A structurally optimal control model for predicting and analyzing human postural coordination

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Abstract

This paper proposes a closed-loop optimal control model predicting changes between in-phase and anti-phase postural coordination during standing and related supra-postural activities. The model allows the evaluation of the influence of body dynamics and balance constraints onto the adoption of postural coordination. For body dynamics, the model minimizes the instantaneous norm of the joint torques with a controller in the head space, in contrast with classical linear optimal models used in the postural literature and defined in joint space. The balance constraint is addressed with an adaptive ankle torque saturation. Numerical simulations showed that the model was able to predict changes between in-phase and anti-phase postural coordination modes and other non-linear transient dynamics phenomena.

1. Introduction

Human stance requires the control of different body segments in a synergetic way. Nashner and McCollum (Nashner and McCollum, 1985) described two preferential postural strategies, i.e. the hip and the ankle strategies, using a popular experimental paradigm based on external postural perturbations. In the ankle strategy, the postural system response is characterized by a large activity and movement of the ankles, whereas the hip strategy

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corresponds to the coordinative activation of hips and ankles with larger movements of the hips. In this context, a classical modelling approach is to assume that humans perform goal-directed movements following certain optimal criteria (He et al., 1991, Kuo, 1995, Tian and He, 1997, Park et al., 2004, Torrence et al., 2008). Kuo (Kuo, 1995), for example, proposed an optimal control model which computes continuous joint feedback responses by explicitly minimizing the quadratic sum of the joint torques, under specific constraints. Most of these approaches used a linearized biomechanical model with an optimal controller defined in the joint space . Actually, the main goal of the Nashner and McCollum paradigm was to restore the joint angle to zero position (vertical position of the body).

Assuming that a minimization of an explicit criterion is required implies a knowledge by the central nervous system (CNS) of complex internal representations of human dynamics and its interaction with the environment. The existence of these internal models is the target of a vivid debate in motor control literature (Mehta and Schaal, 2002, Todorov, 2004). In addition, this concept is unclear as regarding the definition of descriptive variables and the existence of a mixed strategy (Horak and Nashner, 1986, Runge et al., 1999) merging into postural observations which are different from the original strategies.

Based on these observations, Bardy *et al.* (Bardy et al., 2002) proposed that it is not the participation of these different joints per se which determine the organization of the postural system, but rather their coordination. During a visual tracking task, the authors analyzed a collective variable to describe the non-linear postural couplings: the relative phase (Φ) between hips and ankles. In the princeps experiment, standing participants followed in the anteroposterior direction a sinusoidal target with the head. Two coordination modes were observed depending on the target frequency: an in-phase ($\Phi = 0°$) mode for low frequencies and an antiphase ($\Phi = 180°$) mode for high frequencies. This experimentation allowed the observation of non-linear properties of the postural system, such as phase transition, multistability and hysteresis.

Attempts to model postural strategies available so far have used controllers defined in joint space (He et al., 1991, Kuo, 1995, Tian and He, 1997, Park et al., 2004, Torrence et al., 2008). For this reason, they cannot reproduce supra-postural performance such as looking, tracking, or reaching during standing. Only one model (Martin et al., 2006) using a constrained optimization process, based on the minimization of an energetic criterion, was recently proposed to investigate postural coordination (PC) dynamics. The main result was that the location of the center of pressure (CoP) was considered as driving the adoption of coordination modes. However, this model only considered steady state behavior and thus was not able to capture the transient dynamics observed in PC.

Here, we propose a new feedback model able to reproduce some of the most important nonlinear features observed in PC situations. This model is expected to predict *(i)* in-phase and anti-phase postural modes, *(ii)* phase transition between these modes, *(iii)* hysteresis at the transition frequency and *(iv)* to evaluate the influence of environmental (here the length of the support base) and intrinsic (here size and weight of the participant) constraints on postural coordination. Our model has structural features leading to the minimization of the instantaneous norm of the joint torques and guarantees dynamical balance. In addition, the control strategy allows to manage the redundancy between the head and joint space by using the generalized inverse (pseudoinverse) of the Jacobian matrix. This approach was chosen to avoid the classical criticism concerning the complex internal models in motor control litterature.

2. Methods

2.1. Biomechanical model

Human postural dynamics in the anteroposterior plane is represented by a double inverted pendulum (DIP) placed on a triangular foot, with a hip joint and an ankle joint. As shown in Fig. 1 the foot is assumed to be motionless with respect to the support. A simplified representation of the influence of passive muscular viscoelasticity takes the form of passive spring-damping systems at each joint. The inverse dynamics of the body-system is computed with Lagrange's equations:

$$\Gamma = M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) + K_v(\theta - \theta_0) + F_v\dot{\theta} \tag{1}$$

where θ is the vector of ankle (θ_1) and hip (θ_2) angles, and Γ is the vector of joint torques. The passive spring-damping system parameters contributing to restore DIP to the vertical position are in K_v and F_v, respectively the joint stiffness and the viscous friction matrix (Ramos and Stark, 1990). M, C and G are respectively the inertia and Coriolis matrices and the gravity vector of the DIP dynamical model. Balance is described by the position of

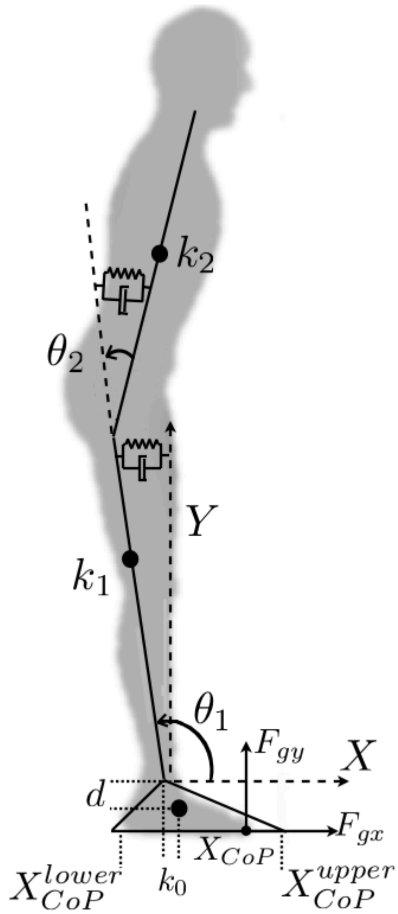

Figure 1: Double inverted pendulum used to model postural coordination. Spring damping components are added at each joint to represent the influence of muscular visocelasticity.

the CoP within the base of support (BoS) in the sagittal plane, which can be expressed as a function of the dynamic parameters (Eq.2):

$$X_{CoP} = \frac{(-\Gamma_1 - F_{gx}d + m_0 k_0 g)}{F_{gy}} \tag{2}$$

where F_{gx} is the horizontal ground reaction force, F_{gy} the vertical one, Γ_1 the ankle torque. m_0 is the mass of the foot, k_0 and d are respectively the distance between the ankle and the foot CoM on X and Y axes (see Fig. 1).

Euler's equations were used for the calculation of the ground reaction forces at each time (Cahouet et al., 2002).

2.2. *Closed-loop modeling of postural coordination*

2.2.1. *Balance constraint*

Balance requires the maintenance of the CoP within the BoS (Pai and Patton, 1997). In equation 2 describing the CoP location, the only control variable is the ankle torque. We propose an adaptive ankle torque saturation to ensure balance constraint. In our case, this saturation is equivalent to a tuning of the ankle response gain depending of the target frequency. Actually, equations 3 and 4 give the ankle torque boundaries as a function of the current joint variables:

$$\Gamma_{1Sat}^{upper} = -F_{gx}(\theta,\dot{\theta},\ddot{\theta})d + m_0 k_0 g - X_{CoP}^{upper} F_{gy}(\theta,\dot{\theta},\ddot{\theta}) \tag{3}$$

where X_{CoP}^{upper} is the upper BoS bound and the lower one is given by:

$$\Gamma_{1Sat}^{lower} = -F_{gx}(\theta,\dot{\theta},\ddot{\theta})d + m_0 k_0 g - X_{CoP}^{lower} F_{gy}(\theta,\dot{\theta},\ddot{\theta}) \tag{4}$$

where X_{CoP}^{lower} is the lower BoS bound.
Note that the use of the saturation loop does not imply the control of instantaneous CoP location, but an instantaneous adaptation of the ankle torque saturation.

2.2.2. *Head tracking task*

To perform the head tracking task in a closed-loop situation, an instantaneous corrective joint torques vector has to be applied to the ankle and hip joints. We propose to use a proportional-derivative (PD) controller, here in the head space, often used in postural modelling (Masani et al., 2006). This PD controller computes a scalar value $\epsilon_X(t)$, which is a linear function of the current error between the head and the target:

$$\epsilon_X(t) = K_p \Delta X_{Head}(t) + K_d \Delta \dot{X}(t)_{Head} \tag{5}$$

where $\Delta X = X_{target}(t) - X_{Head}(t)$ is the tracking error between the target and the horizontal head position, $\Delta\dot{X}(t)$ its derivative, K_p and K_d are the proportional and derivative controller gains in the task space.

The proposed model needs to transform this scalar error into two corrective joint torques, but the actuated system is redundant with respect to the task. Indeed, the CNS needs to manage redundant sets of actuators and sensors to perform the task which sets only the horizontal head position (X axis in Fig. 1). Hence postural control can be assumed as an optimal control problem for the CNS.

To describe the variation of the joint angles in the task space, a classical way is to use the Jacobian matrix, that is the derivative of the direct kinematic model (DKM). The Jacobian matrix of the system is the following:

$$J = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12} \end{bmatrix} \tag{6}$$

where $S_1 = sin(\theta_1)$ and $S_{12} = sin(\theta_1 + \theta_2)$. The trunk length is represented by l_2 and the lower limbs length by l_1.

As already mentioned, previous research (Martin et al., 2006) has shown that the minimization of a dynamic quadratic criterion was able to reproduce some features of the PC. Therefore, we propose in this paper an original approach allowing to reproduce these features by minimizing the instantaneous norm of the torque vector $||\Gamma(t)||_2^2$, under the following linear constraint:

$$\epsilon_X(t) - J\Gamma(t) = 0 \tag{7}$$

The solution of the above well known problem is obtained using the Moore-Penrose pseudoinverse matrix (Penrose, 1955) that provides the minimum-norm solution (Angeles, 2007). The control torque vector is given by:

$$\Gamma(t) = J^+ \epsilon_X(t) \tag{8}$$

With the Jacobian pseudo-inverse vector:

$$J^+ = \begin{bmatrix} \frac{-l_1 S_1 - l_2 S_{12}}{(l_1 S_1 + l_2 S_{12})^2 + l_2^2 S_{12}^2} \\ \\ \frac{-l_2 S_{12}}{(l_1 S_1 + l_2 S_{12})^2 + l_2^2 S_{12}^2} \end{bmatrix} \tag{9}$$

In fact, equation 8 can be read according to two different approaches. On the one hand, from the point of view of robot control theory (Siciliano and Khatib, 2008), equation 8 corresponds to a torque control scheme where joint torques are mapped to joint position errors according to Hooke's law where stiffness matrix is set to an identity matrix. On the other hand, equation 8 can be seen as a pure mathematical object that makes it possible to reproduce the human movement while minimizing the 2-norm of the control vector. Since the objective of our work is the latter, we regard equation 8 as a model that is able to exhibit the different properties of human movement instead of a control scheme for mechanical system. This minimum-norm control vector drives the biomechanical model (Fig 2) to follow the head target under the constraint that dynamical equilibrium is maintained.

The following block diagram takes into account these observations with a non-linear closed-loop model. This model included a DIP with passive spring-damping systems at each joint, a controller in the task space to manage redundancy, and an adaptive ankle torque saturation to ensure balance.

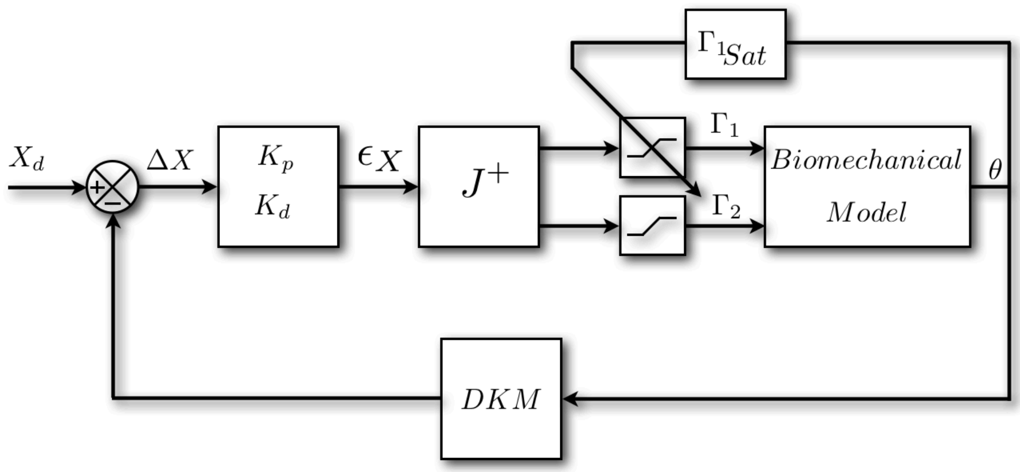

Figure 2: Block diagram of the postural coordination model

2.3. Simulations

Specific input data values were chosen to simulate two different head tracking conditions. In all simulations, parameters were given for a typical subject (height=1.78m; mass=75kg) and the target motion was sinusoidal with 0.1m amplitude. The length of the BoS was 0.1m in agreement with (Martin et al., 2006). The controller gains and the passive spring damping coefficients were constant during the simulation: $K_p = 2000$, $K_d = 2000s$, $K_{v1,2} = 500N.m.rad^{-1}$, $F_{v1,2} = 500N.m.rad^{-1}.s^{-1}$. Note that $K_{v1,2}$ were set in accordance with (Loram and Lakie, 2002) and K_p is a dimensionless coefficient. The other parameters were chosen to allow the reproduction of PC task at all frequencies. The values of the anthropometric parameters were the following: $d = 0.069m$, $l_1 = 0.874m$, $l_2 = 0.836m$, $m_0 = 2.175kg$, $m_1 = 21.97kg$, $m_2 = 50.85kg$, $k_0 = 0.05$, $k_1 = 0.55$, $k_2 = 0.62$, and the inertia were computed by $I_i = m_i(k_i l_i)^2$. In the first simulation, the frequency of the target was increased from 0.1Hz to 0.65Hz by 0.05Hz steps and during

10 periods, reproducing the design employed in human experiments (Bardy et al., 2002). In the second simulation, to further analyze the energetic behavior and task constraint of the PC paradigm, target frequency was up-chirped from 0.2Hz to 0.65Hz, but the dynamical torque saturation could be activated or disabled.

3. Results

3.1. Simulation of the head tracking task

Typical averaged simulation results, in the same conditions than those of the original PC experiment (Bardy et al., 2002), are illustrated in Fig. 3 for several frequency values. The predicted relative phase between ankle

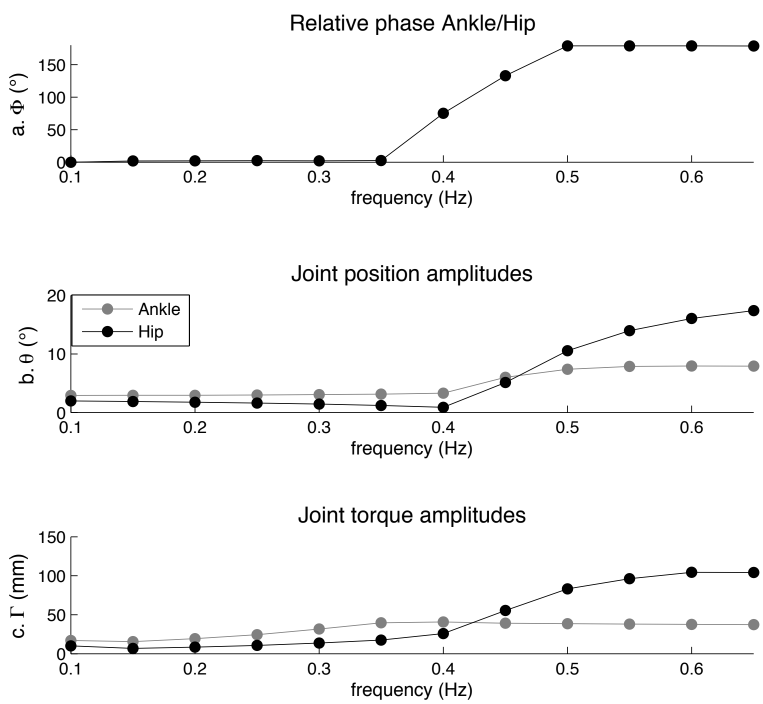

Figure 3: Typical simulation results. (a) Ankle/hip relative phase showing a transition frequency around 0.45Hz (b) Peak-to-peak joint positions. Each point is the mean maximum/minimum value of 10 oscillation periods at a frequency step. Hip position is larger than ankle position for antiphase, and conversely for in-phase (c) Joint torque amplitudes. Ankle torque is larger for in-phase and hip torque is larger for antiphase.

and hip is depicted in Fig. 3a and shows a transition from in-phase to antiphase around 0.45Hz. In-phase coordination, corresponding to $\Phi = 0°$, was obtained for low target motion frequencies (0.1 to 0.45Hz). Antiphase coordination ($\Phi = 180°$) was obtained for higher frequencies (0.5 to 0.65Hz). The simulation showed an increase in hip amplitude for the antiphase mode and an ankle amplitude slightly larger than the hip amplitude for the in-phase mode (Fig. 3b). This is in agreement with the previous measured data (Bardy et al., 2002, Oullier et al., 2002). This observation also holds for ankle and hip torque amplitude (see Fig. 3c). Interestingly, the model is sensitive to body and environmental properties (Fig. 4). In general, the transition frequency increased with increasing the length of the base of support, and decreased with body size and weight.

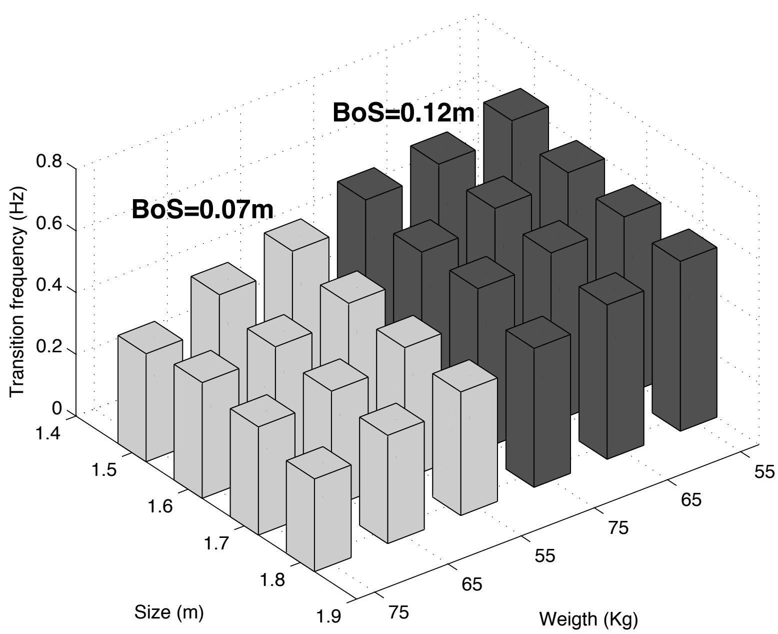

Figure 4: Transition frequencies plotted as a function of BoS length, size and weight of the model.

3.2. Relative phase transition analysis

The results of a continuous variation of target frequency are presented in this subsection. To investigate the balance constraint effect, the adaptive ankle torque saturation was activated and the results are depicted in Fig. 5. The intrinsic dynamics of the DIP itself may have an impact on the adopted coordination mode, and to assess it directly, the adaptive torque saturation was disabled. The corresponding simulation results are given on Fig.6. Fig. 5 shows the Hilbert relative phase on the simulation results, the

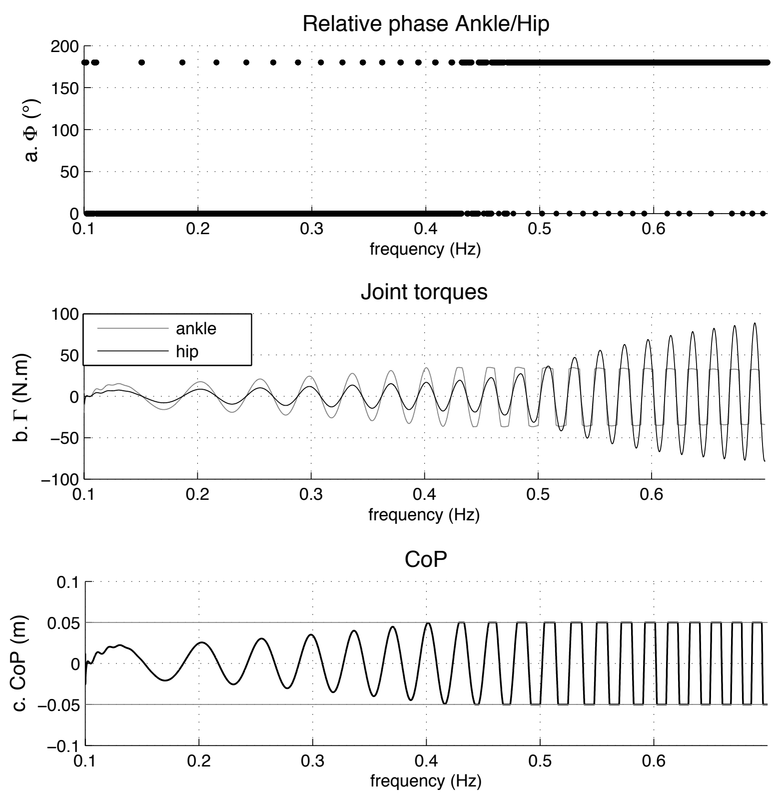

Figure 5: Typical simulation results with an activation of ankle saturation. Ankle/hip relative phase (a), joint torque (b) and CoP motion (c). Transition frequency occurs at 0.42Hz. Hip torque is larger than ankle torque for the antiphase mode. The CoP constraint is able to guide the coordination mode.

joint torques and the CoP location. As illustrated, CoP stays inside the BoS limits (Fig. 5c) and is accompanied by a sudden change from in-phase to antiphase coordination when it reaches BoS limits (Fig. 5a). Finally hip torque is larger than ankle torque (Fig. 5b). Fig. 6 reveals a change in

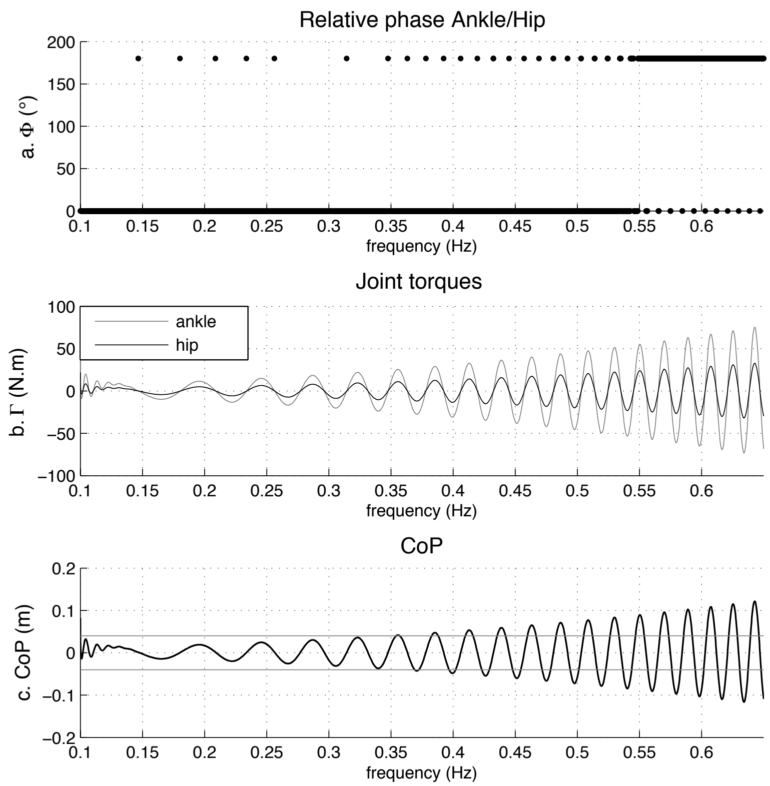

Figure 6: Typical simulation results with the desactivation of ankle saturation. Ankle/hip relative phase (a), joint torque (b) and CoP motion (c). Transition frequency occurs around 0.55Hz. Hip torque is larger than ankle torque for the antiphase mode.

coordination mode even when the CoP constraint is disabled. This change occurs, at a higher target frequency (0.55Hz) compared to Fig. 5 in which the CoP constraint is activated.

3.3. Hysteresis

The model introduced in this article exhibits a hysteresis phenomenon when the target frequency was up-chirped and then down-chirped (Fig. 7). In the PC paradigm, the hysteresis was defined as the tendency for the (postural) system to remain in its current basin of attraction as the control parameter k (target frequency in our case) moves through the transition region, yielding different transition values depending on the direction in which k is changing (i.e., increasing or decreasing); This non-linear behavior was observed in humans (Bardy et al., 2002), and our model (see (Fig. 7)) qualitatively reproduces it. Note that in our model as in humans, in the case of a fixed target frequency (10 cycles at the same frequency), only one coordination mode is adopted that is in agreement with our rate-dependent definition of hysteresis phenomenon. The system stays in the proximity of the attractor during this time of observation, no matter whether it was in-phase when scaling up or in anti-phase when scaling down.

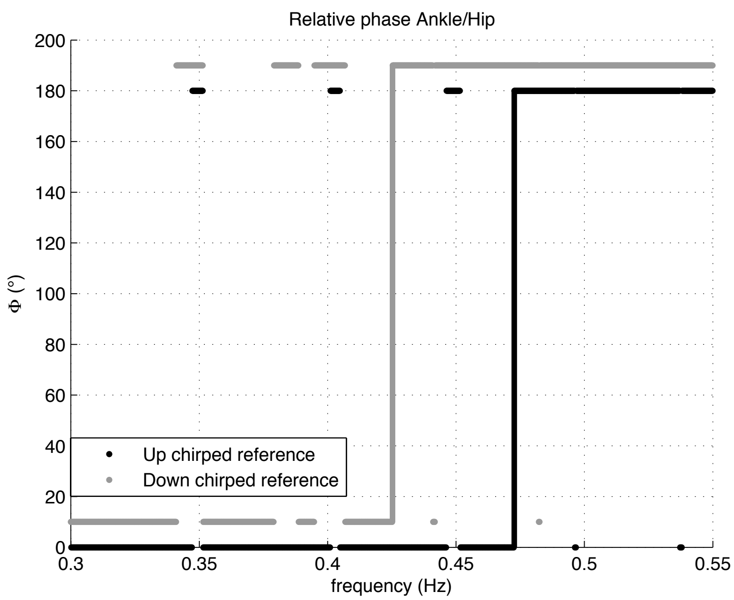

Figure 7: Typical simulation of hysteresis. The relative phase for the up-chirped reference signal (black color) and for the down-chirped reference signal (grey color). Note that the relative phase for the down chirped reference condition is shifted by 10° on the graph.

To obtain this behavior, the dynamical constraint on the equilibrium was disabled, hence the hysteresis phenomenon was only due to the instantaneous minimization of the joint torques.

4. Discussion

We have proposed in this paper a structurally optimal feedback model, which provides realistic predictions of postural sway movements during erected head tracking. Contrary to other phenomenological models (Taga, 1995, Haken et al., 1985), our approach enabled to determine the mechanical parameters involved in postural control. By structure, the model presented here was able to capture transitional non-linear phenomena and hysteresis. Such a result is difficult to obtain with models based on an offline optimization process (Martin et al., 2006). The simulated results shown in Fig. 3 are consistent with observations reported in the literature (Bardy et al., 2002, Oullier et al., 2002).

Examining the simulation results, it appears that the bifurcation emerges from both equilibrium constraint and energy minimization while only the balance constraint was identified by Martin (Martin et al., 2006). Increasing oscillation frequency leads to larger CoP displacement amplitude, which imperils balance. Consequently, the adaptive ankle torque saturation becomes active in the simulation and contributes to the observed phase transition (Fig. 5). To achieve a good tracking performance, and since ankle torque is saturated, the essential part of the motion is produced by the hips (Fig. 5b and 3c). This observation may be connected with the adaptation of the joint gains under task constraints in postural strategies, previously reported in (Park et al., 2004). The hypothesis suggested by Atkeson and Stephens (Atkeson and Stephens, 2007), that the increase in hip torque is the result of a dynamical saturation of the ankle torque, seems to be confirmed. Moreover, when the equilibrium constraint is disabled, or when the BoS is extended, phase transitions continue to exist but are shifted at higher frequencies (Fig. 6). One reason is that the simulated in-phase coordination is less costly at low frequencies than anti-phase coordination in terms of the energetic criterion. The inverse observation holds for higher frequencies.

The sensitivity of the model to modified environmental constraints and subject parameters was explored by manipulating the value of the length of the support base and the size and weight of the subject. In general, an increase of the BOS length resulted in an increase of the estimated transition frequency (Fig. 4). The model also predicted decreasing transition frequency for increased size and weight (Fig. 4). These tendencies are qualitatively in line with the experimental observations of Marin et al. (Marin et al., 1999) and previous modeling (Martin et al., 2006).

Considering the formulation of the control vector, and specially the use of a Jacobian pseudoinverse matrix, our model minimizes the instantaneous joint torque $||\Gamma(t)||_2^2$. The PC experiment supposes a cyclic movement of the body, and since our model works in closed-loop, we can argue that without any perturbation, the minimization due to our controller converges to a steady state behavior. Thus, our model structurally manages the joint redundancy such as an optimal controller. As shown in Figure 7, our model was also able to capture the hysteresis phenomenon, i.e., the delayed frequency transition when target frequency was increased or decreased (see section 3.3 for more explanantions). Hysteresis in our model is a consequence of both the instantaneity of the energetic criterion and the non-linearity of the model. Further versions may examine the energetic cost associated with postural coordination for various values of target frequency around the transition.

The use of a pseudo-inverse matrix is of interest from a human motor control point of view because it avoids the classical criticism regarding the cost for the CNS of several other computational models. In our model the only blocks which could potentially be considered as internal models are the coordinate changing blocks (J^+ and DKM). These blocks relate more to the capacity of neural networks to perform simple geometric transformations of the limb configuration, as proposed earlier by Gurfinkel and Levik (Gurfinkel and Levik, 1979). Such geometrical transformations are generally considered as plausible hypotheses in motor control (Andersen et al., 1993, Kalaska et al., 1997, Green and Angelaki, 2007). Of course, the existence and role of a neural implementation of J^+ and DKM remain open issues in computational neurosciences.

In our model, the feedback loop is not specifically tightened to a particular perceptual modality. In addition to the visual system involved in the tracking task, it is reasonable to assume that the feedback loop integrates information coming from the vestibular system and the somato-sensory system.

A potential extension of the model would be to differentiate the respective contribution of perceptual modalities in the feedback loop, especially if the model is to be applied to pathological situations. Sensor models and their response weighting used by Kuo (1995) are efficient to account for various types of sensory lost. However, the optimal controller in the Kuos model is a Linear Quadratic Regulator in the joint space, and it is not relevant to our coordination situation here defined in the head space. For this reason, direct comparisons between Kuos model and our model are not relevant.

In conclusion, our model seems well suited for the prediction of PC and shows that changes between coordination modes emerge from both balance constraints and energy requirements. Perspectives for future modelling studies include the examination of various intrinsic, environmental, and task constraints, such as those studied on an oscillatory platform for instance

(Buchanan and Horak, 1999), as well as the evaluation of postural relaxation time following an external perturbation.

Conflict of interest statement

All authors have not any financial and personal relationships with other people or organisations that could inappropriately influence (bias) their work.

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