Dual-mode structure digital repetitive control

Keliang Zhou, Danwei Wang, Bin Zhang, Yigang Wang, J.A. Ferreira, S.W.H. de Haan

Abstract

A flexible repetitive control (RC) scheme named “dual-mode structure repetitive control” (DMRC) is presented in this article. A robust stability criterion for DMRC systems is derived in terms of two parameters: odd-harmonic RC gain and even-harmonic RC gain. Several useful corollaries for the stability are addressed to reveal the compatibility of DMRC. The general framework of DMRC offers the flexibility in the development of various RC controllers. Without additional complexity and loss of tracking accuracy, DMRC can achieve faster error convergence rate than conventional RCs. DMRC requires the same data memory size as that of conventional RC one. An application example of DMRC controlled PWM inverter illustrates the validity of our proposed DMRC scheme. Comparisons of DMRC, conventional RC and odd-harmonic RC highlight the advantages of the presented DMRC approach.

Keywords: Internal model principle; Repetitive control; Odd-harmonic; Even-harmonic; Sample-data control

1. Introduction

Repetitive control (RC) (Hara, Yamamoto, Omata, & Nakano, 1988; Inoue, Nakano, Kubo, Matsumoto, & Baba, 1981; Tomizuka, Tsao, & Chew, 1988), which is based on Internal Model Principle (Francis & Wonham, 1976), is developed to track/eliminate periodic signals with a known period by including their generator in a stable closed-loop system. Applications of RC have been widely reported in different fields, which include hard disk drives (Chew & Tomizuka, 1990), robotic manipulators (Cosner, Anwar, & Tomizuka, 1990), PWM inverters (Zhou & Wang, 2001; Zhou, Wang, & Low, 2000), PWM rectifiers (Zhou & Wang, 2003), active power filters (Griñó, Costa-Castelló, & Fossas, 2003), satellites (Broberg & Molyet, 1992), steel castings (Manayathara, Tsao, Bentsman, & Ross, 1996), and so on.

In conventional RC systems, any reference signal with a fundamental period $N$ can be exactly tracked by including a periodic signal generator $1/(z^N - 1)$ in the closed-loop system. Such a periodic signal generator needs at least $N$ memory cells. Conventional RC controllers can eliminate both even and odd harmonics that are below Nyquist frequency by introducing infinite gain at these harmonic frequencies.

For some systems, such as CVCF PWM converters, an odd-harmonic RC controller (Costa-Castelló, Grinó, & Fossas, 2004; Griñó & Costa-Castelló, 2005) is proposed to reduce only the odd-harmonic errors, which are dominant in the errors. The even-harmonic periodic errors are neglected or treated as non-repetitive errors. Compared with a conventional RC controller, an odd-harmonic RC controller occupies less data memory and offers faster convergence rate of the tracking error with identical RC gain (Zhou et al., 2006). However, the even-harmonic errors will reside or might be amplified in an odd-harmonic RC system. The amplified even harmonic errors may lead to some undesired negative impacts on the system, e.g. dc voltage residues may cause the saturation of magnetic components, such as transformers and inductors.

A dual-mode structure RC (DMRC) is proposed in this paper. It can be used to improve the performance (error convergence...
rate and tracking accuracy) of RC controller. More important, without additional complexity, the dual-mode structure is flexible for housing various RC controllers, such as conventional RC controller, odd-harmonic RC controller, and so on. The robust stability condition and error convergence condition of DMRC systems will be derived as an extension of conventional/odd-harmonic RC control system (Griñó & Costa-Castelló, 2005; Hara et al., 1988; Tomizuka et al., 1988). An application example of DMRC controlled PWM inverter is provided to demonstrate the effectiveness and advantages of DMRC.

2. Dual-mode structure digital repetitive control

2.1. Dual-mode periodic signal generator

As shown in Fig. 1, a conventional digital periodic signal generator \( G_g(z) \) (Tomizuka et al., 1988) with period \( N \) can be written as

\[
G_g(z) = \frac{z^{-N}}{1 - z^{-N}} = \frac{1}{z^N - 1},
\]

where \( N = T/T_s \in \mathbb{N} \) with \( T \) and \( T_s \) being the fundamental period of the signals and the sampling time. It is clear that, if \( N \) is even, the \( G_g(z) \) in (1) has its poles at \( z = e^{i2m\pi/N}, m = 0, 1, \ldots, N-1 \), whereas \( z = e^{i(2k+1)\pi/N}, k = 0, 1, \ldots, (N/2) - 1 \) relating to odd-harmonic frequencies and \( z = e^{i(2k)\pi/N}, k = 0, 1, \ldots, (N/2) - 1 \) relating to even-harmonic frequencies. In other words, transfer function \( G_g(z) \) is used to generate fundamental frequency and its harmonics.

Eq. (1) can be rewritten as follows:

\[
G_g(z) = \frac{1}{2} \left( \frac{1}{z^{N/2} - 1} - \frac{1}{z^{N/2} + 1} \right) = \frac{1}{2} (G_{eg}(z) + G_{og}(z)),
\]

where \( G_{eg}(z) = 1/(z^{N/2} - 1) \) is an even-harmonic signal generator, \( G_{og}(z) = -1/(z^{N/2} + 1) \) is an odd-harmonic signal generator. Notice that, the conventional repetitive signal generator \( G_g(z) \) contains two modes of signal generator: an odd-harmonic \( G_{og}(z) \) and an even-harmonic \( G_{eg}(z) \). Hereinafter, such a structure is called “dual-mode structure”.

Furthermore, as shown in Fig. 2, a general dual-mode structure prototype RC controller \( G_{dRp}(z) \) is proposed as follows:

\[
G_{dRp}(z) = (k_o G_{eg}(z) + k_o G_{og}(z)) G_I(z),
\]

where even-harmonic RC gain \( k_o \geq 0 \) and odd-harmonic RC gain \( k_o \geq 0 \); \( G_I(z) \) is a compensation filter. Obviously, if \( k_o = k_e \), Eq. (3) represents a conventional prototype RC controller (Tomizuka et al., 1988) with RC gain \( k_e = 2k_o = 2k_i \); if \( k_o = 0 \), Eq. (3) is a prototype odd-harmonic RC controller (Griñó & Costa-Castelló, 2005) with RC gain \( k_i = k_o \); and if \( k_o = 0 \), Eq. (3) becomes a prototype even-harmonic RC controller with RC gain \( k_e = k_o \). A RC controller, which is based on such a dual-mode periodic signal generator structure, is called “dual-mode repetitive controller” (DMRC).

Remark 1. Fig. 2 clearly indicates that, without additional complexity, DMRC provides a general structure for various RC controllers, such as conventional RC controller, odd-harmonic RC controller and so on. The data memory size occupied by DMRC controller is the same as that of conventional RC one. The tracking error convergence rate of DMRC system can be tuned by adjusting the RC gains of \( k_o \) and \( k_e \). For example, if the odd-harmonic components dominate the errors, the RC gains could be chosen as \( k_o > k_e \) to improve the total error convergence rate, while eliminating both odd-harmonic and even-harmonic errors; and vice versa. What is more, dual-mode structure can not only be applied to the digital RC controller, but also be easily employed to continuous-time RC controller by replacing conventional \( k_i (e^{L/2} - 1) \) (Hara et al., 1988) with \( k_e (e^{L/2} - 1) - k_o (e^{L/2} + 1) \), where \( L \) is the period of the continuous-time periodic signal.

2.2. Dual-mode structure repetitive control system

Fig. 3 shows a typical closed-loop control system with a plug-in DMRC controller, where \( R(z) \) is the reference input; \( Y(z) \) is the output; \( E(z) = R(z) - Y(z) \) is the tracking error; \( D(z) \) is the disturbance; \( G_p(z) \) is the transfer function of the plant; \( G_c(z) \) is the conventional feedback controller; \( G_{dR}(z) \) is a modified DMRC controller; \( k_o \) is the odd-harmonic RC gain; \( k_e \) is the even-harmonic RC gain; \( U_{dR}(z) \) is the output of the RC controller; \( G_I(z) \) is a filter to stabilize the overall closed-loop system; \( Q_{ogm}(z) \) and \( Q_{egm}(z) \) are modified periodic signal generators for odd-harmonic signals and even-harmonic signals, respectively; \( Q_1(z) \) and \( Q_2(z) \) are low-pass filters to enhance the system robustness, with \( |Q_1(e^{i\theta})| \leq 1 \) \((i=1,2,3)|Q_2(e^{i\theta})| \to 1 \) at low frequencies and \( |Q_1(e^{i\theta})| \to 0 \) at high frequencies, e.g. \( Q_1(z) = \alpha_1z + \alpha_0 + \alpha_1z^{-1} \) with \( 2\alpha_1 + \alpha_0 = 1, \alpha_0 \geq 0 \) and \( \alpha_1 \geq 0 \) \((i=1,2)\) (Tomizuka et al., 1988).
The conventional controller $G_c(z)$ is chosen so that the transfer function
\[
H(z) = \frac{G_c(z)G_p(z)}{1 + G_c(z)G_p(z)}
\]
is asymptotically stable. Therefore, there exists an inverse function $G_{fn}(z)$ of $H(z)$ (Tomizuka, 1987) such that
\[
G_{fn}(z)H(z) = 1.
\]
(5)

Since the periodic signal generator introduce a $N$-step delay $z^{-N}$, $G_{fn}(z)$ can be implemented in the RC controller if $G_{fn}(z)$ is a non-causal filter (Note: $N$ is much greater than the relative degree of $H(z)$). And in practice, due to model uncertainties and load variations, it is impossible to obtain the exact transfer function $H(z)$. That is, the practical inverse function $G_{fI}(z)$ of $H(z)$ can be written as
\[
G_{fI}(z) = G_{fn}(z)(1 + A(z)),
\]
(6)

where $A(z)$ denotes the uncertainties which are assumed to be bounded by $|A(e^{j\omega})| \leq \epsilon$ with $\epsilon$ being a positive constant, and $A(z)$ is stable.

From Eqs. (5), (6) and (7), we have
\[
G_{fI}(z)H(z) = G_{fn}(z)H(z)(1 + A(z)) = 1 + A(z).
\]
(7)

Let $G_{fI}(z)H(z) = NGH(\omega)e^{jQH}$ with $z = e^{j\omega}$, from (5), (6) and (7), we have
\[
NGH(\omega) = |G_{fn}(e^{j\omega})H(e^{j\omega})||1 + A(e^{j\omega})| \leq 1 + \epsilon.
\]
(8)

The plug-in DMRC $G_{dr}(z)$ can be expressed as follows:
\[
G_{dr}(z) = (k_o G_{gm}(z) + k_e G_{gm}(z))G_{fI}(z)\left(\begin{array}{c}
\frac{-z^{-N/2}Q_1(z)}{1 + z^{-N/2}Q_1(z)} + k_e \frac{z^{-N/2}Q_2(z)}{1 + z^{-N/2}Q_2(z)}
\end{array}\right) G_{fI}(z).
\]
(9)

**Theorem 1.** For the closed-loop DMRC system shown in Fig. 3 with constraints (4)-(9), if the RC gains $k_e$ and $k_o$ satisfy the following inequalities:
\[
k_e \geq 0, \quad k_o \geq 0,
\]
(10)

and
\[
0 < k_e + k_o < \frac{2}{1 + \epsilon},
\]
(11)

then the closed-loop DMRC system is asymptotically stable.

**Proof.** See Appendix A. □

**Remark 2.** Theorem 1 offers a stability criterion for the closed-loop DMRC system. If the odd-harmonics and even-harmonics are treated as two different periodic signals, DMRC system becomes one special case of multiple-periods RC systems (Chang, Suh, & Oh, 1998; Yamada, Riadh, & Funahashi, 2000).

DMRC offers a general framework to develop various RC controllers, e.g. conventional RC controller, odd-harmonic RC controller, etc. In the following paragraphs, corollaries will show that the stability criterions for conventional RC systems (Cosner et al., 1990) and odd-harmonic RC systems (Griñó & Costa-Castelló, 2005) are compatible to our Theorem 1.

**Corollary 1.** If the DMRC system in Fig. 3 with RC gains $k_o = k_e = k_i/2$ and filters $Q_1(z) = Q_2(z) = Q(z)$ fulfills the following condition:
\[
|Q^2(z)(1 - k_i G_1(z)H(z))| < 1,
\]
(12)

then it is asymptotically stable, and the RC gain $k_i$ satisfies
\[
0 < k_i < \frac{2}{1 + \epsilon}.
\]
(13)

**Proof.** If filters $Q_1(z) = Q_2(z) = Q(z)$ and RC gains $k_o = k_e = k_i/2$, the DMRC system in Fig. 3 becomes a conventional RC system with RC gain $k_i$ and filter $Q(z)$.

From Fig. 3 and Eq. (A.1), the transfer function $G(z)$ from $R(z)$ to $Y(z)$ and the transfer function $G_d(z)$ from $D(z)$ to $Y(z)$ for such a DMRC system can be derived as
\[
Y(z) = G(z)R(z) + G_d(z)D(z)
\]
\[
= H(z)(1 - z^{-N}Q^2(z)(1 - k_i G_1(z)H(z)))\frac{R(z)}{1 - z^{-N}Q^2(z)(1 - k_i G_1(z)H(z))},
\]
\[
+ \frac{(1 + G_c(z)G_p(z))^{-1}(1 - z^{-N}Q^2(z))}{1 - z^{-N}Q^2(z)(1 - k_i G_1(z)H(z))} D(z).
\]
(14)

Obviously, if $|Q^2(z)(1 - k_i G_1(z)H(z))| < 1$, then all the poles $p_i (i = 0, 1, \ldots, N)$ of $G(z)$ and $G_d(z)$ in (14) are inside the unit circle $|z| = 1$. Since $H(z)$ is asymptotically stable, the closed-loop system transfer functions $G(z)$ and $G_d(z)$ in (14) are asymptotically stable. Corresponding criterion can be found in conventional RC systems (Tomizuka et al., 1988).

The formula in (12) can be expressed as
\[
N_Q^2(\omega)|1 - k_i NGH(\omega)e^{jQH}| < 1.
\]
(15)

Or equivalently,
\[
|1 - k_i NGH(\omega) \cos \theta_H - jk_i NGH(\omega) \sin \theta_H| < \frac{1}{N_Q^2(\omega)}.
\]
(16)
Using norm definition and taking square on both sides (Wang & Ye, 2005), we have

\[ 0 < k_t < \frac{1 - N_Q^2(\omega)}{N_Q(\omega)k_t^2 N^2_H(\omega)} + \frac{2 \cos \theta_G}{N_G H(\omega)} \tag{17} \]

Since \((1 - N_Q^2(\omega))/(k_t^2 N^2_H(\omega))\geq 0\), to ensure the system stability conservatively, we can choose

\[ 0 < k_t < \frac{2 \cos \theta_G}{N_G H(\omega)} \leq \frac{2}{N_G H(\omega)}. \]

Similarly, the stability range of \(k_t\) can be obtained as

\[ 0 < k_t < \frac{2}{1 + \varepsilon}. \tag{19} \]

**Corollary 2.** If DMRC in Fig. 3 with RC gains \(k_o > 0\) and \(k_c = 0\) fulfills the following condition:

\[ |Q_1(z)(1 - k_o G_1(z))| < 1 \tag{18} \]

then the DMRC system in Fig. 3 is asymptotically stable, and the RC gain \(k_o\) should satisfy

\[ 0 < k_o < \frac{2}{1 + \varepsilon}. \tag{19} \]

**Proof.** If \(k_c = 0\), the DMRC system in Fig. 3 becomes an odd-harmonic RC system (Griñó & Costa-Castelló, 2005) with RC gain \(k_o\) and filter \(Q_1(z)\). Proof of Corollary 2 is similar to that of Corollary 1. □

**Theorem 2.** If the closed-loop DMRC system with \(Q_1(z) = 1\) \((i = 1, 2)\) in Fig. 3 is asymptotically stable, then the error \(e(k)\) in Fig. 3 converges asymptotically to zero when its spectral content corresponds to the frequencies of the roots of \(z^N = 1\).

**Proof.** The error transfer function \(T(z)\) is given by

\[ T(z) = \frac{E(z)}{R(z) - D(z)} = \frac{1}{1 + G_c(z)G_p(z)} \frac{1}{1 + H(z)G_{dr}(z)} \frac{1}{G_x(z)} \frac{1}{1 + G_c(z)G_p(z)} \frac{1}{G_x(z) + (1 + A(z))G_y(z)}. \tag{20} \]

where

\[ G_x(z) = 1 - z^{-N} Q_1(z) Q_2(z) + z^{-N/2}(Q_1(z) - Q_2(z)), \]

\[ G_y(z) = (k_o + k_c)z^{-N}Q_1(z)Q_2(z) + (k_c Q_2(z) - k_o Q_1(z))z^{-N/2}. \]

Since \(H(z)\) and \(G(z)\) are asymptotically stable, it is clear that if \(Q_1(z) = 1\) \((i = 1, 2)\), then

\[ |T(e^{j\omega})| = 0, \quad \forall \omega = \omega^0 \text{ such that } \omega^0 N = 1. \tag{21} \]

Therefore the error \(e(k)\) in Fig. 3 converges asymptotically to zero. □

**Remark 3.** The proofs of Corollaries 1 and 2 indicate that, Theorem 1 offers a more conservative stability condition for conventional RC systems than Theorem 2 in Cosner et al. (1990); Theorem 1 offers a more conservative stability condition for odd-harmonic RC systems than Proposition 4 in Griñó & Costa-Castelló (2005).

**Remark 4.** Theorem 2 indicates repetitive errors, which include even-harmonic errors and odd-harmonic errors, can be completely eliminated by our proposed DMRC controller with \(Q_1(z) = 1\) \((i = 1, 2)\), even under modeling uncertainty.

**Remark 5.** The proof of Theorem 1 indicates the introduction of low-pass filter \(Q_1(z)\) \((i = 1, 2)\) with \(|Q_1(e^{j\omega})| \leq 1\) will make it easier to ensure all poles \(|D| < 1\) \((i = 0, 1, \ldots, N)\) of \(G(z)\) and \(G_d(z)\) in Eq. (A1), and then enhance the robustness of the DMRC system. On the other hand, as shown in the proof of Theorem 2, the introduction of \(Q_1(z)\) \((i = 1, 2)\) at low frequencies and \(|Q_1(e^{j\omega})| \rightarrow 0\) \((i = 1, 2)\) at high frequencies, will cause the variation of the zeros of \(T(z)\) \((20)\), and yields the imperfect cancellation of periodic errors, especially at the high frequency band. The tracking accuracy will be reduced. Therefore, the introduction of low-pass filter \(Q_1(z)\) \((i = 1, 2)\) brings a trade-off between tracking accuracy and system robustness in the DMRC system.

3. Application example

Consider a single-phase pulse-width modulation (PWM) inverter system as shown in Fig. 4, where \(v_c\) is the output voltage; \(i_o\) is the load current; \(v_{in}\) is the PWM control input; dc voltage \(v_{dc} = 130\) V; \(L_n = 2\) mH, \(C_n = 10\) mF, \(R_n = 30\) Ω are nominal component values of the inductor \(L\) (with actual value 2.2 mH), the capacitor \(C\) (with actual value 12 mF) and the resistive load \(R\) (with actual value 30 Ω), respectively; the reference input

![Fig. 4. Repetitive controlled single-phase PWM inverter.](image-url)
From (8) and Fig. 5, we have $e \approx 1.12 - 1 = 0.12$. According to Theorem 1, we have $k_0 + k_e < 2/(1 + e) \approx 1.79$. In the following simulations, we choose $k_0 + k_e = 0.4 < 1.79$.

Fig. 6(a) shows the steady-state response of only feedback controlled inverter. It can be seen that a 4.7V (amplitude), 50 Hz (fundamental frequency, odd-harmonics) component with a $-0.5$ V dc (even-harmonics) offset dominates the tracking error. Figs. 6(b)–(d) show the steady-state responses of various plug-in RC controlled inverters. Figs. 6(b) and (d) indicate that both conventional RC controller and DMRC controller ($k_e > 0$ and $k_o > 0$) can significantly reduce the feedback controlled tracking error (both odd-harmonics and even-harmonics) to be within negligible ±0.01 V. However, as shown in Fig. 6(c), a 0.25 V (amplitude), 100 Hz (2nd harmonic frequency) component with a $-0.59$ V dc offset still resides in the odd-harmonic RC controlled tracking error. It clearly manifests that odd-harmonic RC controller cannot reduce the even-harmonic errors or might even magnify them.

Fig. 7 shows the transient tracking error responses with various RC controllers ($k_o + k_e = 0.4$) being plugged into the feedback controlled inverter at time $t = 0.06$ s. In terms of error convergence rate, as shown in Fig. 7, odd-harmonic RC ($k_o = 0.4$ and $k_e = 0$) is the fastest, DMRC ($k_o = 0.32$ and $k_e = 0.08$) is the second fastest, DMRC ($k_o = 0.28$ and $k_e = 0.12$) is the third fastest, and conventional RC ($k_e = 0.4$, or $k_o = k_e = 0.2$) is the slowest. Since the odd-harmonic component significantly dominates the tracking error (as shown in Fig. 6(a)), the larger odd-harmonic gain $k_o$ is, the faster the error convergence rate is. In terms of tracking accuracy, as shown in Fig. 7, DMRC ($k_o = 0.32$ and $k_e = 0.08$), DMRC ($k_o = 0.28$ and $k_e = 0.12$) and conventional RC have comparable high accuracy, while there are obvious even-harmonic errors residing in the odd-harmonic RC system. That is to say, odd-harmonic RC is not immune to even-harmonic errors (Zhou et al., 2006). These even-harmonic errors in the odd-harmonic RC system may lead to some undesired negative impacts, e.g. dc voltage residues may cause the saturation of magnetic components.

Remark 6. DMRC is the extension of conventional RC and odd-harmonic RC. The design of $Q_i(z)$ for DMRC can be the same as that of conventional/odd-harmonic ones. In our case, for a CVCF PWM inverter at sample rate 10 kHz, the majority of its total harmonic distortion is below 1 kHz. To enhance the system robustness without significant loss of tracking accuracy, the bandwidth of $Q_i(z)$ ($i = 1, 2$) should be greater than 1 kHz and less than 5 kHz (Nyquist frequency). $Q_1(z)$ and $Q_2(z)$ can be identical or different filters as long as they well meet the above demand. Here $Q_1(z) = Q_2(z) = (z + 2 + z^{-1})/4$ are appropriate filters. Moreover, in practice, since the tracking error components will change significantly with different plants and different loads, and the errors also contain many frequencies, it is almost impossible to determine the optimal gain ratio of $k_o/k_e$ theoretically. Following the basic tuning method described in Remark 1, a good choice of the gains $k_o$ and $k_e$ can be found by experiments.
Fig. 6. Steady-state responses: (a) Feedback controller control; (b) conventional RC control \((k_\theta = 0.4)\); (c) odd-harmonic RC control \((k_\theta = 0.4 \text{ and } k_e = 0)\); (d) DMRC control \((k_\theta = 0.28 \text{ and } k_e = 0.12)\).
Without additional complexity, dual-mode structure offers the flexibility in the development of various RC controllers, such as conventional RC, odd-harmonic RC and so on. A robust stability criterion for DMRC systems is derived. Corollaries for the DMRC yields higher tracking accuracy than odd-harmonic RC.

4. Conclusions

This paper proposed a dual-mode structure for RC controller. Without loss of tracking accuracy, DMRC can offer faster error convergence and the transfer function

\[ G(z)R(z) = \frac{H(z)(1 + G_d(z))}{1 + H(z)G_d(z)} R(z) + \frac{(1 + G_c(z)G_p(z))^{-1}}{1 + H(z)G_d(z)} D(z) \]

to improve the transient response of RC controlled systems, such as PWM converters, hard disks and so on. An application example of DMRC controlled inverter is given to show the promising advantages of the DMRC controller: without loss of tracking accuracy, DMRC can offer faster error convergence rate than conventional RC; DMRC yields higher tracking accuracy than odd-harmonic RC.

Appendix A. Proof of Theorem 1

From Eqs. (4)–(9) and Fig. 3, the transfer function \( G(z) \) from \( R(z) \) to \( Y(z) \) and the transfer function \( G_d(z) \) from \( D(z) \) to \( Y(z) \) can be derived as follows:

\[
Y(z) = G(z)R(z) + G_d(z)D(z) = \frac{H(z)(1 + G_d(z))}{1 + H(z)G_d(z)} R(z) + \frac{(1 + G_c(z)G_p(z))^{-1}}{1 + H(z)G_d(z)} D(z) = \frac{H(z)(1 + G_r(z)G_{ogm}(z) + k_cG_{egm}(z))}{1 + (1 + A(z))(k_cG_{ogm}(z) + k_cG_{egm}(z))} R(z) + \frac{(1 + G_r(z)G_p(z))^{-1}}{1 + (1 + A(z))(k_cG_{ogm}(z) + k_cG_{egm}(z))} D(z),
\]

where

\[
G_{ogm}(z) = \frac{-z^{-N/2}Q_1(z)}{1 + z^{-N/2}Q_1(z)},
\]

\[
G_{egm}(z) = \frac{z^{-N/2}Q_2(z)}{1 - z^{-N/2}Q_2(z)},
\]

\[
Q_1(z) = |Q_1(e^{j\omega})|e^{j\theta_{Q_1}(\omega)} = N_1(\omega)e^{j\theta_{Q_1}(\omega)} (i = 1, 2),
\]

\[
N_1(\omega) = |Q_1(e^{j\omega})| < 1 \quad (i = 1, 2).
\]

Let \( z = |z|e^{j\omega} \) with \( |z| = a \), then we have

\[
\text{Re}[G_{ogm}(z)] = \text{Re} \left[ \frac{-z^{-N/2}Q_1(z)}{1 + z^{-N/2}Q_1(z)} \right] = \text{Re} \left[ -\left( \frac{a^{N/2}}{N_1(\omega)} e^{j(N\omega/2-\theta_{Q_1}(\omega))} + 1 \right) \right]^{-1}
\]

and

\[
\text{Re}[G_{egm}(z)] = \text{Re} \left[ \frac{z^{-N/2}Q_2(z)}{1 - z^{-N/2}Q_2(z)} \right] = \text{Re} \left[ \left( \frac{a^{N/2}}{N_2(\omega)} e^{j(N\omega/2-\theta_{Q_2}(\omega))} - 1 \right) \right]^{-1}.
\]

Let \( b_i = a^{N/2}/N_1(\omega) \) and \( b_i = N_2(\omega)/N_2(\omega) (i = 1, 2) \). If \( |z| = a \geq 1 \), we have \( b_i \geq a^{N/2} \geq a \geq 1 \) \( (i = 1, 2) \). Eq. (A.2) can be rewritten as

\[
\text{Re}[G_{ogm}(z)] = \text{Re} \left[ -\frac{1}{b_i e^{\theta_1}} \right] = \text{Re} \left[ -\frac{(b_1 \cos \theta_1 + 1) + j b_1 \sin \theta_1}{(b_1 \cos \theta_1 + 1)^2 + (b_1 \sin \theta_1)^2} \right] = \frac{-b_1 \cos \theta_1 - 1}{b_1^2 + 2b_1 \cos \theta_1},
\]

If \( -(b_1 \cos \theta_1 + 1) \geq 0 \), we have

\[
\text{Re}[G_{ogm}(z)] = \frac{-(b_1 \cos \theta_1 + 1)}{b_1^2 + 2b_1 \cos \theta_1} \geq 0;
\]

If \( -(b_1 \cos \theta_1 + 1) < 0 \), we have

\[
\frac{b_1^2 + 1 + 2b_1 \cos \theta_1}{-(b_1 \cos \theta_1 + 1)} = \frac{b_1^2 - 1}{-(b_1 \cos \theta_1 + 1)} = \frac{a^2 - 1}{-(b_1 \cos \theta_1 + 1)} \leq a^2 - 1, \quad \forall |z| = a \geq 1,
\]

Fig. 7. Various RC controlled transient tracking errors.
and then
\[ \text{Re}[G_{\text{egm}}(z)] = \frac{-(b_1 \cos \theta_1 + 1)}{b_1^2 + 1 + 2b_1 \cos \theta_1} \geq -\frac{1}{2} |z| \geq 1. \]  
(A.4)

Eq. (A.3) can be rewritten as
\[ \text{Re}[G_{\text{egm}}(z)] = \text{Re} \left[ \frac{1}{b_2 \cos \theta_2 - 1} \right] \]
\[ = \text{Re} \left[ \frac{(b_2 \cos \theta_2 - 1) - jb_2 \sin \theta_2}{(b_2 \cos \theta_2 - 1)^2 + (b_2 \sin \theta_2)^2} \right] \]
\[ = \frac{b_2 \cos \theta_2 - 1}{b_2^2 + 1 - 2b_2 \cos \theta_2} \]

If \( b_2 \cos \theta_2 - 1 \geq 0 \), we have
\[ \text{Re}[G_{\text{egm}}(z)] = \frac{b_2 \cos \theta_2 - 1}{b_2^2 + 1 - 2b_2 \cos \theta_2} \geq 0; \]
If \( b_2 \cos \theta_2 - 1 < 0 \), we have
\[ \frac{b_2^2 - 1}{b_2 \cos \theta_2 - 1} - 2 \leq \frac{a^2 - 1}{b_2 \cos \theta_2 - 1} - 2 \leq -2 \forall |z| = a \geq 1, \]

and then
\[ \text{Re}[G_{\text{egm}}(z)] = \frac{b_2 \cos \theta_2 - 1}{b_2^2 + 1 - 2b_2 \cos \theta_2} \geq -\frac{1}{2} \forall |z| \geq 1. \]  
(A.5)

From Eqs. (4)–(8), (A.4), (A.5) and \( k_o \geq 0, k_e = 0, 0 < k_o + k_e < 2/(1 + \epsilon) \), we have
\[ \min_{|z| \geq 1} \text{Re}[k_o G_{\text{ogm}}(z) + k_e G_{\text{egm}}(z)] \]
\[ = \min_{|z| \geq 1} (k_o \text{Re}[G_{\text{ogm}}(z)] + k_e \text{Re}[G_{\text{egm}}(z)]) \]
\[ \geq \min_{|z| \geq 1} \left[ \left( -\frac{1}{2} \right) k_o + k_e \left( -\frac{1}{2} \right) \right] > -\frac{1}{2} \frac{2}{1 + \epsilon} \]
\[ \geq -\left| \frac{1}{1 + \Delta(z)} \right|, \forall |z| \geq 1. \]  
(A.6)

This implies that
\[ \frac{1}{1 + \Delta(z)} + (k_o G_{\text{ogm}}(z) + k_e G_{\text{egm}}(z)) \neq 0, \forall |z| \geq 1. \]  
(A.7)

Thus, all the poles \( p_i \ (i = 0, 1, \ldots, N) \) of the transfer functions \( G(z) \) and \( G_D(z) \) in (A.1) are inside the unit circle \( |z| = 1 \), i.e. \( |p_i| < 1 \). Finally, the asymptotical stability of \( H(z) \) implies the asymptotical stability of the closed-loop DMRC system in Fig. 3.

References


Keliang Zhou received his B.E. degree from the Huazhong University of Science and Technology, Wuhan, China, in 1992, the M.Eng. degree from Wuhan University of Transportation, Wuhan, China, in 1995, and the Ph.D. degree from Nanyang Technological University, Singapore, in 2002. Currently he is a professor with Department of Electrical Engineering, Southeast University, Nanjing, China. His research interests mainly involve power electronics and electric machines drives, advanced control and its applications and renewable energy generation.

Dr. Zhou has authored or co-authored more than 30 published technical articles in the relevant areas.
Danwei Wang received his Ph.D. and MSE degrees from the University of Michigan, Ann Arbor in 1989 and 1985, respectively. He received his B.E. degree from the South China University of Technology, China in 1982. Since 1989, he has been with the School of EEE, Nanyang Technological University, Singapore. Currently, he is an associate professor of the School of EEE, head of the division of Control and Instrumentation and director of the Centre for Intelligent Machines, NTU. He has served as general chairman, technical chairman and various positions in international conferences, such as ICARCVs, IEEE RAM and ACCV. He is an associate editor of Conference Editorial Board, IEEE Control Systems Society, an associate editor of International Journal of Humanoid Robotics, and deputy chairman of IEEE Singapore Robotics and Automation Chapter. He was a recipient of Alexander von Humboldt fellowship, Germany. His research interests include robotics, control theory and applications. He has published many technical articles in the areas of iterative learning control, repetitive control, robust control and adaptive control systems, as well as manipulator/mobile robot dynamics, path planning and control.

Bin Zhang received his BE and MSE degrees from Nanjing University of Science and Technology, China, in 1993 and 1999, respectively. He is currently working toward the Ph.D. degree at Nanyang Technological University, Singapore. His current research interests are ILC/RC, intelligent control, digital signal processing, and their applications to robot manipulators, power electronics and vibration suppression.

Yigang Wang received the B.E. and MSE degrees from Harbin Institute of Technology, Harbin, China, in 2001 and 2003, respectively. His current research interests are repetitive and learning control, robust, adaptive and multirate filtering and control, with applications to PWM inverters, mechatronic system.

J.A. Ferreira received the BSc Eng., MSc Eng., and Ph.D. degrees from Rand Afrikaans University, Johannesburg, South Africa, in 1981, 1983, and 1988, respectively, all in electrical engineering. In 1981, he was with the Institute of Power Electronics and Electric Drives, Technical University of Aachen, and worked in industry at ESD (Pty.) Ltd. from 1982 to 1985. From 1986 to 1997, he was with the Faculty of Engineering, Rand Afrikaans University, where he held the Carl and Emily Fuchs Chair of power electronics in later years. Since 1998, he has been a Professor with Delft University of Technology, Delft, The Netherlands. Dr. Ferreira was the Chairman of the South African Section of the IEEE from 1993 to 1994. He is the Founding Chairman of the IEEE Joint Industry Applications Society/Power Electronics Society (IAS/PELS) Benelux chapter. He served as the IEEE Transactions on Industry Applications Review Chairman for the IEEE Industry Applications Society Power Electronic Devices and Components Committee and is an Associate Editor of the IEEE Transactions on Power electronics. He was a member of the IEEE Power Electronics Specialists Conference Adcom and is currently the Treasurer of the IEEE PELS. He served as the Chairman of the CIGRE SC14 National Committee of the Netherlands and was a Member of the Executive Committee of the European Power Electronics Society.

Sjoerd de Haan is associate professor in power electronics within the Electrical Power Processing group of the Delft University of Technology since 1995. He obtained his MSc degree in Physics at the Delft University of technology in 1973. He started his carrier at TPD-TNO (applied physics research centre) and was subsequently employed by both Delft University of Technology and Eindhoven University of technology. From 1993 until 1995 he was senior researcher with ECN (The Netherlands Energy Research Foundation) in Petten, where he was in charge of research on electrical conversion for renewable energy systems. His current research interests concerns electrical systems for dispersed generation, pulsed power and compact converters. De Haan was author and co-author of over a hundred papers.