Estimation of the Center of Mass: From Humanoid Robots to Human Beings

S. Cotton, *Member, IEEE*, A. Murray, *Member, ASME*, P. Fraisse, *Member, IEEE*

***Abstract*—This paper presents a new technique for estimating the center of mass of articulated rigid body systems. This estimation technique uses the statically equivalent serial chain, a serial chain representation of any multi-link branched chain whose end-effector locates directly the center of mass. This technique works based on a knowledge of only the kinematic architecture of the system and does not require the total mass, or any of the individual body's mass or length properties. This constitutes an advance in center of mass estimation, providing an alternate to techniques requiring a force plate. A comparison of these estimation techniques is presented. The modeling and estimation technique is then implemented on a human subject.**

***Index Terms*— Center of Mass, Estimation, Humanoids.**

I. INTRODUCTION

THE need for predicting the location of the center of mass (CoM) is well established in both robotics and the life sciences. In humanoid robotics, controlling the robot's CoM provides significant aid in maintaining static balance [1]. For robotic systems generally, an estimation of the CoM provides an additional source of information in the identification process. For human beings, the CoM provides an indicator of stability and is an essential parameter in human postural control systems which use visual, vestibular and somatosensory information to maintain balance [2]-[5]. Moreover, calculating the CoM can prove critical to assessing rehabilitation success [6], in pathology detection [2]-[4], and in describing gaits [7], [8]. From the kinematic and static models, perfect knowledge of the mechanical parameters of the studied system allows for the exact prediction of the CoM. Although anthropometric tables have been compiled [9], their accuracy when applied to a specific person, especially for estimating the individual's CoM, are readily questioned. Ultimately, if the system's mechanical parameters have errors, or are completely unknown, the CoM must be estimated.

With the goal of predicting the CoM location of an articulated system of bodies, especially while the system is moving, the literature contains several methods dedicated to this task. The most common approach is to estimate the horizontal location of the CoM by recording the center of pressure (CoP), generally using data generated via a force platform, and then using this information in manipulations of Newton's equations. Following Shimba's work [10], King et al. have proposed a method utilizing the double integration of the horizontal ground reaction forces [11]. The challenge posed by this method is the determination of the initial constants of integration, a difficult problem in light of force platform sampling rates. Brenière et al. detail the relationship between the CoM and the CoP in the frequency domain, but it is a relationship best suited to addressing periodic motions [12], [13]. Other methods for CoM estimation include the genetic sum-of-sines model [14] or the neural network model [15]. These methods produce acceptable CoM estimation error but remain sensitive to the complexity of the task or motion.

Another approach has been proposed to estimate the CoM of jointed mechanical systems [16]. Unlike the previous approaches which seek an estimation of CoM location based on current data, this approach involves an initial experiment to construct a model to predict the CoM location, and a method for updating this model based on current joint angles in the system. This approach is based on a manipulation of the equation for determining the CoM originally detailed by Espiau [17]. Espiau showed that the CoM of any branched architecture could be written to resemble the forward kinematics of a serial chain. Since termed the statically equivalent serial chain, or SESC, this modeling technique has been applied to many examples and generalized to address the potential challenges encountered in developing the SESC for a sufficiently complex chain of bodies as in, say, a humanoid [18].

The SES chain discussed here, and the original work by Espiau on which it is based, are by no means the only use of a virtual chain useful in CoM discussions. For work prior to Espiau's, the work on virtual manipulators by Vafa and Dubowsky [19] and Dubowsky and Papadopoulus [20] should be consulted. For more recent work in which this virtual chain concept has proved useful, see Agrawal et al. [21].

The modeling presented in Section II and the method for estimating the CoM presented in Section III have been successfully employed on robotic systems [18]. Using neutrally stable postures of the studied architecture, the SESC of a HOAP-3 humanoid robot was accurately estimated. A significant advantage of CoM estimation using this technique is that it does not require any knowledge of the system's static parameters including the total mass, total height, the masses of any of the individual bodies, or the relative center of mass locations of the individual links. Additionally, the technique requires no knowledge of any lengths in the system. Only the

Manuscript submitted February 14, 2009.

S. Cotton is with the CNRS and the LIRMM, Robotics Department, 34392 Montpellier, France. (Phone: +33 467418565; e-mail: cotton@lirmm.fr).

A. Murray is with the University of Dayton, Dayton, OH 45469 USA (e-mail: murray@udayton.edu).

P. Fraisse is with the University of Montpellier and the LIRMM, Robotics Department, 34392 Montpellier, France (e-mail: fraisse@lirmm.fr).

system's kinematic architecture needs to be understood, including the joint types and the order in which they are connected. A potential challenge to using the SESC to estimate the CoM is its lack of invertibility. That is, the static and kinematic parameters, just listed as unnecessary, of the articulated system cannot be uniquely determined from the SESC model even though an accurate estimation of the system's CoM can be produced for an arbitrary configuration.

On living subjects, like humans, the base of support is not as clearly defined as in mechanical, specifically robotic, systems. As such, using neutrally stable postures to reconstruct the subject's SESC and estimate its CoM cannot be performed accurately enough. For this reason, the previous estimation technique using these neutrally stable postures must be adapted to human beings. For this purpose, additional equipment like motion capture devices and a force platform have been utilized to collect the required data and to build the SESC as presented in Section IV. The result is capacity to perform CoM estimation on healthy people. Section V then compares the results of SESC modeling to the other commonly used techniques, and the conclusions may be found in Section VI.

II. Statically Equivalent Serial Chain Modeling

This section contains a review of the notation, equations, and previous results that will prove useful to a discussion of the SESC of a human subject. Additional details may be found in the references.

A. Kinematic and Static Parameters

The systems studied are assumed to be composed of rigid bodies, called links, connected by revolute or spherical joints. As such, each link is fully described by its geometric and mass properties. Thus, for each link, the mass and the location of the center of mass are known, as are the locations of all joints. Homogeneous transforms, denoted T_i, are used to relate the reference frames attached to any two bodies in the system,

$$T_i = \begin{bmatrix} A_i & \vec{d}_i \\ 0 & 1 \end{bmatrix} \qquad (1)$$

where A_i is a 3-by-3 rotation matrix, $\vec{d}_i$ is a 3-by-1 displacement vector, and the 0 represents a 1-by-3 vector of zeros. The 3-by-1 vector $\vec{c}_i$ is used to locate the CoM of an individual body in the local reference frame attached to body i, or relative to T_i. Finally, the mass of body i is given by m_i where the total mass of the system is $M = \sum m_i$.

B. SESC Modeling

A brief review of the main steps in the development of the statically equivalent serial chain of the example chain depicted in Fig. 1 is now presented. The CoM of any multi-link chain, $\vec{C}_M$, with a serial or a branched chain structure, can be expressed as the end-effector of a SES chain. Fig. 2 illustrates this point for the branched chain depicted in Fig. 1. The process begins with the definition of the CoM of a collection of bodies, or the weighted sum of each body's center of mass location, Eqn. (2).

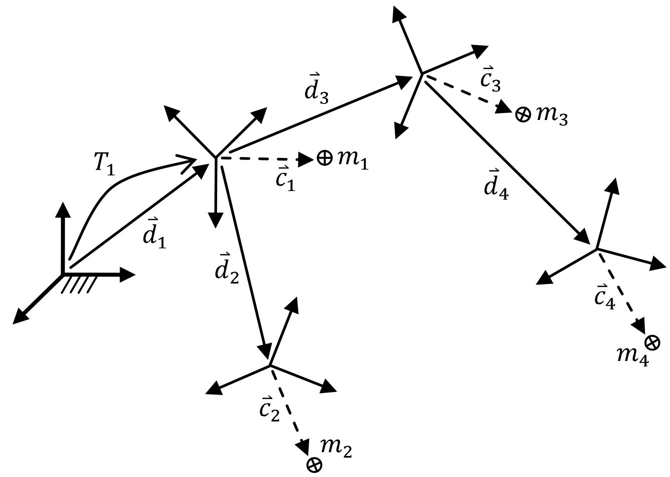

Fig. 1. The kinematic and static parameters of a spatial multi-link chain.

$$\begin{Bmatrix} \vec{C}_M \\ 1 \end{Bmatrix} = \frac{m_1 T_1 \begin{Bmatrix} \vec{c}_1 \\ 1 \end{Bmatrix}}{M} + \frac{m_2 T_1 T_2 \begin{Bmatrix} \vec{c}_2 \\ 1 \end{Bmatrix}}{M} + \frac{m_3 T_1 T_3 \begin{Bmatrix} \vec{c}_3 \\ 1 \end{Bmatrix}}{M} + \frac{m_4 T_1 T_3 T_4 \begin{Bmatrix} \vec{c}_4 \\ 1 \end{Bmatrix}}{M} \qquad (2)$$

Expanding,

$$\vec{C}_M = \vec{d}_1 + A_1\vec{r}_2 + A_1A_2\vec{r}_3 + A_1A_3\vec{r}_4 + A_1A_3A_4\vec{r}_5 \qquad (3)$$

where

$$\vec{r}_2 = \frac{m_1\vec{c}_1 + m_2\vec{d}_2 + (m_3 + m_4)\vec{d}_3}{M}, \vec{r}_3 = \frac{m_2\vec{c}_2}{M}, \quad \vec{r}_4 = \frac{m_3\vec{c}_3 + m_4\vec{d}_4}{M}, \vec{r}_5 = \frac{m_4\vec{c}_4}{M} \qquad (4)$$

Observe that with a complete knowledge of the kinematic and static parameters of the system, the $\vec{r}_i$ vectors in Eqn. (4) are known. Moreover, for a system connected by only revolute and spherical joints, the $\vec{d}_i$ are constant and, thus, the $\vec{r}_i$ are too. Letting $\hat{A}_3 = A_2^{-1}A_3$, the similarity between the expression in Eqn. (3) and the forward kinematics of the following serial chain is noted,

$$\begin{Bmatrix} \vec{C}_M \\ 1 \end{Bmatrix} = \begin{bmatrix} A_1 & \vec{d}_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_2 & \vec{r}_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{A}_3 & \vec{r}_3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_4 & \vec{r}_4 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \vec{r}_5 \\ 1 \end{Bmatrix}. \qquad (5)$$

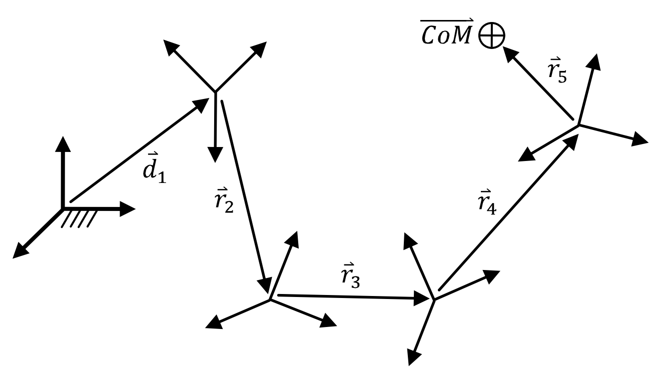

Fig. 2. The Statically Equivalent Serial Chain for the spatial system in Fig. 1.

The result is that the CoM location of the original branched chain is modeled by the end-effector location of an appropriately sized spatial serial-chain, as given in Eqn. (5) and shown in Fig. 2, where this SESC maintains the same DOF as the original branched chain.

C. Manipulating the SESC for CoM Estimation

For the purposes of estimating the CoM location, Eqn. (3) is manipulated in yet another way,

$$\vec{C}_M = \vec{d}_1 + A_1^*\vec{r}_2 + A_2^*\vec{r}_3 + A_3^*\vec{r}_4 + A_4^*\vec{r}_5 \tag{6}$$

where $A_1^* = A_1$, $A_2^* = A_1A_2$, $A_3^* = A_1A_3$, and $A_4^* = A_1A_3A_4$. Finally, for this example,

$$\vec{C}_M = [I\ A_1^*\ A_2^*\ A_3^*\ A_4^*]\begin{Bmatrix}\vec{d}_1\\ \vec{r}_2\\ \vec{r}_3\\ \vec{r}_4\\ \vec{r}_5\end{Bmatrix} \tag{7}$$

where I is the 3-by-3 identity matrix.

This concept can be applied generally to any multi-link chain. Observing that if the multi-link chain contains only revolute and spherical joints, the vector composed from the concatenation of $\vec{d}_1$ and the $\vec{r}_i$ vectors is a constant. Thus, for the study of an arbitrary branched chain with only revolute and spherical joints, we can realize a statement of the form

$$\vec{C}_M = [I \quad A_1^* \quad \cdots \quad A_n^*]\begin{Bmatrix}\vec{d}_1\\ \vec{r}_2\\ \vdots\\ \vec{r}_{n+1}\end{Bmatrix} = B\vec{R} \tag{8}$$

where n is the degrees of freedom of the original chain, the vector $\vec{R}$ is constant, and the matrix B is 3-by-$3(n+1)$ for the spatial case.

D. The Non-Uniqueness of the Generating Chain for a SESC

Given the vectors $\vec{d}_1$ and the $\vec{r}_i$ in the SESC, note that a unique chain generating the SESC is not realizable. Consider, again, the example in Eqn. (4). Given the values of $\vec{r}_2$, $\vec{r}_3$, $\vec{r}_4$, and $\vec{r}_5$, the values in m_i, $\vec{d}_i$, $\vec{c}_i$, and M constitute 26 unknowns in 12 relationships (4 equalities where each contains 3 components). Even under the reasonable assumption that the $\vec{d}_i$ and the value of M are known accurately, Eqn. (4) still contains 15 unknowns. Thus, given only the values in the SESC and the original architecture, the mechanical parameters cannot be uniquely determined.

E. Chains with Prismatic Joints

For chains including prismatic joints, the distance between two successive frames can vary. As such, the $\vec{d}_i$ vectors (corresponding to the links containing prismatic joints) vary and, hence, the corresponding $\vec{r}_i$ vary. This represents no difficulty in developing a SESC, as was shown in [18] where an algorithm was presented to efficiently compute a SESC on a high degree of freedom robot under the simplifying assumption that the $\vec{r}_i$ may vary. There is a challenge posed by prismatic joints for this CoM estimation work in that $\vec{R}$ in Eqn. (8) will not be constant.

III. CoM Estimation Method

A. Constructing the SESC from CoM Data

Given a system with unknown mass (and perhaps kinematic) parameters, $\vec{R}$ in Eqn. (8) is unknown. For simplicity, assume that the fixed frame of the system is aligned with the first joint, or $\vec{d}_1 = 0$, and Eqn. (8) simplifies to:

$$\vec{C}_M = [A_1^* \quad \cdots \quad A_n^*]\begin{Bmatrix}\vec{r}_2\\ \vdots\\ \vec{r}_{n+1}\end{Bmatrix} = B\vec{R}\,. \tag{9}$$

For this case, the matrix B is 3-by-$3n$. For a given configuration of the robot, configuration *i*, we write $\vec{C}_{M,i} = B_i\vec{R}$. For *n* configurations of the robot,

$$\begin{Bmatrix}\vec{C}_{M,1}\\ \vdots\\ \vec{C}_{M,n}\end{Bmatrix} = \begin{bmatrix}B_1\\ \vdots\\ B_n\end{bmatrix}\vec{R} = D\vec{R}\,. \tag{10}$$

The matrix D is $3n$-by-$3n$. Even though D is not full rank, the vector containing the center of mass locations is in its range space, and there exists a solution for $\vec{R}$ or, to be more precise, many solutions for $\vec{R}$. One of these solutions may be determined using the pseudo-inverse, and

$$\vec{R} = D^{-P}\begin{Bmatrix}\vec{C}_{M,1}\\ \vdots\\ \vec{C}_{M,n}\end{Bmatrix} \tag{11}$$

determines the parameters of a SES chain. Given $\vec{R}$, Eqn. (9) could then be used to determine the CoM for any other configuration of the robot. The ramifications of the existence of multiple SESC chains have not been studied.

B. Constructing the SESC from Partial CoM Data

The problem with the previous procedure is that the general CoM is not readily known. To remedy this, we focus on the instant at which a part of the CoM is known at, say, the condition of static neutral stability or when a force plate is used. Consider the known component to be in the *x*-direction (where there is no indication of the *y*- and *z*- components of the CoM). Ignoring the two unknown components, Eqn. (9) becomes

$$C_{Mx} = [A_{1,x}^* \quad \cdots \quad A_{n,x}^*]\begin{Bmatrix}\vec{r}_2\\ \vdots\\ \vec{r}_{n+1}\end{Bmatrix} = B_x\vec{R} \tag{12}$$

where the matrix B_x is 1-by-$3n$. Similarly, the system may have a known *y*- component (with the *x*- and *z*- components of the CoM unknown), or known *x*- and *y*- components. In fact, a distribution in the two directions is necessary for the process to yield usable results. For 3*n* known CoM components,

$$\begin{Bmatrix} C_{Mx,1} \\ \vdots \\ C_{My,3n} \end{Bmatrix} = \begin{bmatrix} B_{x,1} \\ \vdots \\ B_{y,3n} \end{bmatrix} \vec{R} = D\vec{R} \text{ and } \vec{R} = D^{-P} \begin{Bmatrix} C_{Mx,1} \\ \vdots \\ C_{My,3n} \end{Bmatrix}. \quad (13)$$

Again, given $\vec{R}$, Eqn. (9) determines the CoM for any other configuration of the robot.

The collection of $3n$ pieces of data, in theory, is enough for the procedure to work. Due to the vagaries of the actual data collection in practice, many more such readings are needed. Additionally, with planar systems the focus of the current work, the multiplier of 3 in the above equations is replaced by 2. That is, in Eqns. (12) and (13), B_x is 1-by-$2n$, and D is $2n$-by-$2n$. Intriguingly, this matrix in the planar case is full rank. Again, due to vagaries in the collection of data, many more points are used and the pseudo-inverse remains a necessity.

C. Validation on the HOAP-3 Humanoid Robot

This estimation technique was used in [16] to predict the CoM location in the medio-lateral and vertical directions, using the six joints that produced motion the frontal plane: the two shoulders, two hips, and two ankles. Fourteen neutrally stable postures on the right foot were recorded, using both the left and right side of the base of support, see Fig. 3. At the static and neutrally stable condition, one component of the CoM is known. Fig. 4 shows the comparison of values in the SESC determined by the methodology presented here to the manufacturer provided mechanical parameters. This estimation has proven useful in several control algorithms for the HOAP-3.

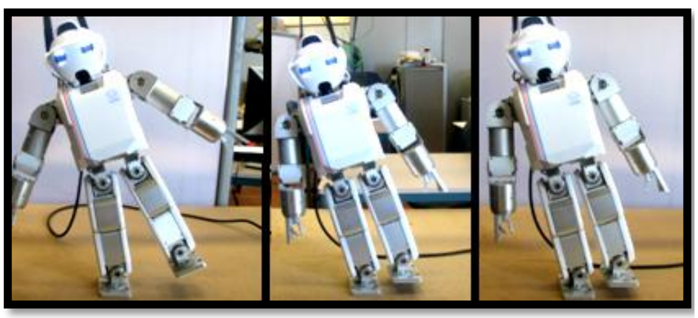

Fig. 3. Some neutrally stable postures used to reconstruct HOAP-3 SESC.

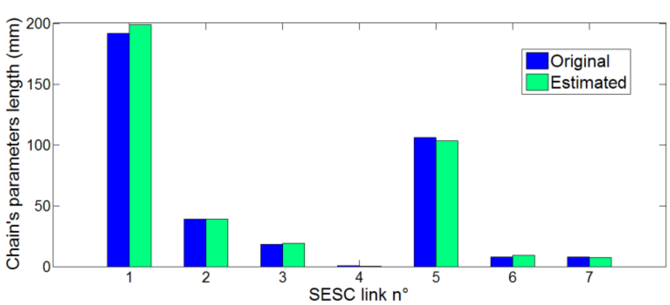

Fig. 4. A comparison of parameters determined via this technique versus those from the manufacturer.

D. Extension to Living Subjects

On a living subject, a human being say, there are additional challenges posed by collecting the joint information to construct D, and determining all of part of the CoM at any given instant. To solve the latter problem, a force platform can be utilized. As a force platform measures the forces and moments acting on it, if the system being monitored is in a static configuration, the horizontal (x, y) location of the CoM is readily obtained from the center of pressure (CoP). To obtain joint values for a human, motion capture equipment can be used. In brief, the subject is covered with markers. Multiple cameras provide enough information to calculate a spatial location for each marker. With enough markers, the current kinematics of the subject is entirely known.

IV. Experimental Results on Humans Beings

This section deals with the results of our estimation method on a healthy person to estimate the CoM in the frontal plane. The similarity of this experiment to that done with the HOAP-3 is intentional.

A. Experimental Setup and Collected Data

Motion capture markers were used in order to identify "rigid" segments, shown in Fig. 5, useful in computing the joint values at the ankles, hips and shoulders. With the joint values identified, B in Eqn. (9), or B_x in Eqn. (12), is known.

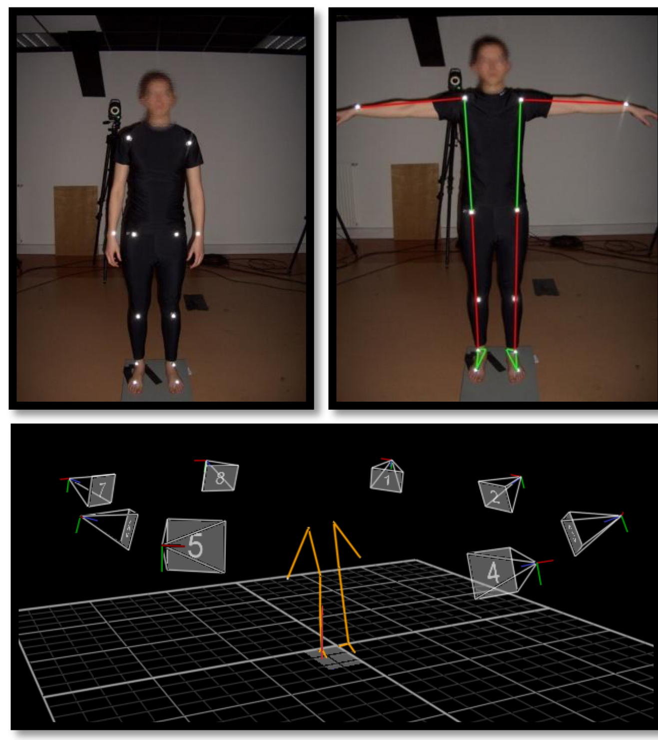

Fig. 5. The marker locations and segments used to record postures and generate joint values in a human subject.

An AMTI force platform was used to record center of pressure locations at a sampling rate of 1 KHz. Given that the CoP and projection of the CoM are the same for a static posture, the CoP was tracked for five seconds. For a period of at least two hundred milliseconds, if the CoP maintained a standard deviation of less than one millimeter, the posture was deemed static. Fig. 6 shows a typical CoP sample, with a static period identified by a red box from 2.2 to 2.4 seconds. For this static period, the mean of the (x-component of the) CoP horizontal location was used as C_{Mx} and the mean of the corresponding joint values, obtained with Vicon Nexus motion capture equipment, over this time were used to populate B_x.

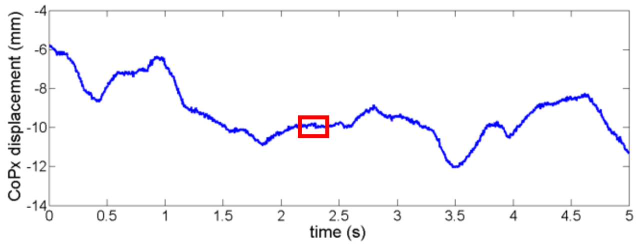

Fig. 6. The CoP sample of a posture during a 5 second interval. The red box from 2.2 to 2.4 seconds identifies a posture deemed static.

The data collected from static postures with motion capture equipment and force platform are given in Table I.

TABLE I
THE NINETEEN STATIC POSTURES USED TO ESTIMATE THE CoM

pose	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	C_{Mx}
1	0	0	0	0	-87	90	107
2	-3	-8	-7	3	-83	80	17
3	-5	-2	10	-20	-99	16	1
4	0	0	2	-1	-15	15	108
5	0	0	1	4	-44	44	103
6	-5	-7	0	-12	-41	32	0
7	-6	-14	-2	-14	-29	62	-18
8	8	-9	-4	-8	-25	21	209
9	-4	-32	3	-26	-98	46	-2
10	-3	-12	-1	-18	-47	24	2
11	-5	-14	8	-26	-100	30	-23
12	-6	-15	9	-27	-37	57	-7
13	-5	-6	6	-22	-73	51	-8
14	0	-24	-22	-7	-48	45	39
15	0	18	23	-4	-72	24	158
16	-5	-15	-2	-22	-99	39	-23
17	10	-27	-25	-8	-92	36	179
18	-3	-13	30	-27	-86	49	5
19	-4	-14	4	-24	-109	22	-11

The joint angles are rounded to the nearest degree and the C_{Mx} is rounded to the nearest millimeter.

B. Estimated SESC

Using fourteen of the nineteen postures, the estimated SESC of the subject, the vector $\vec{R}$, was calculated via Eqn. (13). Fig. 7 shows the SESC estimated from the data.

Fig. 7. The estimated SESC of the subject.

C. Static Validation

To validate this CoM estimation technique, the C_{Mx} values of the remaining five static postures, not used in determining the SESC, were compared to the C_{Mx} value predicted by Eqn. (9).

TABLE II
ESTIMATED CoM VS MEASURED CoM

C_{Mx} ***estimated***	C_{Mx} ***measured from force platform***
113	108
-20	-18
213	209
51	39
156	158

The comparison between *x*-components of the CoM measured from the force platform and the CoM estimated through the SESC, in millimeters.

V. COMPARISON TO OTHER ESTIMATION METHODS

Two other common methods of CoM estimation are the Low Pass Filter (LPF) and the Second Integral (SI) method. To validate during motion the method presented in section IV, the SESC, LPF and SI methods were used under the condition of an oscillating subject in the frontal plane. Results are compared in Fig. 8. The estimated CoM horizontal locations (*x*-component) for the three methods are very close.

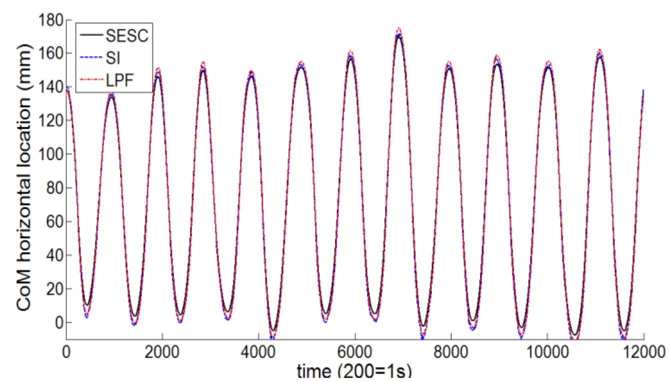

Fig. 8. The C_{Mx} values estimated by three different methods for an osciallting human subject.

The real advantage of the estimation process presented here is that, once the SESC has been estimated from the static posture data, CoM estimation can be performed in real time from the joint value data. As a result, the subject is no longer required to be on a force platform to produce a CoM estimate. This also introduces the potential drawback to this method as, for living systems, motion capture or goniometry equipment is the typical way of generating this joint value data. Of course, for mechanical systems, encoders would readily provide this data.

Although all three methods, LPF, SI, and SESC, require the use of a force platform on living subjects, the two others utilize the force platform data to continuously update the CoM estimate. The SESC method can operate independently of the force platform once the SESC is estimated.

The LPF and SI methods utilize information about the mass and height of the subject in order to make the estimation. The method proposed here does not require any knowledge of the kinematic or static parameters, but does require an understanding of the architecture or topology of the subject. Moreover, LPF and SI make assumptions about the vertical component of the CoM (which is estimated from anthropometric tables).

The LPF and SI methods are also best suited for estimation of components of the CoM under the assumption of oscillating motions. Finally, the SESC method estimates, for any motion, the CoM location including the vertical component.

VI. Conclusion

This paper summarized work on estimating the center of mass location of a living subject using the statically equivalent serial chain. The SESC is a serial chain whose end-effector locates the CoM of any branched chain composed of rigid bodies. The theory underlying the derivation of the SESC, and justifying this technique for estimating the CoM, were presented. We showed that if the only joints in the system are revolute and prismatic, then the vectors in the SESC are constant and may be readily solved for. The results of previous work in which only neutrally stable postures were used on a mechanical system, a HOAP-3 humanoid, to generate an accurate CoM estimator was reviewed. Then the methods were extended in order to perform CoM estimation on humans. Finally, the data gathered on a human subject, and the corresponding SESC, were also presented. The results on the living subject indicate that the method is promising. Moreover, with the SESC estimated, the CoM of any configuration for the subject may be determined so long as the appropriate joint data is available.

References

[1] L. Sentis, "Synthesis and control of whole-body behaviors in humanoid systems", *Thesis*, Stanford University, 2007.

[2] K.F. Zabjek, C. Coillard, C.H. Rivard and F.Prince, "Estimation of the center of mass for the study of postural control in Idiopathic Scoliosis patients: a comparison of two techniques", *European Spine Journal*, n°17, p355-p360, 2008.

[3] H.Corriveau, R.Hebert and F.Prince, "Postural control in the ederly: an analysis if test-retest and interrater reliability of the COP-COM variable", *Archives of Physical Medicine and Rehabilitation*, n°82, p80-p85, 2001.

[4] F.De la Huerta, M.A.Leroux, K.F.Zabjek, "Stereovideographic evaluation of the postural geometry of healthy and scoliotic patients", *Annales de Chirurgie*, n°52, p776-p783, 1998.

[5] S.K.Banala, S.K. Agrawal and A.Fattah, "A gravity balancing leg orthosis for robotic rehabilitation", *Proceedings of the IEEE ICRA Conference*, p2474-2479, 2004.

[6] J. J. Buchanan and F. B. Horak, "Emergence of postural patterns as a function of vision and translation frequency", *Journal of Neurophysiology*, n°6 , p2325-p2339, 1999.

[7] S.E.Halliday, D.A.Winter and J.S.Frank, "The initiation of gait in young, elderly and Parkinson's disease subjects", *Journal of Gait and Posture* , n°8, p8-p14, 1998.

[8] Y.Jian, D.A.Winter and M.G.Ishac, "Trajectory of the body COG and COP furing initiation and termination of gait", *Journal of Gait and Posture* , n°1, p9-p22, 1993.

[9] D. Winter, "Biomechanics and motor control of human movement", *Book*, Wiley Edition, 2005

[10] T. Shimba, "An Estimation of center of gravity from force platform data", *Journal of Biomechanics*, n°17, p53-p60, 1984.

[11] D. King and V. Zatiorsky, "Extracting gravity line displacement from stabilographic recordings", *Journal of Gait Posture*, n°6, p27-p38, 1997.

[12] Y. Brenière, "Why we walk the way we do?", *Journal of Motor Behavior*, n°28, p291-p298, 1996.

[13] O.Caron, B. Faure and Y. Brenière, "Estimating the center of gravity of the body on the basis of the center of pressure in standing posture", *Journal of Biomechanics*, n°30, p1169-p117 1, 1997.

[14] A.L. Betker, Z. Moussaviand and T.Szturm, "Center of mass function approximation", *Proccedings of the 26th IEEE EMBS*, p687-p690, 2004.

[15] A.L. Betker, Z. Moussaviand and T.Szturm, "Application of feedward propagation neural network to center of mass estimation for use in clinical environment", *Proccedings of the IEEE EMBS Conference*, p2714-p2717, 2003.

[16] S. Cotton, A.Murray and P.Fraisse, "Estimation of the center of mass using static equivalent serial chain modeling", *Proceedings of the ASME IDETC/CIE Conference*, 2009.

[17] B. Espiau and R. Boulic, "On the Computation and control of the mass center of articulated chains", Research Report INRIA, n°3479, 1998.

[18] S. Cotton, A.Murray and P. Fraisse, "Statically equivalent serial chain for modeling the center of mass of humanoid robots", *Proceedings of the IEEE-RAS Humanoids Conference*, p138-p144, 2008.

[19] Z.Vafa and S.Dubowsky, "The kinematics and dynamics of space manipulators: the virtual manipulator approach", *The International Journal of Robotics Research* , n°9, p3-p21, 1990.

[20] S.Dubowsky and E.Papadopoulos, "The Kinematic, dynamics and control of free-flying and free-floating space robotic systems" , *IEEE Journal – Transactions on Robotics and Automation*, n°9, p531-p543, 1993.

[21] S. Agrawal, G. Gardner and S. Pledgie, "Design and Fabrication of a Gravity Balanced Planar Mechanism Using Auxiliary Parallelograms", *ASME Journal of Mechanical Design*, 2001, vol. 123, n°4, pp. 525-528

[22] B. El Ali, "Contribution à la commande du centre de masse d'un robot bipède", *Thesis*, INPG INRIA, 1999.