control system belongs to an event-based linear controller. There are larger normalised MSE values under the occurrence of load variations; therefore the control gains should be redesigned for different situations. Moreover, the FNN control system could be designed successfully without complex mathematical model and possesses smaller normalised MSE values. However, this scheme is more complicated than the PID control system. In addition, the adaptive control system is presented to solve this problem, and it not only has good tracking response but also makes this system more robust under different step commands and load variations. From these performance comparisons the adaptive control system is more suitable to the position control of the hybrid maglev system than the PID and FNN control systems. Note that this study just demonstrates three specific model-free control strategies, not all possible control methods suggested in the literature. The main contributions of this application-oriented study are summarised as follows.

- Derivation of the dynamic model of a hybrid maglev system.
- Development of PID, FNN and adaptive control strategies for this hybrid maglev system.
- Implementation of these model-free control schemes for the position control of the hybrid maglev system considering the possible occurrence of uncertainties.
- Comparisons of individual control performances to provide designers with preliminary guideline for manipulating the hybrid maglev system efficiently.

6 Acknowledgments

The authors would like to acknowledge the financial support of the National Science Council of Taiwan, R.O.C. through grant NSC 93-2213-E-155-014 and express their gratitude to the referees and Associate Editor for their useful comments and suggestions. In addition, the authors would like to thank Prof. Seng-Chi Chen for his assistance in supporting the maglev equipment.

7 References


Table 1: Performance comparisons of PID, FNN and adaptive control systems

<table>
<thead>
<tr>
<th>Performance</th>
<th>PID control</th>
<th>FNN control</th>
<th>Adaptive control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised MSE</td>
<td>6.041 × 10⁻³ mm</td>
<td>6.902 × 10⁻³ mm</td>
<td>1.108 × 10⁻³ mm</td>
</tr>
<tr>
<td></td>
<td>1.492 × 10⁻³ mm</td>
<td>6.150 × 10⁻⁴ mm</td>
<td>8.061 × 10⁻⁴ mm</td>
</tr>
<tr>
<td>Robustness</td>
<td>poor</td>
<td>fair</td>
<td>good</td>
</tr>
<tr>
<td>Control framework</td>
<td>simple</td>
<td>complex</td>
<td>simple</td>
</tr>
<tr>
<td>Learning ability</td>
<td>none</td>
<td>online</td>
<td>online</td>
</tr>
</tbody>
</table>