

Atomic Object Composition

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Abstract. A worthwhile approach to achieve transaction atomicity within object-based distributed systems is to confine concurrency control and recovery mechanisms within the shared objects themselves. Such objects, called *atomic objects*, enhance their modularity and can increase transaction concurrency. Nevertheless, when designed *independently*, atomic objects can be *incompatible*, and if combined, do not ensure transaction atomicity anymore. It has been shown that atomic objects can be incompatible when they assume different *Global Serialization Protocols (GSPs)*.

We deal with the *incompatibility* problem by introducing a property of atomic objects' specifications, named *o-atomicity*, which is orthogonal to the GSP. Objects that guarantee this property achieve transaction atomicity, *whatever* the GSP may be. Such objects are compatible, not only with each others, but also with atomic objects that guarantee previously defined GSP-dependent properties, i.e., *static atomicity*, *dynamic atomicity* or *hybrid atomicity*. This is very desirable since most of existing object-based distributed systems rely on these properties. To show how *o-atomicity* can be ensured, we propose a generic implementation within an object-oriented framework, which we illustrate through a simple banking application.

1 Introduction

One of the basic claim underlying object-orientation is that of object composition [26]. Objects are modular software components that can be designed and tested independently, and then combined within the same application. While this claim has been supported in a sequential context, much still to be done in concurrent and distributed contexts [20]. In this paper we discuss the problem of composing objects in the context of transaction-based distributed systems.

1.1 Atomic Objects

To preserve the consistency of distributed systems despite concurrent accesses and failures, it is helpful to compose applications out of atomic activities called transactions [11,21]. Atomicity (in the sense of [32]) means that incomplete transactions appear as having no effects, and complete transactions appear as executing sequentially according to a *Serialization Order (SO)*, determined by some

Global Serialization Protocol (GSP). A worthwhile approach to achieve transaction atomicity is to confine concurrency control and recovery mechanisms within the shared objects themselves. Such objects, called *atomic objects* [29], enhance their modularity since they can be designed and tested locally, and can increase transaction concurrency by providing appropriate mechanisms to their use and semantics [1,2,3,6,14,15,19,22,25,27,30,31,33]. Nevertheless, when designed *independently*, atomic objects are sometimes *incompatible*, and when combined, do not ensure transaction atomicity anymore [32].

1.2 The Incompatibility Problem

Weihl has discussed the incompatibility problem and pointed out its deep relation with GSPs [32]. He grouped GSPs in three classes: *static* GSPs that determine SOs according to timestamps assigned to transactions at creation time, e.g., in [24]; *dynamic* GSPs that determine SOs during transaction execution, e.g., in [7]; and *hybrid* GSPs that determine SOs according to timestamps assigned to transactions at commit time, e.g., in [13]. Weihl showed that objects are incompatible if they assume GSPs belonging to different classes. For example, an object which schedules transactions according to timestamps assigned to them at creation (assuming a static GSP) is incompatible with an object which schedules transactions according to a two phase locking protocol [7] (assuming a dynamic GSP). When used together, these objects do not ensure atomicity anymore. As a consequence, applications cannot be composed out of atomic objects that assume GSPs belonging to different classes.

Since in most of existing object-based distributed systems, atomic objects assume particular GSPs (static, dynamic, or hybrid), their compatibility is limited. In this paper we propose a way to specify atomic objects, so that they do not assume any particular GSP, i.e., they are compatible with any other atomic object.

1.3 Composing Atomic Objects Through *O-atomicity*

We present a property of objects' specifications named *o-atomicity* which is *orthogonal*¹ to the GSP. Roughly speaking, an object which guarantees *o-atomicity* assumes that there exists some GSP (whatever it may be) and schedules transactions so that they appear to execute in the SO, determined by the GSP. We show that *o-atomicity* is sufficient to achieve transaction atomicity, and that, given only the assumption of the existence of some GSP, *o-atomicity* is also necessary.

¹ Hence its name *orthogonal-atomicity*.

As long as atomic objects guarantee *o-atomicity*:

- They are compatible not only with each others, but also with objects that assume particular GSPs. In other words, atomic objects that guarantee *o-atomicity* can be reused in most of existing object-based distributed systems [3,8,12,17,22,33].
- They can be designed independently and provide their own concurrency control and recovery mechanisms. To increase transaction concurrency, objects with *high risks* conflict can for example provide semantic-based pessimistic mechanisms, whereas objects with *low risks* conflict can provide optimistic mechanisms [1,2,6,13].
- They ensure transaction atomicity whatever the GSP may be. This enables to substitute a GSP by another without affecting the objects. Such a modularity feature is very convenient since the optimality of the GSP is application-dependent [19,32], and it would be worthwhile to modify it according to the application evolution.

To show how *o-atomicity* can be guaranteed, we propose a generic implementation of an abstract atomic class. We show how to customize this implementation within subclasses of which objects guarantee *o-atomicity*. We present two examples of such subclasses: a subclass of which objects guarantee *o-atomicity* through a pessimistic concurrency control, and a subclass of which objects guarantee *o-atomicity* through an optimistic concurrency control. To illustrate the feasibility of this implementation, we describe a simple banking application composed of bank-account objects from different subclasses.

The remainder of the paper is organized as follows. Section 2 describes our computational model and defines objects' specifications and transaction atomicity. Section 3 defines *o-atomicity* and presents its characteristics through three main theorems: *sufficiency*, *relation with GSP-dependent properties*, and *necessity*. Section 4 presents an abstract atomic class, and describes subclasses that provide adequate mechanisms to guarantee *o-atomicity*. Section 5 summarizes our main contribution and suggests some complementary work. Appendix I and Appendix II contain the theorems' proofs and the algorithms' details.

2 System Model

This section describes our model and defines atomic objects' specifications and transaction atomicity. As in [13,14,31,32], our definition of atomicity is based on *serial* specifications of atomic objects, and integrates both *recoverability*, and *serializability* [4].

We consider a system compound of atomic objects (called simply objects in the following, and noted $O, O1, O2, ..Om$) and transactions (noted $T, T1, T2, ..Tn$).

Objects and transactions are called components. Transactions are sequential monolithic² processes communicating through shared objects. A transaction invokes an operation on an object³, receives the matching reply, invokes another operation etc. The transaction ends either by committing or aborting. In both cases, invoked objects are notified about the transaction outcome through specific operations *commit()* and *abort()*⁴. When a *commit()* (resp. *abort()*) operation for a transaction T is invoked on an object O , T is said to commit (resp. abort) at O . A transaction is *terminated* when it commits or aborts at all objects it has invoked.

2.1 Histories

To depict computations, we use the simple event-based model presented in Weihl's thesis [29]. This model is quite intuitive and has been successfully used in various works on atomic objects [13,14,19,31,32]. It represents computations as finite sequences of *events* called *histories* (noted $H, H1, H2, ..Hk$). Events are either an operation invocation of an object, or a reply to an invocation. Each event involves exactly one transaction and one object.

For our purpose (i.e., to define *o-atomicity*), we consider histories that contain only terminated transactions. We assume that a transaction is not allowed to commit at some objects and abort at others [11]; a transaction waiting for a reply (pending transaction) cannot commit; and a transaction cannot invoke any operation after it commits. We say that a history H *involves* a component C (object or transaction), if some event in H involves C . Given $\{C1, ..Ck\}$ a set of components involved in a history H , we note $H_{\{C1,..Ck\}}$ the subsequence of H containing all events involving components in $\{C1, ..Ck\}$. We say that a history is *local* to an object if it involves only this object; a history is *global* if it involves at least two objects; a history is *failure-free* if all the involved transactions commit; and a history is *sequential* if events involving different transactions are not interleaved.

Example 1 (history): Figure 1 (a) shows a history H representing a bank transfer. H involves a transaction T , and two bank-account objects $O1$ and $O2$, providing *balance()*, *deposit()* and *withdrawal()* operations. T invokes *deposit(5)* on $O1$, *withdrawal(5)* on $O2$, and then commits at both $O1$ and $O2$. Figure 1 (b) shows the history $H_{\{O1\}}$, which is local to $O1$, and figure 1 (c) shows the history $H_{\{O2\}}$, which is local to $O2$.

Events in H are:

– $\langle \textit{deposit}(5), T, O1 \rangle$: T invokes *deposit(5)* on $O1$.

² Nested transactions in the sense of [9,18] are not considered here.

³ We will also say that the transaction invokes the object.

⁴ These operations are assumed to be provided by the objects, and invoked by transaction managers.

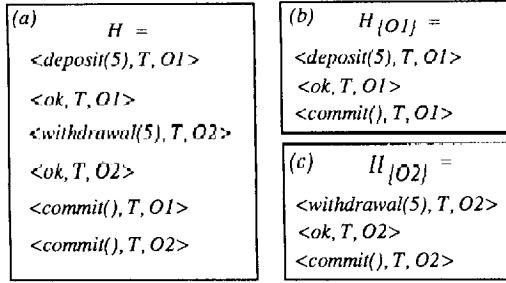


Fig.1. Histories

- $\langle ok, T, O1 \rangle$: $O1$ returns a positive reply ok to T .
- $\langle withdrawal(5), T, O2 \rangle$: T invokes $withdrawal(5)$ on $O2$.
- $\langle ok, T, O2 \rangle$: $O2$ returns a positive reply ok to T .
- $\langle commit(), T, O1 \rangle$: $O1$ is notified about the commit of T .
- $\langle commit(), T, O2 \rangle$: $O2$ is notified about the commit of T .

2.2 Serial Specifications

A specification of an object is a set of local histories involving the object. The specification describes how the object interacts with the transactions that invoke it. The *serial* specification of an object O , noted $O.serial$, describes how O interacts with transactions that perform sequentially in a failure-free way (i.e., they all commit). Such a specification contains all local sequential failure-free histories involving the object, and *compatible* with its semantic⁵.

Definition 1 (serially possible history): A history H is *serially possible*⁶ if for each object O , involved by H , $H_{\{O\}} \in O.serial$.

Example 2 (serial specification): Consider a bank-account object O which returns a positive answer to a $withdrawal()$ if there is enough money in the balance, and returns a negative answer otherwise. Figure 2 (a) shows a local history $H1$ that belongs to $O.serial$, whereas figure 2 (b) shows a history $H2$ that does not. $H2$ is not compatible with the semantic of O , because O refuses $withdrawal(5)$ (i.e., returns no), although it is supposed to have enough money after $deposit(5)$.

2.3 Atomicity

We consider atomicity in the sense of [13,14,19,31,32,33]. A transaction is atomic if it appears to execute in an *all-or-nothing*, and *isolated* manner⁷. An atomic history is one of which all transactions are atomic. Informally, given $committed(H)$

⁵ See [32] for how to derive a serial specification from an object state machine representation.

⁶ This notion is similar to the notions of *acceptability* in [31] and *legality* in [14].

⁷ This definition encompasses both *recoverability* and *serializability* [4].

<p>(a) $H1 =$</p> <p>$\langle deposit(5), T1, O \rangle$</p> <p>$\langle ok, T1, O \rangle$</p> <p>$\langle commit(), T1, O \rangle$</p> <p>$\langle withdrawal(5), T2, O \rangle$</p> <p>$\langle ok, T2, O \rangle$</p> <p>$\langle commit, T2, O \rangle$</p>	<p>(b) $H2 =$</p> <p>$\langle deposit(5), T1, O \rangle$</p> <p>$\langle ok, T1, O \rangle$</p> <p>$\langle commit(), T1, O \rangle$</p> <p>$\langle withdrawal(5), T2, O \rangle$</p> <p>$\langle no, T2, O \rangle$</p> <p>$\langle commit, T2, O \rangle$</p>
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Fig.2. $H1$ is compatible with the semantic of O whereas $H2$ is not

the set of all the committed transactions involved by a history H , H is atomic if the transactions of $committed(H)$ appear to perform in a sequential way, i.e., they can be executed in a sequential way and have the same effects. We give a precise definition of atomicity through hypothetical sequential failure-free histories. Given a history H , and a total order γ , called a serialization order (SO), on $committed(H)$, we introduce $perm(H, \gamma)$ as the sequential failure-free history made of H 's events involving $committed(H)$'s transactions in the SO γ . We then define atomicity as follows:

Definition 2 (atomicity): A history H is atomic if there exists a SO γ on $committed(H)$ such that $perm(H, \gamma)$ is serially possible.

The fact that $perm(H, \gamma)$ is serially possible means that the transactions of $committed(H)$ can be executed in the SO γ , and have the same effects. H is said to be atomic in the SO γ .

Example 3 (atomicity): Figures 3 (a) and 3 (d) show two global histories $H1$ and $H2$ involving two bank-account objects $O1$ and $O2$, and two transactions $T1$ and $T2$. Intuitively, $H1$ is atomic because the transactions of $committed(H)$, i.e., $T1$ and $T2$, can be executed in the SO $(T2; T1)$ and have the same effects. More accurately, $H1$ is atomic in the SO $(T2; T1)$ because $perm(H1, (T2; T1))$ is serially possible, i.e., $perm(H1, (T2; T1))_{\{O1\}} \in O1.serial$, and $perm(H1, (T2; T1))_{\{O2\}} \in O2.serial$ (figures 3 (b) and 3 (c)). $H2$ is not atomic because neither $perm(H2, (T2; T1))$ nor $perm(H2, (T1; T2))$ is serially possible, i.e., $perm(H2, (T2; T1))_{\{O1\}} \notin O1.serial$, and $perm(H2, (T1; T2))_{\{O2\}} \notin O2.serial$ (figures 3 (e) and 3 (f)).

2.4 Behavioral Specifications

The behavioral specification of an object O , noted $O.behavior$, is a set of histories that describes how O behaves in case of concurrency and failures⁸.

Definition 3 (possible history): A history H is possible if for each object O involved by H , $H_{\{O\}} \in O.behavior$.

⁸ It is obvious that the behavioral specification of an object contains its serial specification.

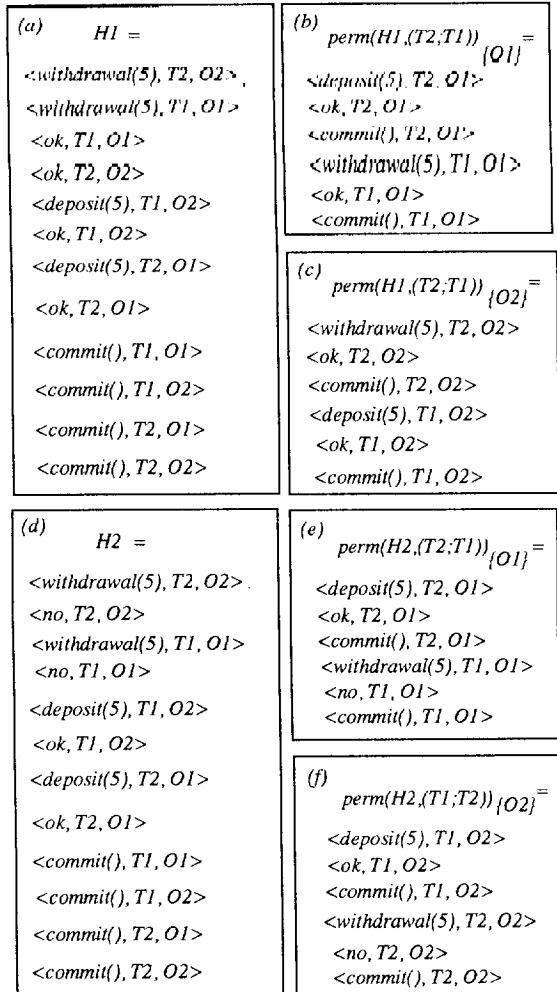


Fig.3. H1 is atomic whereas H2 is not

Our goal is to draw a property of objects' behavioral specifications which ensures that all possible histories are atomic⁹. Such a property must require more than the atomicity of local histories (see the example below). In fact, objects must ensure the atomicity of local histories according to some common SO.

Example 4 (possible history): Figure 3 (d) shows a global history $H2$ which is not atomic although local histories are so. The local history $H2_{\{O1\}}$ is atomic in the order $(T1; T2)$ but not in the order $(T2; T1)$ (i.e., $\text{perm}(H2, (T2; T1))_{\{O1\}} \notin O1.\text{serial}^{10}$), whereas $H2_{\{O2\}}$ is atomic in the or-

⁹ This would imply that all transactions are atomic.

¹⁰ Note that $\text{perm}(H_{\{O\}}, \gamma) = \text{perm}(H, \gamma)_{\{O\}}$.

der $(T2; T1)$ but not in the order $(T1; T2)$ (i.e., $perm(H2, (T1; T2))_{\{O2\}} \notin O2.serial$). Therefore, to ensure atomicity, $O1$ and $O2$ must avoid such a history (i.e., $H2$ must be impossible). On the contrary, figure 3 (a) shows a history $H1$ which is atomic because local histories (i.e., $H2_{\{O1\}}$ and $H2_{\{O2\}}$) are atomic according to a common SO, i.e., $(T2; T1)$.

The global protocol in charge of establishing a common SO is noted GSP (global serialization protocol). To ensure atomicity, objects must follow the same GSP. Weihl has distinguished (in [32]) three classes of GSPs: *static* GSPs that determine SOs according to timestamps assigned to transactions at creation, e.g., in [24]; *dynamic* GSPs that determine SOs during transaction execution, e.g., in [7]; and *hybrid* GSPs that determine SOs according to timestamps assigned to transactions at commit time, e.g., in [13]. Given this classification, he defined three properties of objects' behavioral specifications: *static atomicity*, *dynamic atomicity* and *hybrid atomicity*. Objects that guarantee *static atomicity* (resp. *dynamic atomicity* and *hybrid atomicity*) assume a static GSP (resp. a dynamic GSP and a hybrid GSP). Objects are incompatible if they guarantee different GSP-dependent properties.

In the following, we introduce a property of objects' behavioral specifications, named *o-atomicity* (*orthogonal atomicity*), which enables to ensure atomicity, whatever the GSP may be¹¹.

3 The *O-atomicity* Property

In this section we define the *o-atomicity* property and we present its characteristics through three theorems.

3.1 Definition of *O-atomicity*

We assume that there exists some GSP, whatever it may be, which determines for each history H in the system, a SO, noted $\tau[H]$, on all the transactions involved by H . We note $\tau[H]_{committed(H)}$, the restriction of $\tau[H]$ to $committed(H)$.

Definition 4 (*o-atomicity*): **The behavioral specification $O.behavior$ of an object O , satisfies the *o-atomicity* property if each local history H in $O.behavior$, is atomic in the SO $\tau[H]_{committed(H)}$.**

Example 5 (*o-atomicity*): Consider a bank-account object O such that $O.behavior$ satisfies *o-atomicity*. Assume $H1$ is a local history involving O , and two transactions $T1$ and $T2$ (figure 4 (a)), such that the GSP establishes that $T1 \xrightarrow{\tau[H1]} T2$ ($T1$ must be serialized before $T2$). $H1$ belongs to $O.behavior$ since it is atomic in the order $(T1; T2)$, i.e., $perm(H1, (T1; T2))$ is serially possible (figure 4 (b)). $H1$ would not belong to $O.behavior$ if the GSP established that $T2 \xrightarrow{\tau[H1]} T1$, because $H1$ is not atomic in the order $(T2; T1)$, i.e., $perm(H1, (T2; T1))$ is not serially possible (figure 4 (c)).

¹¹ Without assuming any particular GSP.

<p>(a) $H1 =$</p> <p>$\langle \text{deposit}(3), T2, O \rangle$ $\langle \text{ok}, T2, O \rangle$ $\langle \text{withdrawal}(5), T1, O \rangle$ $\langle \text{no}, T1, O \rangle$ $\langle \text{deposit}(2), T2, O \rangle$ $\langle \text{ok}, T2, O \rangle$ $\langle \text{commit}(), T1, O \rangle$ $\langle \text{commit}(), T2, O \rangle$</p>	<p>(b) $\text{perm}(H1, (T1;T2)) =$</p> <p>$\langle \text{withdrawal}(5), T1, O \rangle$ $\langle \text{no}, T1, O \rangle$ $\langle \text{commit}(), T1, O \rangle$ $\langle \text{deposit}(3), T2, O \rangle$ $\langle \text{ok}, T2, O \rangle$ $\langle \text{deposit}(2), T2, O \rangle$ $\langle \text{ok}, T2, O \rangle$ $\langle \text{commit}(), T2, O \rangle$</p>	<p>(c) $\text{perm}(H1, (T2;T1)) =$</p> <p>$\langle \text{deposit}(3), T2, O \rangle$ $\langle \text{ok}, T2, O \rangle$ $\langle \text{deposit}(2), T2, O \rangle$ $\langle \text{ok}, T2, O \rangle$ $\langle \text{commit}(), T2, O \rangle$ $\langle \text{withdrawal}(5), T1, O \rangle$ $\langle \text{no}, T1, O \rangle$ $\langle \text{commit}(), T1, O \rangle$</p>
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Fig.4. H1 belongs to the behavioral specification of O only if T1 is serialized before T2

3.2 Characteristics of *O-atomicity*

The following theorem implies that *o-atomicity* is a sufficient condition to achieve history atomicity.

Theorem 1 (sufficient condition): if each object's behavioral specification satisfies *o-atomicity*, then every possible history is atomic.

Proof outline: we show that if H is a possible history, then it is atomic in the SO $\tau[H]_{\text{committed}(H)}$ (see Appendix I).

Since *o-atomicity* does not rely on any particular GSP, this theorem means that objects which guarantee *o-atomicity*, ensure history atomicity¹² whatever the GSP may be. Such objects can thus be designed and tested within applications relying on different GSPs, and then reused within the same application. The most efficient GSP can be chosen according to the application's semantic [19,32]. The GSP can be changed for another, e.g., according to the evolution of the application, without affecting the objects.

The following theorem points out the compatibility relation between *o-atomicity* and GSP-dependent properties, i.e., *static atomicity*, *dynamic atomicity*, and *hybrid atomicity*.

Theorem 2 (relation with GSP-dependent properties): if each object's behavioral specification satisfies either *o-atomicity*, or *static atomicity* (resp. *o-atomicity* or *dynamic atomicity*, *o-atomicity* or *hybrid atomicity*), then every possible history is atomic.

Proof outline: we show that every possible history H is atomic in the SO $\tau[H]_{\text{committed}(H)}$ determined by the static (resp. dynamic or hybrid) GSP (see Appendix I).

This theorem implies that atomic objects which guarantee *o-atomicity* are compatible, not only with each others, but also with objects which guarantee

¹² *O-atomicity* is thus a *local atomicity property* in the sense of [32].

GSP-dependent properties, i.e. *static atomicity*, *dynamic atomicity*, or *hybrid atomicity*. Note that these objects must all guarantee the same GSP-dependent property¹³. The theorem is quite intuitive since *static atomicity* (resp. *dynamic atomicity*, or *hybrid atomicity*) implies history atomicity assuming a static (resp. dynamic, or hybrid) GSP, whereas *o-atomicity* imply history atomicity, whatever the GSP may be.

The following theorem implies that, given only the assumption of the existence of some GSP, *o-atomicity* is also a necessary condition to achieve history atomicity.

Theorem 3 (necessary condition): **If *o-atomicity* is not satisfied by an object's behavioral specification, then there exists a possible history that is not atomic.**

Proof outline: we suppose that there exists an object of which the behavioral specification does not satisfy *o-atomicity*, and we show that we can build a global possible history that is not atomic (see Appendix I).

This theorem means that there is no property of objects' specifications weaker¹⁴ than *o-atomicity*, which, based only on the assumption of the existence of a GSP, suffices to ensure the atomicity of global histories.

4 An Abstract Atomic Class

In this section we propose examples of how to implement objects that guarantee *o-atomicity*. We consider an object-oriented framework (in the sense of [28]): objects are instances of classes and classes are organized in an inheritance hierarchy. An object O , instance of a class A , is said to be derived from a class A' if A is a subclass of A' . We describe an abstract class, named *AtomicObject*, which provides the basic state variables and operations of a generic implementation. From this class, we derive a subclass of which objects guarantee *o-atomicity* through a pessimistic concurrency control, and a subclass of which objects guarantee *o-atomicity* through an optimistic concurrency control. To illustrate these implementations, we describe a simple banking application composed of objects from the two subclasses. To point out the GSP orthogonality of *o-atomicity*, we will first assume a static GSP, then a dynamic GSP.

4.1 Basic State Variables and Operations

The state variables provided by the basic abstract class *AtomicObject* are: a log of invocations and a set of transactions' identifiers lists. For an object O , derived from *AtomicObject*, the log, noted Log_O , contains accepted invocations, i.e., invocations for which O has performed the invoked operations (we note $T(inv)$ the invoking transaction). The status of an invocation inv in Log_O , noted $status(inv)$,

¹³ Because the GSP-dependent properties are incompatible with each others.

¹⁴ In the sense that it allows more histories.

is either *active* if $T(inv)$ is not terminated yet, or *committed* if $T(inv)$ is committed. The set of transactions' identifiers lists is used to maintain dependencies between transactions. For each transaction T , of which an invocation has been accepted by O , a *dependency list*, noted $D_O(T)$, is created.

The main operations provided by the class *AtomicObject* are *abort()*, *ask-commit()*, and *commit()* invoked by transaction managers, and *access()* and *order()* invoked internally by objects. These operations are defined for an object O , derived from *AtomicObject*, as follows:

- *order*(T_i, T_j): This operation implements the GSP. It is internally invoked by O to know the SO between any pair of different transactions (T_i, T_j): *order*(T_i, T_j) = *true* means that T_i is serialized before T_j ; *order*(T_i, T_j) = *false* means that T_j is serialized before T_i ; *order*(T_i, T_j) = *unknown* means that the SO is yet unknown. Since every subclass of *AtomicObject* inherits the operation *order()*, all objects derived from *AtomicObject* follow the same GSP. Changing the GSP comes down to change the implementation of the operation *order()* within the class *AtomicObject* (see examples below (4.5)).
- *abort*(T): This operation is invoked on O , when a transaction T , that invoked O , is aborted. O discards the effects of T by performing appropriate *reverse operations*¹⁵, and deletes its invocations and its dependency list.
- *ask-commit*(T): This operation is invoked on O when a transaction T , that invoked O , is about to commit. O returns *ok* if every transaction in $D_O(T)$ is committed and serialized before T ; O returns *no* if there exists a transaction in $D_O(T)$ which is either aborted or serialized after T ; otherwise O delays the operation *ask-commit()* (until the notification of another transaction termination), since some transactions in $D_O(T)$ are not yet terminated, or their SO with T is still unknown.
- *commit*(T): This operation is invoked on O when a transaction T , that invoked O , is committed. O changes the status of T 's invocations in its log (from *active* to *commit*).
- *access*(inv): This operation is the core of the concurrency control algorithm. It is internally invoked by O every time a transaction $T(inv)$ invokes O . The *access()* operation is based on three predicates: *aCond()*, *wCond()*, and *cCond()*. *aCond()* is evaluated for each invocation inv , and represents the condition under which $T(inv)$ must be aborted; *wCond()* is also evaluated for each invocation inv , but represents the condition under which $T(inv)$ must be delayed (until a termination notification); *cCond()* is evaluated for every pair of invocations (inv, inv') (where inv' is an invocation in Log_O), and

¹⁵ We assume here that each operation has a corresponding reverse operation, e.g., the reverse operation of *deposit*(x) is *withdrawal*(x).

represents the condition under which $T(inv')$ must be added to $D_O(T(inv))$. Customizing the generic concurrency control implementation comes down to define the three predicates within subclasses of *AtomicObject*.

Given the three predicates $aCond()$, $wCond()$, and $cCond()$, the *access()* algorithm is the following:

```

access(inv)
{
  await := true;
  while (await)
  {
    if (aCond(inv)) then abort(T(inv));
    if (wCond(inv)) then wait(T(inv));
    else await := false;
  }
   $\forall inv' \in Log_O$ 
  if (cCond(inv, inv') = true) then add(T(inv'),  $D_O(T(inv))$ );
}

```

4.2 Atomic Subclass with Pessimistic Concurrency Control

The class *PessAtomicObject* is built by inheriting the state variables and operations from the basic class *AtomicObject*, and by defining the predicates $aCond()$, $wCond()$ and $cCond()$ as follows:

- $aCond(inv) = (\exists inv' \in Log_{O1}$ **such that**
 $((T(inv) \neq T(inv')) \wedge (order(T(inv), T(inv')) = true));$

- $wCond(inv) = (\exists inv' \in Log_{O1}$ **such that**
 $((T(inv) \neq T(inv')) \wedge ((order(T(inv), T(inv')) = unknown) \vee (status(inv') = active)));$

- $cCond(inv, inv') = false$.

Broadly speaking, an object $O1$, instance of *PessAtomicObject*, aborts a transaction $T(inv)$ if $T(inv)$ cannot be performed in the SO ($T(inv)$ arrives too late). $T(inv)$ is delayed if the SO is yet unknown, or if $T(inv)$ can interfere with another transaction which invoked $O1$; otherwise, inv is accepted. (The resulting algorithm is described in Appendix II)

Theorem 4 (*o-atomicity*): every object of the class *PessAtomicObject* guarantees the *o-atomicity* property.

Proof outline: we show that the object forces, *a priori*, a sequential execution of the invoking transactions in the SO given by the operation *order()* (see Appendix II).

4.3 Atomic Subclass with Optimistic Concurrency Control

The class *OptAtomicObject* is built by inheriting the the state variables and operations from the basic class *AtomicObject*, and by defining the predicates *aCond()*, *wCond()* and *cCond()* as follows:

- $aCond(inv) = (\exists inv' \in Log_{O2} \text{ such that } ((T(inv) \neq T(inv')) \wedge (order(T(inv), T(inv')) = true)));$
- $wCond(inv) = false;$
- $cCond(inv, inv') = (((T(inv) \neq T(inv')) \wedge ((status(inv) = active) \vee (order(T(inv), T(inv')) = unknown))))$

An object *O2*, instance of *OptAtomicObject*, accepts every invocation *inv* unless *T(inv)* arrives too late. Therefore, *inv* can be accepted although an invocation *inv'*, of another transaction is still active, or the SO between *T(inv)* and *T(inv')* is yet unknown. Nevertheless, at the invocation of *ask-commit(T(inv))*, *O2* requires *T(inv')* to commit and to be serialized before *T(inv)*; otherwise *T(inv)* is aborted. (The resulting algorithm is described in Appendix II).

Theorem 5 (*o*-atomicity): every object of the class *OptAtomicObject* guarantees the *o*-atomicity property.

Proof outline: we show that the object ensures, *a posteriori*, a sequential execution of the invoking transactions in the SO given by the operation *order()* (see Appendix II).

4.4 A Simple Banking Application

To illustrate the proposed implementations, we describe a simple banking application where a transaction *T1* executes a funds transfer from a bank-account object *O1* to a bank-account object *O2*, while another transaction *T2* executes a funds consultation of *O1* and *O2*. *O1* is derived from *PessAtomicObject* and *O2* is derived from *OptAtomicObject*¹⁶. Figure 5 (a) depicts a non-atomic history *H1*, whereas figure 5 (b) depicts an atomic one *H2*. In the following we show how *O1* and *O2* guarantee that only *H2* is possible. To illustrate the GSP orthogonality, we consider first a static GSP, and second a dynamic GSP.

Static GSP: Suppose the SO is known statically, upon transaction creation, i.e., the operation *order()* within the abstract class *AtomicObject* implements a

¹⁶ In [10], we give more details on bank-account subclasses of *PessAtomicObject* and *OptAtomicObject*. We describe concrete implementations of these subclasses within an object-based distributed system. In both subclasses, we increase concurrency by taking into account the bank-accounts' semantics. More precisely, we refine the predicates *aCond()*, *wCond()* and *cCond()* by considering the commutativity of two *deposit()* executions, and the commutativity of two *balance()* executions.

<p>(a) $H_1 =$</p> <p>$\langle withdrawal(5), T1, O1 \rangle$ $\langle ok, T1, O1 \rangle$ $\langle balance(), T2, O2 \rangle$ $\langle 5, T2, O2 \rangle$ $\langle balance(), T2, O1 \rangle$ $\langle 0, T2, O1 \rangle$ $\langle deposit(5), T1, O2 \rangle$ $\langle ok, T1, O2 \rangle$ $\langle commit(), T1, O1 \rangle$ $\langle commit(), T1, O2 \rangle$ $\langle commit(), T2, O1 \rangle$ $\langle commit(), T2, O2 \rangle$</p>	<p>(b) $H_2 =$</p> <p>$\langle withdrawal(5), T1, O1 \rangle$ $\langle ok, T1, O1 \rangle$ $\langle deposit(5), T1, O2 \rangle$ $\langle ok, T1, O2 \rangle$ $\langle balance(), T2, O2 \rangle$ $\langle 10, T2, O2 \rangle$ $\langle balance(), T2, O1 \rangle$ $\langle 0, T2, O1 \rangle$ $\langle commit(), T1, O1 \rangle$ $\langle commit(), T1, O2 \rangle$ $\langle commit(), T2, O1 \rangle$ $\langle commit(), T2, O2 \rangle$</p>
--	---

Fig.5. Transfer and consultation histories

static GSP. Each transaction holds a timestamp which is assigned to it at the time of creation. The timestamps reflect the SO. Hence, when a transaction T invokes an operation on an object, the latter knows the SO between T and any transaction of which an invocation has already been accepted by O . H_1 is not possible: (1) if the SO is $(T_2; T_1)$, O_1 will accept $\langle withdrawal(5), T_1, O_1 \rangle$, and will abort T_2 when receiving $\langle balance(), T_2, O_1 \rangle$, since T_2 will arrive too late; (2) if the SO is $(T_1; T_2)$, O_2 will accept $\langle balance(), T_2, O_2 \rangle$, and then will abort T_1 when receiving $\langle deposit(5), T_1, O_2 \rangle$, since T_1 will arrive too late. H_2 is possible if the SO is $(T_1; T_2)$ (sequential execution). If the SO is $(T_2; T_1)$, O_2 will accept $\langle deposit(5), T_1, O_2 \rangle$, then will abort T_2 as $\langle balance(), T_2, O_2 \rangle$ will arrive too late.

Dynamic GSP: Suppose now the SO is determined dynamically, according to how transactions invoke operations on objects, i.e., the operation *order()* within the abstract class *AtomicObject* implements a dynamic GSP. An object knows the SO between two transactions T_i and T_j if either they are both committed or T_i has committed and T_j invokes the object later. H_1 is not possible: O_1 will delay $\langle balance(), T_2, O_1 \rangle$, waiting for T_1 to commit whereas O_2 will require T_2 to commit before allowing T_1 to do so; this situation will lead to a deadlock and one of the transactions will be aborted (we suppose here that deadlocks are detected and resolved by aborting transactions). H_2 is possible: O_1 will accept $\langle withdrawal(5), T_1, O_1 \rangle$, will delay $\langle balance(), T_2, O_1 \rangle$ (and block T_2), and then will resume T_2 at the commit time of T_1 .

5 Concluding Remarks

In this paper, we dealt with the problem of composing atomic objects in the context of transaction-based distributed systems. More precisely, we discussed how to specify atomic objects, so that they can be combined with any other atomic object, and ensure transaction atomicity. Atomic objects can be incompatible when they assume different GSPs. When combined, such objects do not ensure transaction atomicity anymore. Starting from this observation, we proposed a property of objects' specifications, named *o-atomicity*, which ensures transaction atomicity, whatever the GSP may be. Broadly speaking, an object's behavioral specification satisfies the *o-atomicity* property, if each of its local histories is atomic in the order determined by the GSP. *O-atomicity* represents exactly what must be ensured by an atomic object, which assumes only that there exists some GSP (without assuming any particular one).

O-atomicity enhances atomic object compatibility. It enables atomic objects to be designed and tested within various applications relying on different GSPs, and then reused within the same application. Within an application, a GSP can be changed for a more efficient one, e.g., according to the application's semantic, without affecting atomic objects. Furthermore, atomic objects that guarantee *o-atomicity* are compatible, not only with each others, but also with objects that guarantee GSP-dependent properties. This is very desirable since most of existing transaction-based systems rely on these properties [3,8,12,17,22,33]. Our work can be viewed as a generalization of [19], where Ng proposed a way to design atomic objects, without assuming any particular GSP. Whilst he proposed a particular pessimistic mechanism, we considered a more general specification framework, that enables pessimistic and optimistic implementations.

We believe that designing efficient concurrency control and recovery algorithms that ensure *o-atomicity*, is an important issue to be addressed before *o-atomicity* can be practically applied to object-based distributed systems. Indeed, the aim of the algorithms proposed in the paper was rather to show the feasibility of our approach. An interesting, and complementary, question, is how to modify existing algorithms that ensure GSP-dependent properties, so that they ensure *o-atomicity*. Such a facility would be of great help in *federative* distributed systems, where a major problem is how to safely combine *pre-existing* components [5].

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References

1. R. Agrawal, M. Carey and M. Livny - Concurrency Control Performance Modelling: Alternatives and Implications - ACM Transactions on Database Systems - Vol 12, Num 4 - 1987.
2. M.S. Atkins and M.Y. Coady - Adaptable Concurrency Control for Atomic Data Types - ACM Transactions on Computer Systems - Vol 10, Num 3 - 1992.
3. B.R. Badrinath and K. Ramamritham - Semantic-Based Concurrency Control: Beyond Commutativity - ACM Transactions on Database Systems - Vol 17, Num 1 - 1987.
4. A.J. Bernstein, V. Hadzilacos and N. Goodman - Concurrency Control and Recovery in Distributed Database Systems - Addison Wesley - 1987.
5. Y. Breitbart, H. Garcia Molina and A. Silberschatz - Overview of Multidatabase Transaction Management - The VLDB Journal - Vol 1, Num 2 - 1992.
6. P.K. Chrysanthis, S. Raghuram and K. Ramamritham - Extracting Concurrency From Objects: A Methodology - ACM Proceedings of the SIGMOD International Conference on Management of Data - pp 108.117 - 1991.
7. K.P. Eswaran, J.N. Gray, R.A.Lorie and I.L. Traiger - The notion of consistency and predicate locks in a database system - Communications of the ACM - Vol 19, Num 11 - 1976.
8. R. Guerraoui, R. Capobianchi, A. Lanusse and P. Roux - Nesting Actions through Asynchronous Message Passing: the ACS protocol - Proceedings of the European Conference on Object Oriented Programming - LNCS, Springer, pp 170.184 - 1992.
9. R. Guerraoui - Nested Transactions: Reviewing The Coherency Contract - Proceedings of the International Symposium on Computer and Information Science - pp 152.160 - 1993.
10. R. Guerraoui - Towards Modular Concurrency Control for Distributed Object Oriented Systems - IEEE Proceedings of the International Workshop on Future Trends in Distributed Computing Systems - pp 240.247 - 1993.
11. J.N. Gray - Notes on database operating systems - In Operatings Systems: An Advanced Course - LNCS, Springer, pp 393.481 - 1978.
12. M.P. Herlihy and J. Wing - Avalon: language support for reliable distributed systems - IEEE Proceedings of the International Symposium on Fault-Tolerant Computing - pp 89.95 - 1987.
13. M.P. Herlihy and W.E. Weihl - Hybrid Concurrency Control for Abstract Data Types - ACM Proceedings of the Symposium on Principles of Database Systems - pp 201.220 - 1988.
14. M.P. Herlihy - Apologizing Versus Asking Permissions: Optimistic Concurrency Control for Abstract Data Types - ACM Transactions on Database Systems - Vol 15, Num 1 - 1990.
15. T. Hirotsu and M. Tokoro - Object-Oriented Transaction Support for Distributed Persistent Objects - IEEE Proceedings of the International Workshop on Object Orientation in Operating Systems - 1992.
16. S. Mehrotra, R. Rastogi, H.F. Korth and A. Silberschatz - The Concurrency Control Problem In Multidatabases: Characteristics And Solutions - Technical Report 91-37, Department of Computer Science, Univ of Texas at Austin - 1991.
17. M. Mock, R. Kroeger and V. Cahill - Implementing Atomic Objects with the Relax Transaction Facility - Computing Surveys - Vol 5, Num 3 - 1992.
18. J.E.B. Moss - Nested Transactions: An Approach to Reliable Distributed Computing - MIT Press, MA - 1985.

19. T.P. Ng - Using Histories to Implement Atomic Objects - ACM Transactions on Computer Systems - Vol 7, Num 4 - 1989.
20. O. Nierstrasz and M. Papathomas - Viewing Objects as Patterns of Communicating Agents - ACM Proceedings of the International Conference on Object-Oriented Programming Systems, Languages and Applications - pp 38.43 - 1990.
21. C.H. Papadimitriou - The Theory of Database Concurrency Control - Computer Science Press - 1987.
22. G.D. Parrington and S.K. Shrivastava - Implementing Concurrency Control in Reliable Distributed Object-Oriented Systems - Proceedings of the European Conference on Object Oriented Programming - LNCS, Springer - pp 233.247 - 1988.
23. Y. Raz - The Principle of Commitment Ordering - Proceedings of the International Conference on Very Large Data Bases - pp 292.312 - 1992.
24. D.P. Reed - Naming and Synchronization in a Decentralized Computer System - Ph.D Thesis, MIT - 1978.
25. P.M. Schwarz and A.Z. Spector - Synchronizing Shared Abstract Data Types - ACM Transactions on Computer Systems - Vol 2, Num 3 - 1984.
26. D. Tsichritzis - Object Composition - Centre Universitaire d'Informatique, Univ of Geneva - 1991.
27. K. Wakita and A. Yonezawa - Linguistic supports for development of distributed organizational information systems in object-oriented computation frameworks - ACM Proceedings of the International Conference on Organizational Computing Systems - pp 185.198 - 1990.
28. P. Wegner - Dimensions of Object-based Language Design - ACM Proceedings of the International Conference on Object-Oriented Programming Systems, Languages and Applications - pp 168.182 - 1987.
29. W. E. Weihl - Specification and Implementation of Atomic Data Types - Ph.D Thesis, MIT - 1984.
30. W. E. Weihl and B.H. Liskov - Implementation of Resilient, Atomic Data Types - ACM Transactions on Programming Languages and Systems - Vol 7, Num 2 - 1985.
31. W. E. Weihl - Commutativity-based Concurrency Control for Abstract Data Types - IEEE Transactions on Computing - Vol 37, Num 12 - 1988.
32. W. E. Weihl - Local Atomicity Properties: Modular Concurrency Control for Abstract Data Types - ACM Transactions on Programming Languages and Systems - Vol 11, Num 2 - 1989.
33. W. E. Weihl - Linguistic Support for Atomic Data Types - ACM Transactions on Programming Languages and Systems - Vol 12, Num 2 - 1990.

Appendix I

Theorem 1 (sufficient condition): when satisfied by each object's behavioral specification, *o-atomicity* implies that every possible history is atomic.

Proof: Consider a possible history H (i) (we note $\tau[H]$ the SO (established by the GSP) of the transactions involved by H , and we note $\tau[H]_{committed(H)}$ the restriction of $\tau[H]$ to $committed(H)$). Assume that, $\forall O$ involved by H , O .behavior satisfies *o-atomicity* (ii).

1: By definition 2, assumption (i) $\Rightarrow \forall O$ involved by H , $H_{\{O\}} \in O$.behavior (1).

2: By definition 4, assumption (ii) $\Rightarrow \forall O$ involved by H , $H_{\{O\}}$ is atomic in the order $\tau[H_{\{O\}}]_{committed(H_{\{O\}})}$ (2).

3: By definition 3, (2) $\Rightarrow \forall O$ involved by H , $perm(H_{\{O\}}, \tau[H_{\{O\}}]_{committed(H_{\{O\}})}) \in O$.serial (3).

4: Since $perm(H_{\{O\}}, \tau[H_{\{O\}}]_{committed(H_{\{O\}})}) = perm(H, \tau[H]_{committed(H)})_{\{O\}}$, then by definition 1, (3) $\Rightarrow perm(H, \tau[H]_{committed(H)})$ is serially possible (4).

5: By definition 3, (4) $\Rightarrow H$ is atomic (in the order $\tau[H]_{committed(H)}$). \square

Theorem 2 (relation with GSP-dependent properties): if each object's behavioral specification satisfies either *o-atomicity*, or *static atomicity* (resp. *dynamic atomicity*, or *hybrid atomicity*), then every possible history is atomic.

Proof: Consider a possible history H (i), and assume that A is the set of objects involved by H of which behavioral specification satisfy *o-atomicity* (ii), and A' is the set of objects involved by H of which behavioral specification satisfy *static atomicity* (iii) (resp. *dynamic atomicity*, or *hybrid atomicity*).

1: By definition 2, assumption (i) $\Rightarrow \forall O \in (A \cup A')$, $H_{\{O\}} \in O$.behavior (1).

2: By reformulating the definition of *static atomicity* (resp. *dynamic atomicity*, or *hybrid atomicity*), given in [32], (assumption (ii) and (1)) $\Rightarrow \exists \tau 1[H_A]$, a SO (determined by a static (resp. dynamic or hybrid) GSP) of the transactions involved by H_A , such that $\forall O_j \in A$, $H_{\{O_j\}}$ is atomic in the order $\tau 1[H_{\{O_j\}}]_{committed(H_{\{O_j\}})}$ (2).

3: By definition 3, (2) $\Rightarrow \forall O_j \in A$, $perm(H_{\{O_j\}}, \tau 1[H_{\{O_j\}}]_{committed(H_{\{O_j\}})}) \in O_j$.serial (3).

Using the same static (resp. dynamic or hybrid) GSP, we can extend the SO $\tau 1[H_A]$ (of the transactions invoking objects in A), so that to obtain a SO $\tau[H]$, of the transactions invoking also objects in A' .

4: By definition 4, (assumption (iii) and (1)) $\Rightarrow \forall O_i \in A'$, $H_{\{O_i\}}$ is atomic in the order $\tau[H_{\{O_i\}}]_{committed(H_{\{O_i\}})}$ (4).

5: By definition 3, (4) $\Rightarrow \forall O_i \in A'$, $perm(H_{\{O_i\}}, \tau[H_{\{O_i\}}]_{committed(H_{\{O_i\}})}) \in O_i$.serial (5).

6: ((3) and (5)) $\Rightarrow \forall O \in (A \cup A')$, $perm(H_{\{O\}}, \tau[H_{\{O\}}]_{committed(H_{\{O\}})}) \in O$.serial (6).

7: By definition 1, (6) $\Rightarrow perm(H, \tau[H]_{committed(H)})$ is serially possible (7).

8: By definition 3, (7) $\Rightarrow H$ is atomic (in the order $\tau[H]_{committed(H)}$). \square

Theorem 3 (necessary condition): If *o-atomicity* is not satisfied by an object's behavioral specification, then there exists a possible history that is not atomic.

Proof: Suppose that there exists an object O , such that $O.behavior$ does not satisfy

o-atomicity (i).

1: By definition 4, assumption (i) $\Rightarrow \exists H1 \in O.behavior$, such that $H1$ is not atomic in the order $\tau[H1]$ (1) (where $\tau[H1]$ is the SO determined by the GSP; we note $T1, T2..Tn$ the series of the transactions of $committed(H1)$, in the order $\tau[H1_{committed}(H1)]$).

2: By definition 3, (1) $\Rightarrow perm(H1, \tau[H1]_{committed(H1)}) \notin O.serial$ (2).

Now assume another object Oc representing a counter, such that $Oc.behavior$ satisfies *o-atomicity* (ii). Oc provides an incrementation operation that returns its current value. We can build a local history $H2$, involving the transactions of $committed(H1)$, such that $\tau[H2] = \tau[H1_{committed}(H1)]$ ($H2$ is the sequential failure-free history involving the transactions of $committed(H1)$, in the order $\tau[H2]$; $H2 = perm(H2, \tau[H2])$ (figure 6). An important characteristic of $H2$ and the counter semantic is that the only order γ , such that $perm(H2, \gamma) \in Oc.serial$ is $\tau[H2]$ (iii).

3: By definition 3, assumption (iii) \Rightarrow the only order γ in which $H2$ is atomic, is the order $\tau[H2]$ (3).

4: By definition 4, (assumption (ii) and (3)) $\Rightarrow H2$ belongs to $Oc.behavior$ (4).

5: By definition 2, ((1) and (4)) $\Rightarrow (H1 \cup H2)$ is a possible history (5).

6: By definition 3, ((2) and assumption (iii)) $\Rightarrow (H1 \cup H2)$ is not atomic (6) (because the only order γ , such that $perm((H1 \cup H2), \gamma)_{Oc} \in Oc.serial$ is $\tau[H2]$, whereas

$perm(H1, \tau[H2]) \notin Oc.serial$).

7: $(H1 \cup H2)$ is thus possible and not atomic. \square

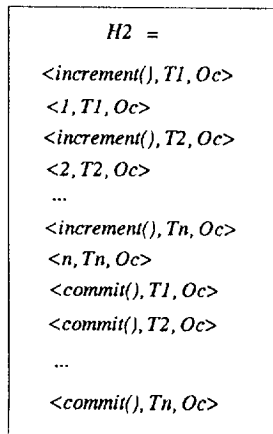


Fig.6. A counter history

Appendix II

Theorem 4 (*o-atomicity*): every object of the class *PessAtomicObject* guarantees the *o-atomicity* property.

An object $O1$, instance of the class *PessAtomicObject*, has the following *access()* algorithm:

```

access(inv)
{
  await := true;
  while (await)
  {
    if ( $\exists inv' \in Log_{O1}$  such that
         $((T(inv) \neq T(inv')) \wedge (order(T(inv), T(inv')) = true))$ )
      then abort( $T(inv)$ );
    if ( $\exists inv' \in Log_{O1}$  such that
         $((T(inv) \neq T(inv')) \wedge$ 
           $((order(T(inv), T(inv')) = unknown) \vee (status(inv') = active)))$ )
      then wait( $T(inv)$ );
    else await := false;
  }
}

```

Proof of theorem 4: Assume $O1$ is an instance of *PessAtomicObject*.

1: When $O1$ receives an invocation inv from a transaction $T(inv)$, $O1$ checks whether there exists an invocation inv' in Log_{O1} , of a different transaction $T(inv')$, such that inv' is still active, i.e. $status(inv') = active$. In this case $T(inv)$ is delayed until a termination invocation (*commit* or *abort*) arrives at O . Therefore, $O1$ delays interfering transactions and forces a strict sequential execution **(1)**.

2: if $T(inv)$ invokes $O1$ after a transaction $T(inv')$ serialized before it ($T(inv)$ comes too late), $T(inv)$ is either delayed if inv' is still active, or aborted if inv' is already committed. As a result, $O1$ schedules transactions according to the SO given by the operation *order()* **(2)**.

3: **((1) and (2))** $\Rightarrow \forall H_{O1} \in O1.behavior$, $H_{\{O1\}}$ corresponds to a serial execution of transactions in the SO $\tau[H_{\{O1\}}]$ **(3)**.

4: **(3)** $\Rightarrow \forall H_{\{O1\}} \in O1.behavior$, $H_{\{O1\}}$ is atomic in the SO $\tau[H_{\{O1\}}]$. \square

Theorem 5 (*o-atomicity*): every object of the class *OptAtomicObject* guarantees the *o-atomicity* property.

An object, *O2*, instance of the class *OptAtomicObject*, has the following *access()* algorithm:

```

access(inv)
{
  if ( $\exists inv' \in Log_{O2}$  such that
       $((T(inv) \neq T(inv')) \wedge (order(T(inv), T(inv')) = true))$ )
    then abort( $T(inv)$ );
   $\forall inv' \in Log_{O2}$ 
    if  $((T(inv) \neq T(inv')) \wedge ((status(inv) = active) \vee (order(T(inv), T(inv')) = unknown)))$ 
      then add( $T(inv')$ ,  $D_{O2}(T(inv))$ );
}

```

Proof of theorem 5: Assume *O2* is an instance of *OptAtomicObject*.

1: *O2* accepts every invocation *inv*, unless it has already accepted an invocation *inv'*, of a transaction $T(inv')$, that is serialized before $T(inv)$; in which case $T(inv)$ is aborted (it arrived too late).

2: When *O2* accepts an invocation *inv*, after it has accepted an invocation *inv'* which is still active, *O2* requires that, at commit time, $T(inv')$ should have committed and serialized before $T(inv)$. If $T(inv')$ comes to invoke *O2* later, $T(inv')$ will be attached a contradictory termination condition which might not be satisfied at commit time. In this case, both $T(inv)$ and $T(inv')$ will be aborted. Therefore, out of order and interfering transactions will be aborted (*a posteriori*).

3: ((**1**) and (**2**)) $\Rightarrow \forall H_{\{O2\}} \in O2.behavior$, $H_{\{O2\}}$ corresponds to a serial execution of transactions in the SO $\tau[H_{\{O2\}}]$ (**3**).

4: (**3**) $\Rightarrow \forall H_{\{O2\}} \in O2.behavior$, $H_{\{O2\}}$ is atomic in the SO $\tau[H_{\{O2\}}]$. \square