

AlgoB seminar, LIGM
26/05/2015 - Champs-sur-Marne

Finding a tree in a phylogenetic network

Philippe Gambette

Main results: joint work with Andreas Gunawan, Anthony Labarre, Stéphane Vialette and Louxin Zhang.



Outline

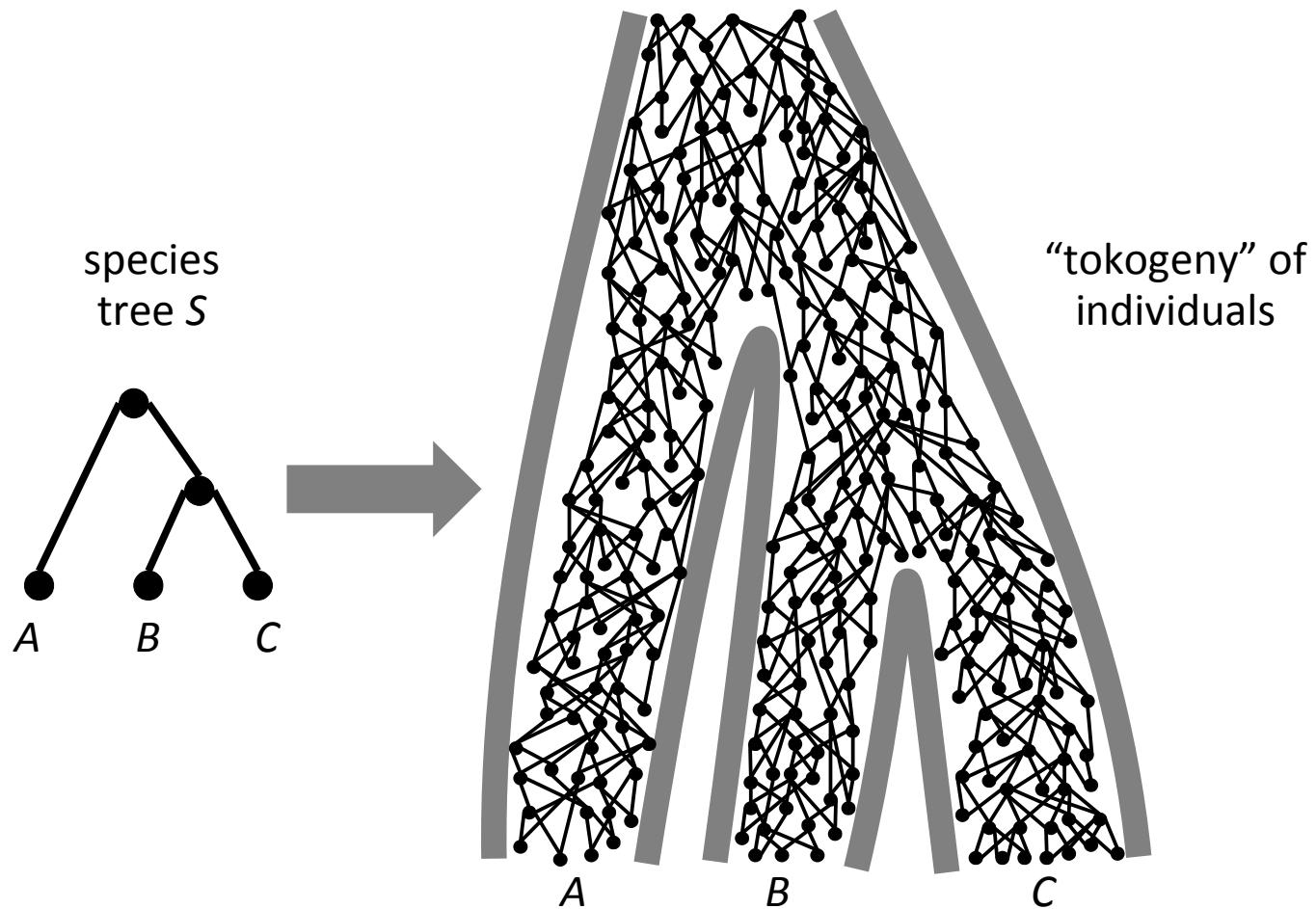
- Phylogenetic networks
- Who is who in phylogenetic networks?
- Classes of phylogenetic networks
- The Tree Containment Problem
- A quadratic-time algorithm on quasi-stable networks

Outline

- Phylogenetic networks
- Who is who in phylogenetic networks?
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Phylogenetic trees

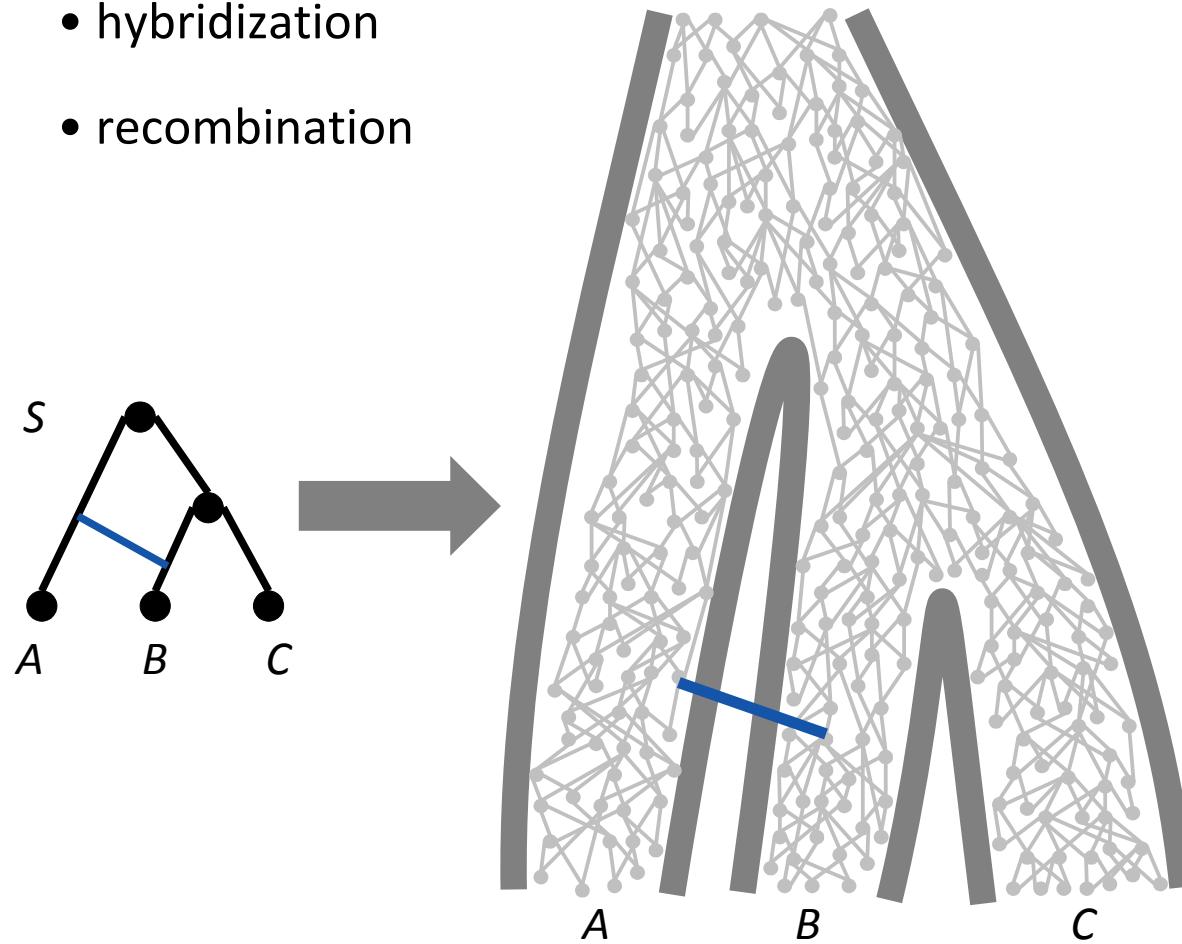
Phylogenetic tree of a set of species



Genetic material transfers

Transfers of genetic material between coexisting species:

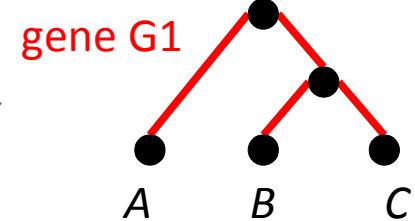
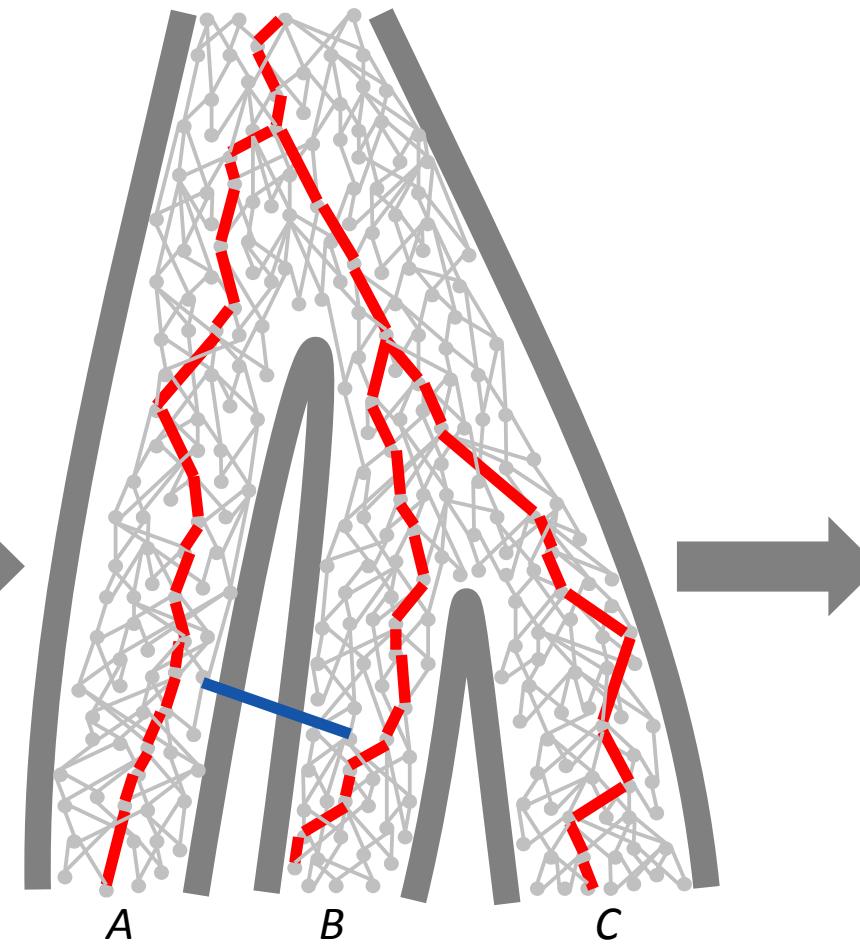
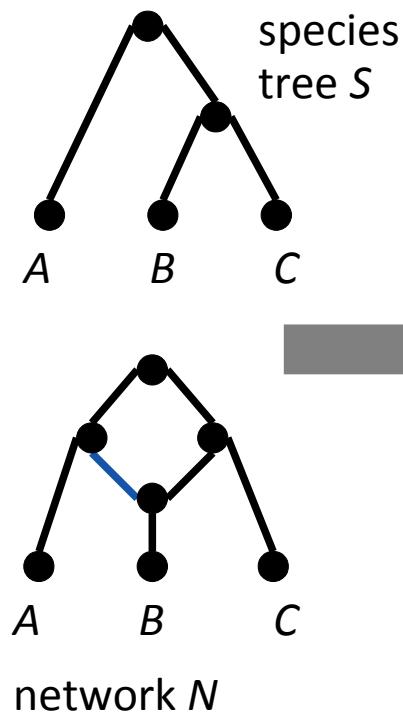
- lateral gene transfer
- hybridization
- recombination



Genetic material transfers

Transfers of genetic material between coexisting species:

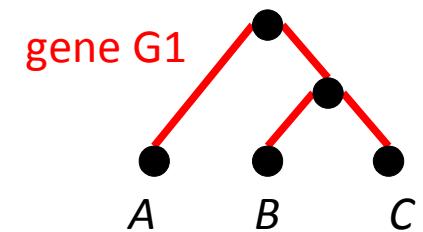
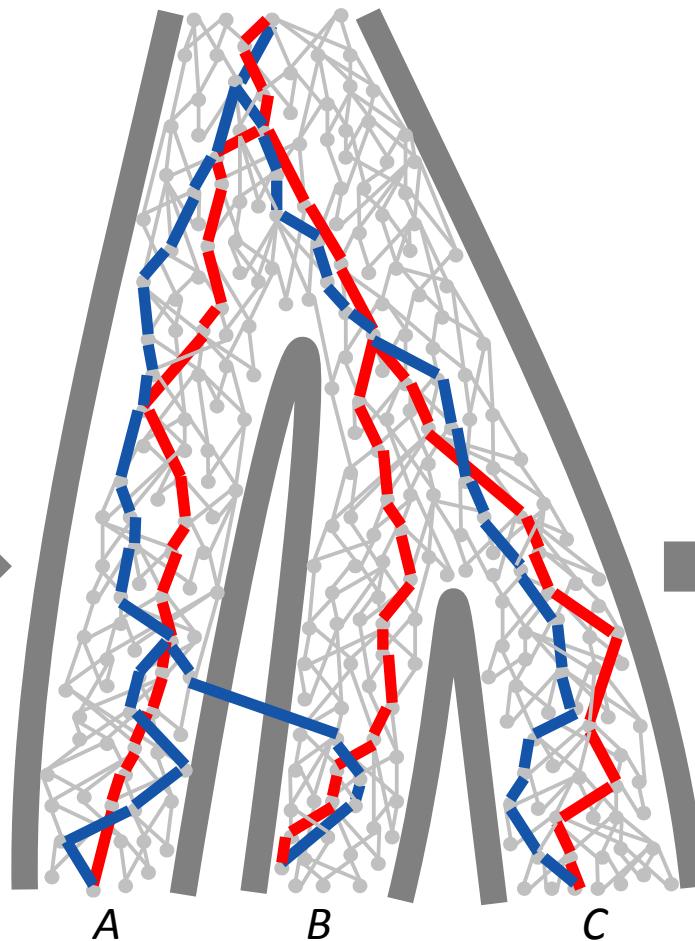
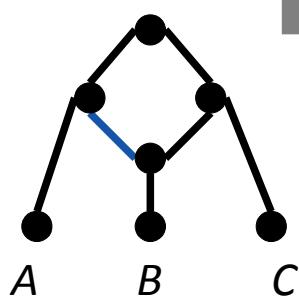
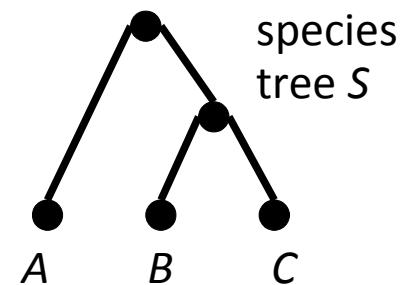
- lateral gene transfer
- hybridization
- recombination



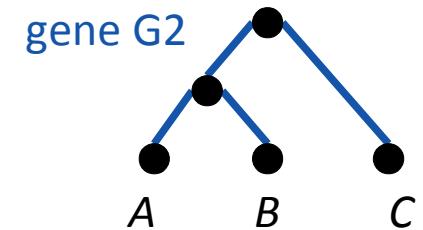
Genetic material transfers

Transfers of genetic material between coexisting species:

- lateral gene transfer
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- recombination



incompatible
gene trees

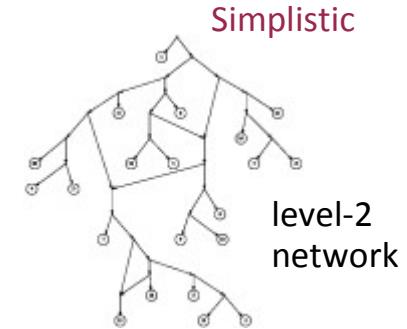
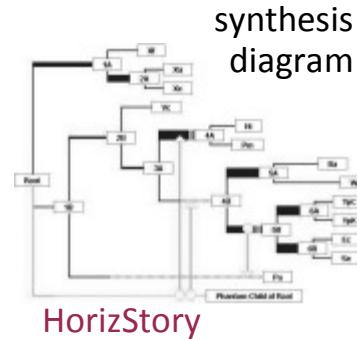
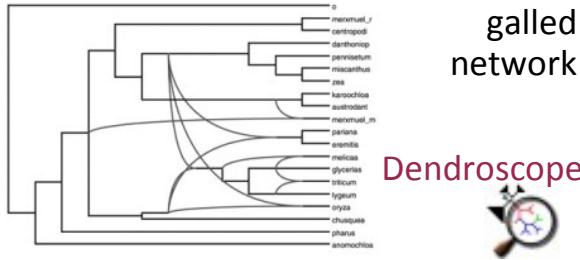


Phylogenetic networks

Phylogenetic network: network representing evolution data

- **explicit** phylogenetic networks

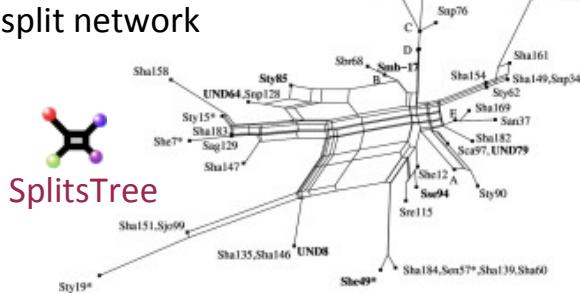
model evolution



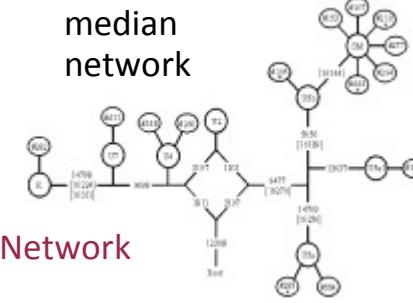
- **abstract** phylogenetic networks

classify, visualize data

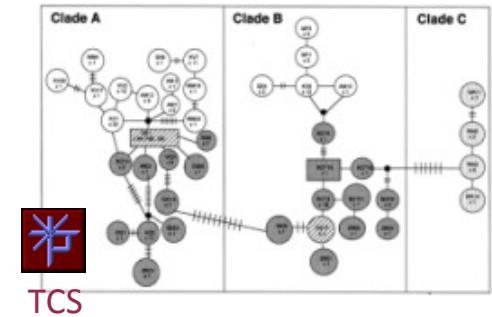
split network



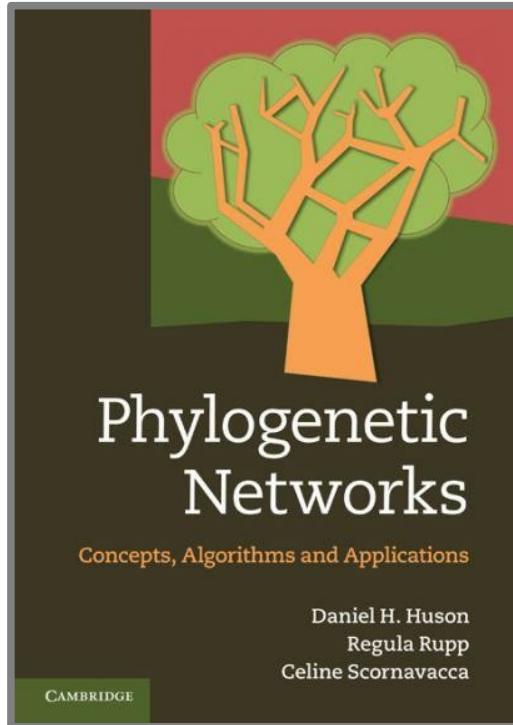
median network



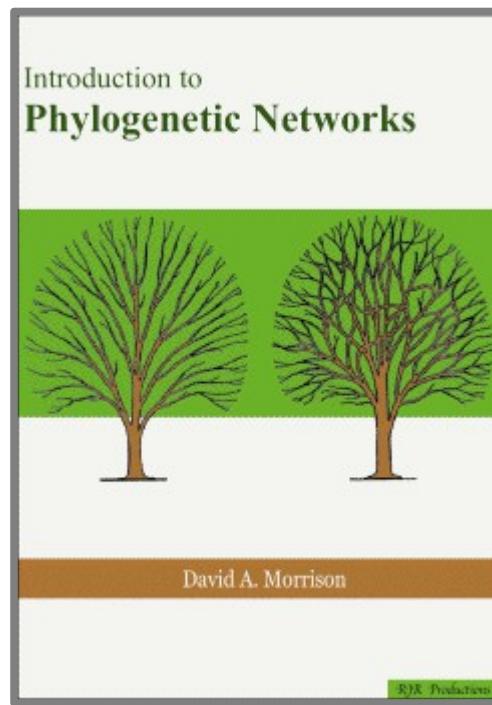
minimum spanning network



Books about phylogenetic networks

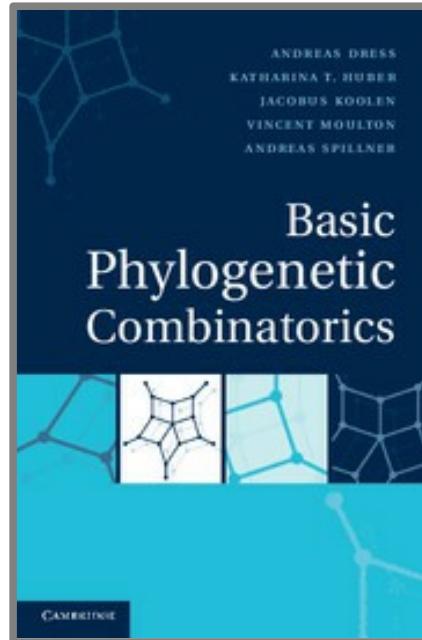


Huson, Rupp,
Scornavacca, 2011

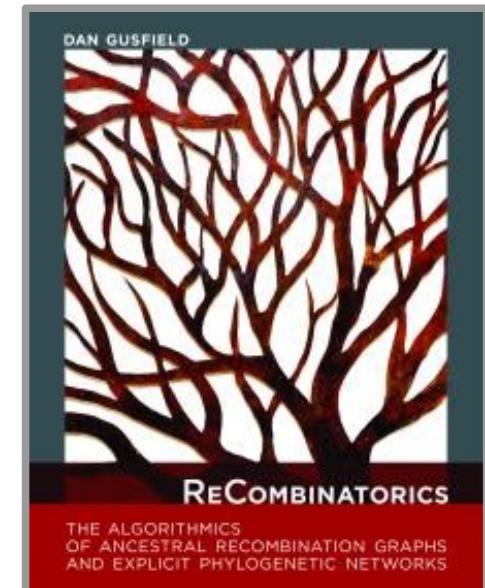


Morrison, 2011

Dress, Huber,
Koolen, Moulton,
Spillner, 2012



Gusfield, 2014



Workshops about phylogenetic networks



*The Future of Phylogenetic Networks, 15-19 October 2012,
Lorentz Center,
Leiden, The Netherlands*

*Utilizing Genealogical
Phylogenetic Networks in
Evolutionary Biology:
Touching the Data,
7-11 July 2014, Lorentz Center,
Leiden, The Netherlands*



*The Phylogenetic Network
Workshop, 27 - 31 Jul 2015,
Institute for Mathematical Science
(National University of Singapore)*

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Who is who in Phylogenetic Networks?

[Who is Who in Phylogenetic Networks - Articles, Authors & Programs](#) [RSS](#)

[Index](#) [Browse](#)
 Search: in All Go (word length ≥ 3) [Contribute!](#) [My selection](#) [Login](#)

Publications - Index (All 448 publications) Selection by: Year | Category | Keyword | Author

Selection by Year
Number of publications per year on phylogenetic networks
Click on a year to display the publications

Year	Number of Publications
1975	1
1980	1
1985	5
1990	3
1995	3
2000	6
2001	5
2002	4
2003	6
2004	17
2005	29
2006	41
2007	34
2008	47
2009	43
2010	47
2011	38
2012	40
2013	30

See how the community working on phylogenetic networks evolved in the last 10 years with the coauthor graphs!

Selection by Category

Category	Count
Article (Journal)	(255)
Book	(3)
Misc	(30)
InProceedings	(104)
PhdThesis	(30)
Programs	(62)
InBook	
Masters	

Selection by Keyword

abstract-network(71) approximation(13) APX-hard(2) ARG(5) bayesian(4)
branch-and-bound(1) cactus-graph(1) characterization(9) circular-split-system(12) consistency(2) cophylogeny(1) distance-between-networks(25) diversity(4) enumeration(4) evaluation(25) **explicit-network(135)** exponential-algorithm(1) continuous-characters(1) from-distances(37) from-multilabeled-tree(9) from-networks(1) **rooted-trees(81)** from-sequences(46) from-species-tree(32) from-triplets(22) from-unrooted-trees(14) galled-network(7) galled-tree(3)

Based on BibAdmin by Sergiu Chelcea
+ tag clouds, date histograms, journal lists,
keyword definitions, co-author graphs

Analysis of the co-author and
keyword graphs: internship of
Tushar Agarwal

Who is Who in
Phylogenetic
Networks, Articles,
Authors &
Programs

[Who is Who in Phylogenetic Networks - Articles, Authors & Programs](#) [RSS](#)

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Programs to compute, evaluate, compare, visualize... **phylogenetic networks**.
This page is automatically built from all publications tagged by Program* in the database.

Program Arlequin
The goal of *Arlequin* is to provide the average user in population genetics with quite a large set of basic methods and statistical tests, in order to extract information on genetic and demographic features of a collection of population samples. In particular, *Arlequin* implements a Minimum Spanning Network algorithm to embed the set of all minimum spanning trees computed from a distance matrix of haplotypes (<http://cmpg.unibe.ch/software/arlequin3/>).
 [5 publications in the database mention Program Arlequin](#)

Program Beagle
Beagle is a small collection of related programs for analysing the minimum number of recombinations required for a SNP data set under the infinite sites model. Available at <http://www.stats.ox.ac.uk/~lyngsoe/beagle/>.
[3 publications in the database mention Program Beagle](#)

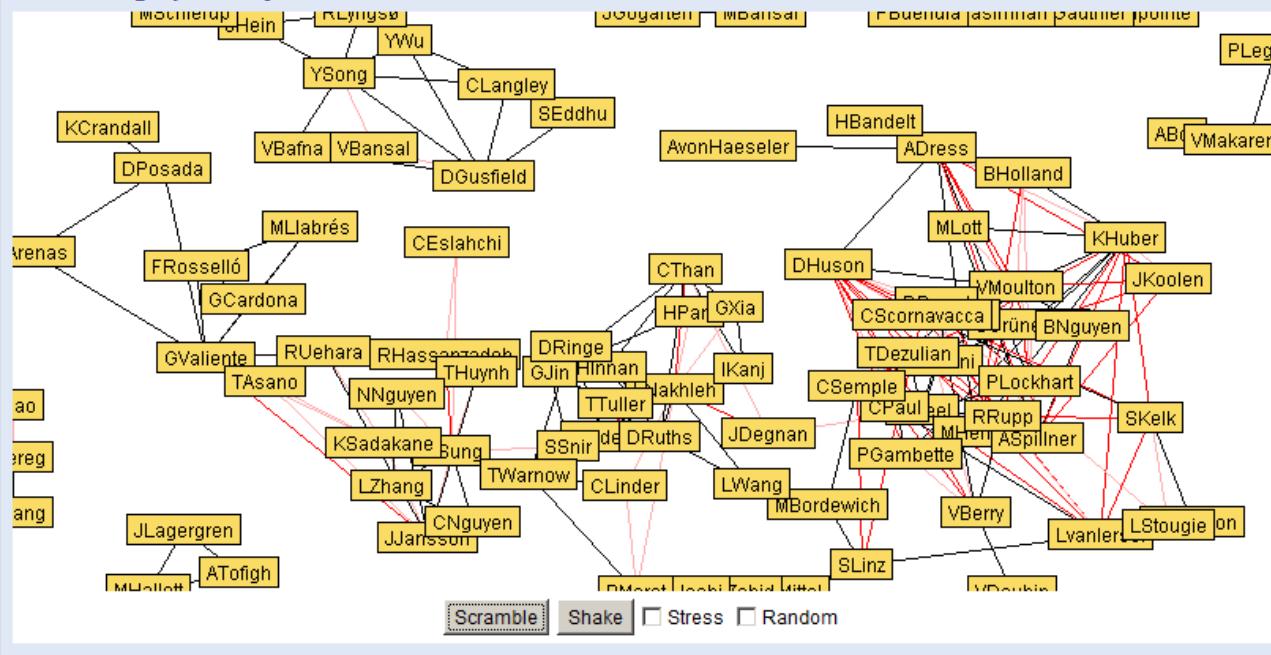
Program Bio PhyloNetwork
Bio-PhyloNetwork is a Perl package that relies on the BioPerl bundle and implements many algorithms on phylogenetic networks (<http://dmi.uib.es/~gcardona/BiolInfo/Bio-PhyloNetwork.tgz>). It is used in a Java Applet which can compare and draw two phylogenetic networks entered in eNewick format with the same set of leaves (<http://dmi.uib.es/~gcardona/BiolInfo/alignment.php>)
[4 publications in the database mention Program Bio PhyloNetwork](#)

Who is who in Phylogenetic Networks?

Coauthor Graphs

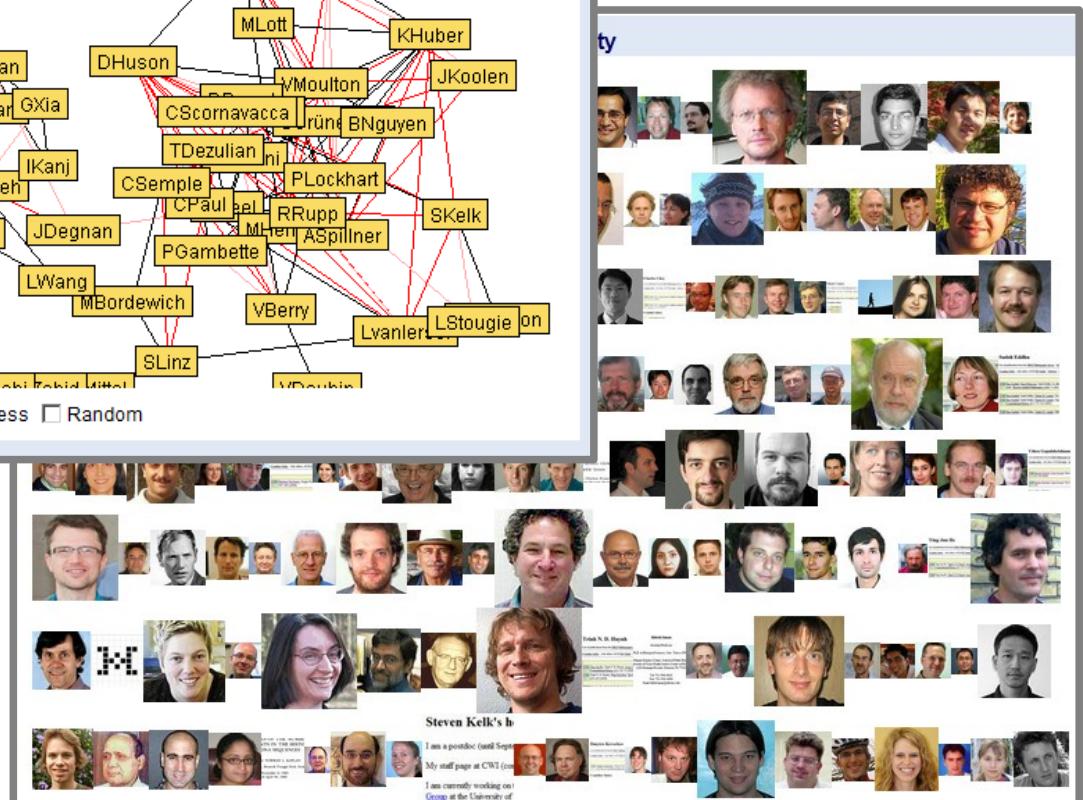
The graphs below link authors of at least two publications who published on phylogenetic networks at least twice together. They show how the community of mathematicians and computer scientists working on phylogenetic networks evolved and structured itself over the last 10 years.

Coauthor graph today

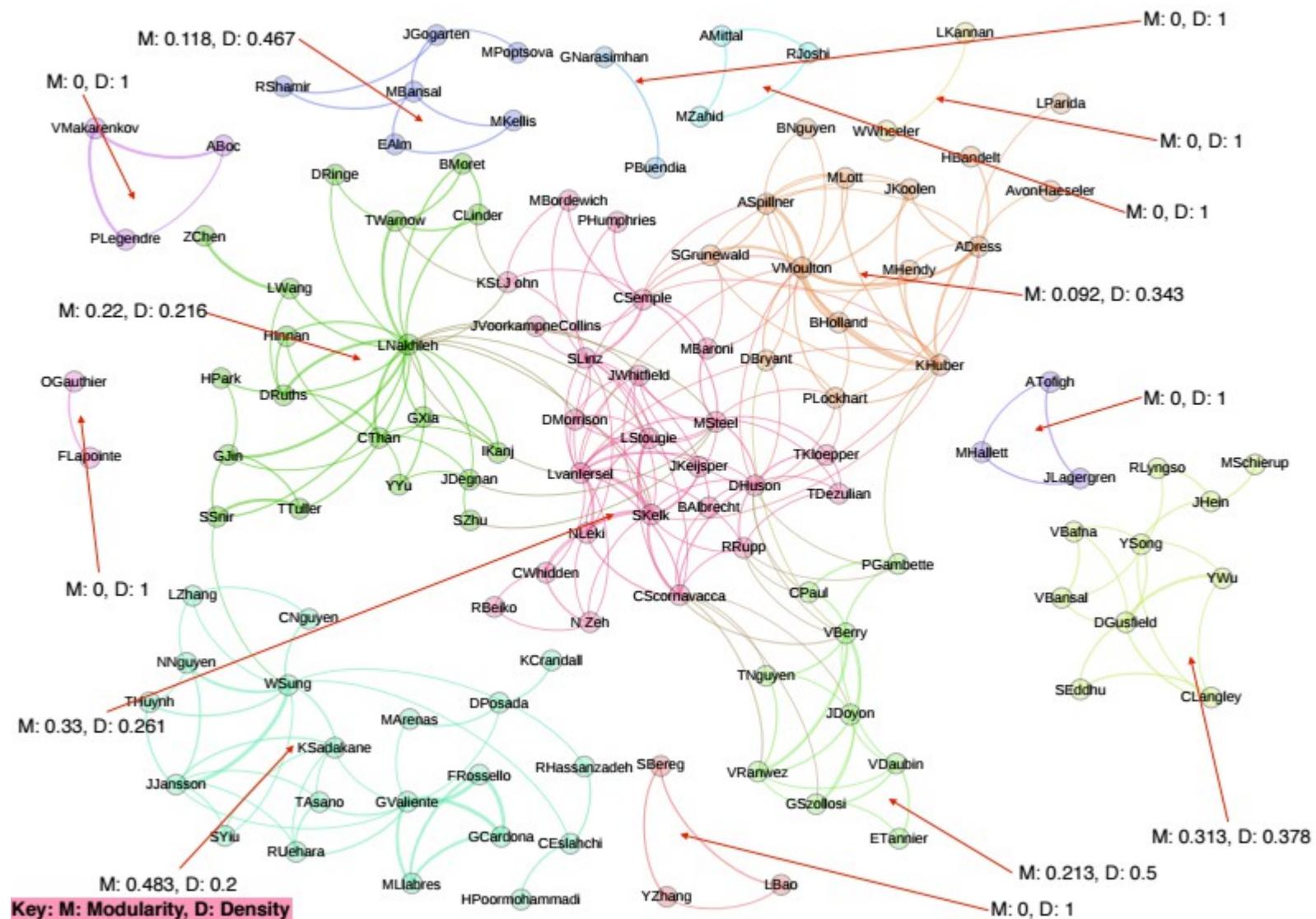


Who is Who in Phylogenetic Networks, Articles, Authors & Programs

Analysis of the co-author and keyword graphs: internship of Tushar Agarwal



Who is who in Phylogenetic Networks?



Gephi visualization of the coauthor network of main authors by Tushar

Who is who in Phylogenetic Networks?

Who is Who in Phylogenetic Networks - Articles, Authors & Programs

Index | Browse | Search: _____ in All | Go (word length ≥ 3) | RSS | Contribute! | My selection | Login

Publications of Daniel H. Huson   

Order by: Type | Year

Associated keywords

abstract-network circular-split-system consensus **explicit-network** FPT from-clusters from-distances from-network from-rooted-trees from-sequences **from-splits** from-trees from-unrooted-trees galled-network galled-tree heuristic hybridization level-k-phylogenetic-network minimum-number NP-complete **phylogenetic-network** **phylogeny** polynomial Program-Beagle **Program-Dendroscope** Program-HybridInterleave Program-HybridNumber Program-Spectronet **Program-SplitsTree** Program-SPNet recombination **reconstruction** software split split-decomposition **split-network** supernet survey tanglegram **visualization**

<< 2012 >>

1 

[Benjamin Albrecht](#), [Celine Scornavacca](#), [Alberto Cenci](#) and [Daniel H. Huson](#). Fast computation of minimum hybridization networks. In *Bioinformatics*, Vol. 28(2):191–197, 2012. [Comment]  

Keywords: explicit network, from rooted trees, minimum number, phylogenetic network, phylogeny, Program Dendroscope, reconstruction. Note: <http://dx.doi.org/10.1093/bioinformatics/btr618>.

<< 2011 >>

2 

Basé sur BibAdmin
par Sergiu Chelcea
+ nuages de mots, histogramme
des dates, liste des journaux,
graphes de co-auteurs,
définition des mots-clés.

Who is Who in
Phylogenetic
Networks, **Articles**,
Authors &
Programs

munity



Steven Kelk's bio

I am a postdoc (until Sept 2013). My staff page at CWI (one of the top 50 mathematics research institutions in the world). I am currently working on  Group at the University of

<http://phylnet.univ-mlv.fr>

Who is who in Phylogenetic Networks?

abstract-network(88) agreement-forest(29) approximation(18) APX-hard(4) ARG(6) bayesian(4) block-realization(1) bootstrap(4) bound(4) branch-and-bound(1) cactus-graph(1) characterization(11) circular-split-system(13) clustering(3) coalescent(10) consensus(8) consistency(2) cophylogeny(1) counting(3) database(1) distance-between-networks(30) diversity(5) duplication(33) dynamic-programming(10) enumeration(4) evaluation(25) **explicit-network(191)** exponential-algorithm(6) FPT(31) from-clusters(16) from-continuous-characters(1) from-distances(45) from-gene-order(1) from-multilabeled-tree(9) from-network(39) from-NGS-data(1) from-quartets(16) **from-rooted-trees(122)** from-sequences(51) from-species-tree(46) from-splits(13) from-trees(7) from-trinets(2) from-triplets(24) from-unrooted-trees(16) galled-network(7) galled-tree(38) generation(12) haplotype-network(2) haplotyping(1) heuristic(32) HMM(2) hybridization(52) inapproximability(5) integer-linear-programming(3) isomorphism(3) k-reticulated(1) kernelization(2) labeling(4) lateral-gene-transfer(56) level-k-phylogenetic-network(30) likelihood(16) lineage-sorting(10) loss(16) MASN(4) median-network(16) MedianJoining(2) Minimal-lateral-network(1) minimum-contradiction(2) **minimum-number(33)** minimum-spanning-network(2) model-selection(2) mu-distance(2) NeighborNet(14) nested-network(2) netting(3) normal-network(9) NP-complete(33) optimal-realization(3) parsimony(39) pedigree(1) perfect(5) **phylogenetic-network(402)** **phylogeny(387)** **polynomial(70)** population-genetics(5) Program-AdmixTools(1) Program-ALE(1) Program-Angst(2) Program-Arlequin(5) Program-Beagle(3) Program-BIMLR(1) Program-Bio-PhyloNetwork(4) Program-Clustistic(2) Program-CMPT(1) Program-CombineTrees(2) Program-ConsensusNetwork(1) Program-constNJ(1) Program-CycleKiller(3) **Program-Dendroscope(14)** Program-EEEP(3) Program-FastHN(1) Program-FlatNJ(1) Program-Fylogenetica(2) Program-GalledTree(1) Program-GraphDTL(2) Program-HapBound(1) Program-HGT_simul(1) Program-HiDe(1) Program-HorizStory(2) Program-Hybrid-Lambda(1) Program-HybridInterleave(5) Program-HybridNET(2) Program-HybridNumber(3) Program-Hybroscale(3) Program-JML(1) Program-LatTrans(5) Program-LEV1ATHAN(1) Program-Lev1Generator(1) Program-Level2(2) Program-lingpy(1) Program-LNetwork(2) Program-MaaF(2) Program-Marlon(3) Program-MC-Net(1) Program-McKitTsche(1) **Program-Mowgli(5)** Program-MowgliNNI(2) Program-MPNet(1) Program-MY-CLOSURE(1) **Program-Nepal(7)** Program-NetGen(3) Program-NetTest(1) Program-NetView(1) **Program-Network(5)** Program-Notung(1) **Program-PADRE(7)** Program-Phangorn(2) Program-PhippsNetwork(2) **Program-PhyloNet(11)** Program-PhyloNet-HMM(1) Program-PIRN(3) Program-Prunier(2) Program-Pyramids(3) Program-QNet(4) Program-Quartet(1) Program-Quartet-Decomposition(1) Program-QuartetMethods(1) Program-QuartetNet(2) Program-QuasiDec(1) Program-QuickCass(1) Program-RANGER-DTL(4) Program-RecMin(1) Program-Reodon(3) Program-RecPars(1) Program-Reticlad(2) Program-SAGE(2) Program-SAQ-Net(1) Program-Serial-NetEvolve(1) Program-SHRUB(3) Program-Simplistic(3) Program-Sliding-MinPD(1) Program-SNSA(2) Program-Spectronet(4) **Program-SplitsTree(36)** Program-SPNet(5) Program-SPRDist(1) Program-SuperQ(1) **Program-T-REX(13)** Program-TCS(8) Program-TERA(2) Program-TerminusEst(2) Program-TreeFix-DTL(1) Program-TreeMix(1) Program-Treeevolve(2) Program-TripNet(2) Program-ultra-Net(1) Program-Ultranet(1) Program-WeakHierarchies(2) Program-Xscape(1) pyramid(8) quasi-median-network(3) realization(4) **recombination(29)** recombination-detection(4) **reconstruction(255)** regular-network(7) reticulogram(10) serial-evolutionary-networks(1) simulated-annealing(4) simulation(5) site-consistency(1) **software(66)** split(27) split-decomposition(13) split-network(54) SPR-distance(13) spread(2) statistical-model(32) statistical-parsimony(3) supernetwork(4) survey(31) tanglegram(1) time-consistent-network(12) tree-child-network(15) tree-sibling-network(10) tripartition-distance(9) triplet-distance(3) unicyclic-network(3) visualization(30) weak-hierarchy(8) weakly-compatible(3)

Who is who in Phylogenetic Networks?

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hierarchy(8) weakly-compatible(3)

input

software

Who is who in Phylogenetic Networks?



problems

algorithmic properties

input

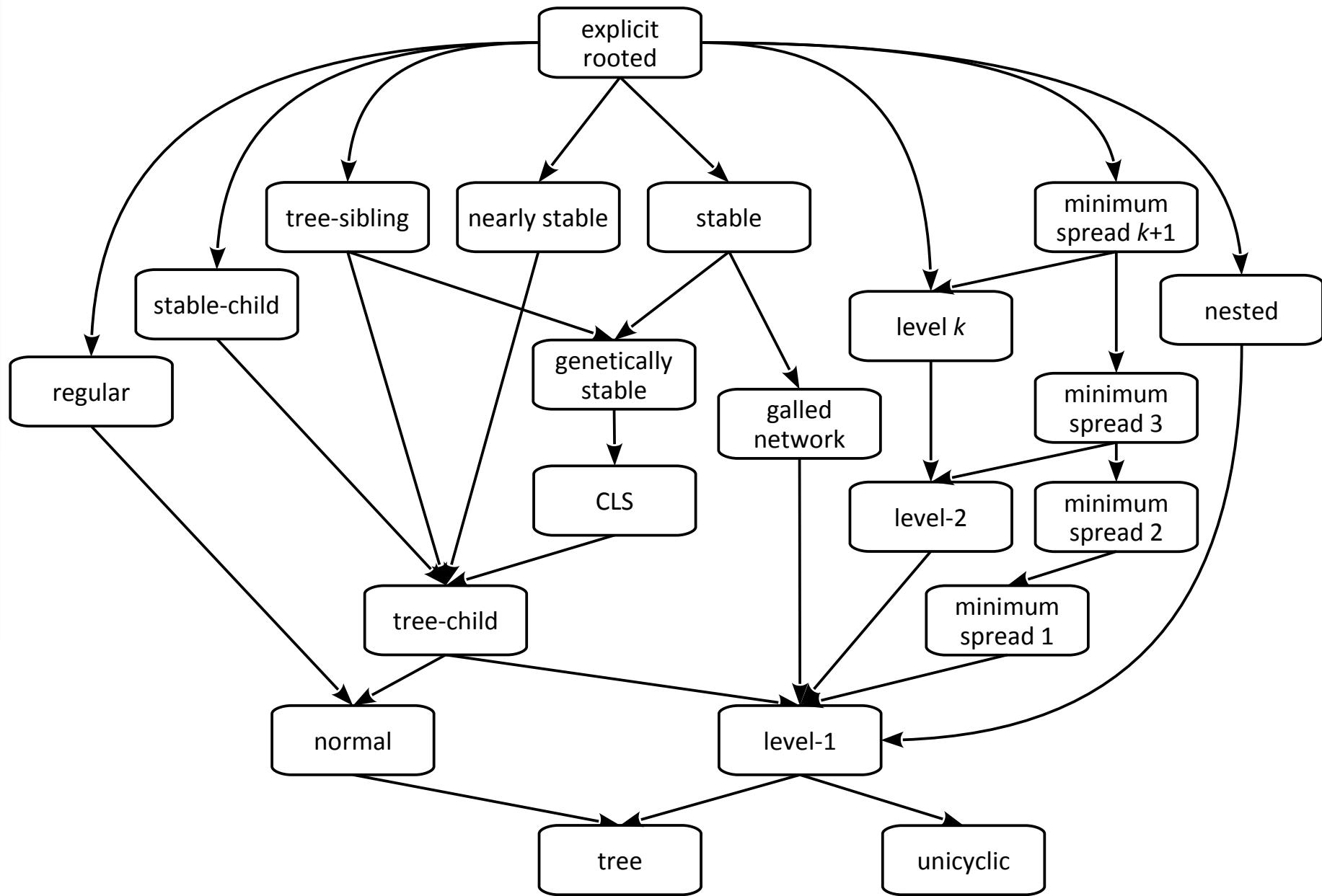
software

classes

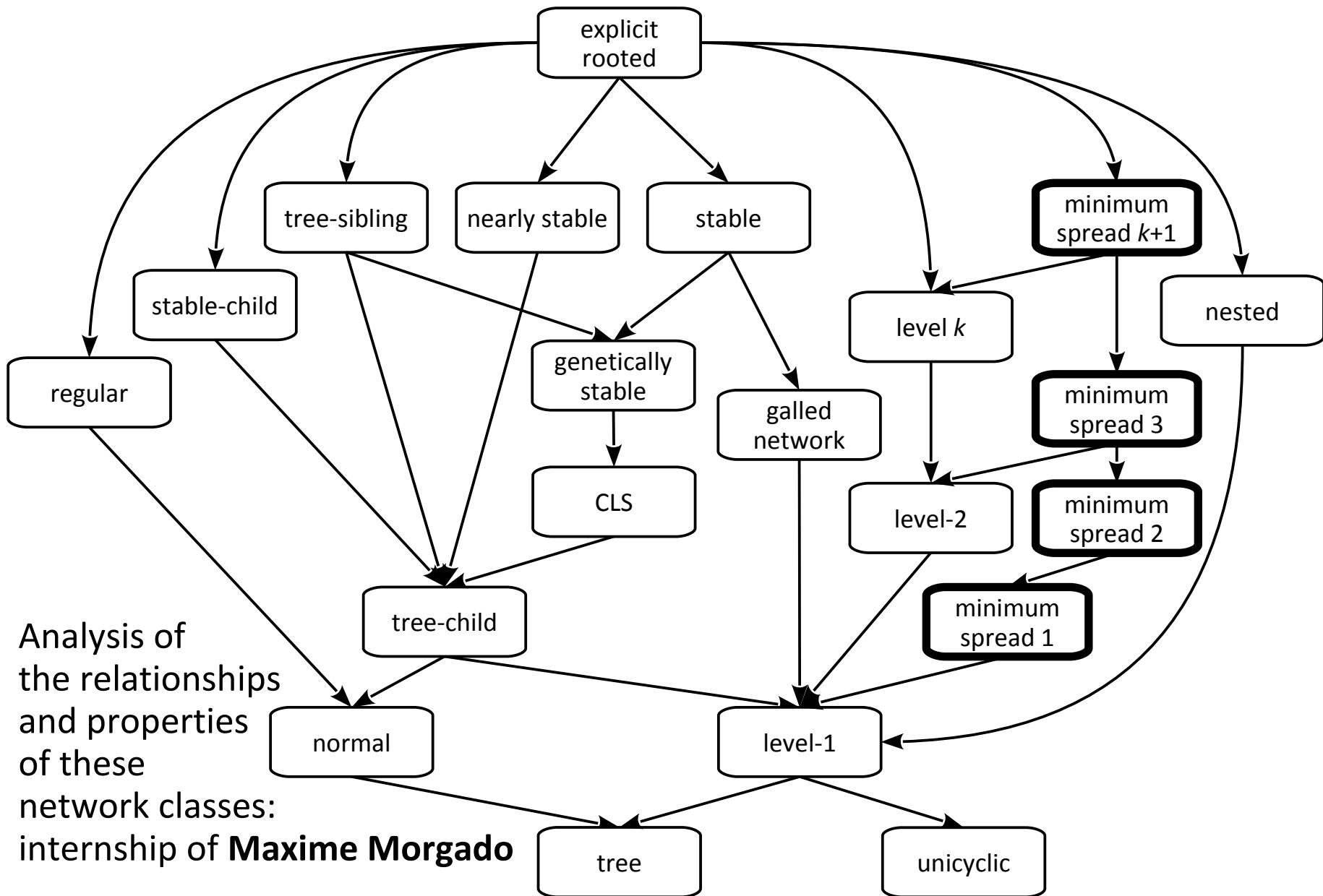
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Classes of Phylogenetic Networks



Classes of Phylogenetic Networks



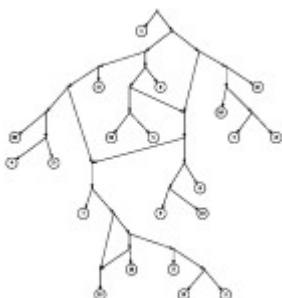
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Phylogenetic network reconstruction

```
espèce 1 : AATTGCAG TAGCCCAAAAT  
espèce 2 : ACCTGCAG TAGACCAAT  
espèce 3 : GCTTGCAG TAGACAAGAAT  
espèce 4 : ATTTCAG AAGACCAAAT  
espèce 5 : TAGACAAGAAT  
espèce 6 : ACTTGAG TAGCACAAAAAT  
espèce 7 : ACTGGTG TAAAAT
```

G1 G2



{gene sequences}

distance methods

Bandelt & Dress 1992 - Legendre & Makarenkov 2000 -
Bryant & Moulton 2002 - Chan, Jansson, Lam & Yiu 2006

parsimony methods

Hein 1990 - Kececioglu & Gusfield 1994 - Jin, Nakhleh,
Snir, Tuller 2009 - Park, Jin & Nakhleh 2010 - Kannan &
Wheeler, 2012

likelihood methods

Snir & Tuller 2009 - Jin, Nakhleh, Snir, Tuller 2009 -
Velasco & Sober 2009 - Meng & Kubatko 2009

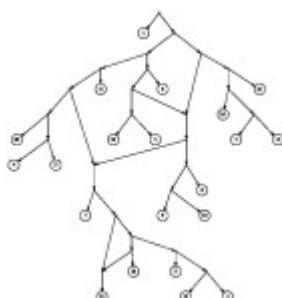
network N

Phylogenetic network reconstruction

Problem: methods are usually slow,
especially with rapidly increasing sequence length.

```
espèce 1 : AATTGCAG TAGCCCAAAAT  
espèce 2 : ACCTGCAG TAGACCAAT  
espèce 3 : GCTTGCAG TAGACAAGAAT  
espèce 4 : ATTTCAG AAGACCAAAT  
espèce 5 : TAGACAAGAAT  
espèce 6 : ACTTGCAG TAGCACAAAAAT  
espèce 7 : ACTTGGTG TAAAAT
```

G1 G2



{gene sequences}

distance methods

Bandelt & Dress 1992 - Legendre & Makarenkov 2000 -
Bryant & Moulton 2002 - Chan, Jansson, Lam & Yiu 2006

parsimony methods

Hein 1990 - Kececioglu & Gusfield 1994 - Jin, Nakhleh, Snir, Tuller 2009 - Park, Jin & Nakhleh 2010 - Kannan & Wheeler, 2012

likelihood methods

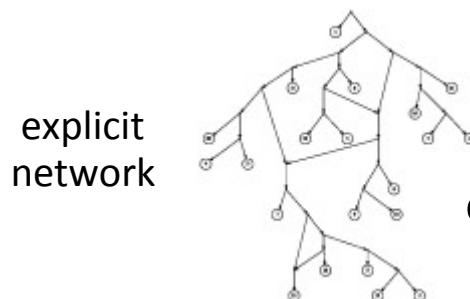
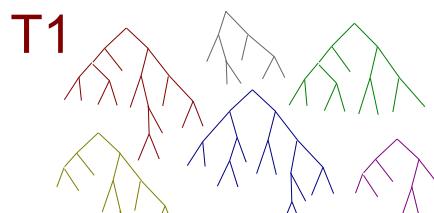
Snir & Tuller 2009 - Jin, Nakhleh, Snir, Tuller 2009 -
Velasco & Sober 2009 - Meng & Kubatko 2009

network N

Phylogenetic network reconstruction

espèce 1 : AATTGCAG TAGCCCAAAAT
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{gene sequences}

Reconstruction of a tree for each gene present in several species

Guindon & Gascuel, SB, 2003

{trees}

HOGENOM Database

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Tree reconciliation or consensus

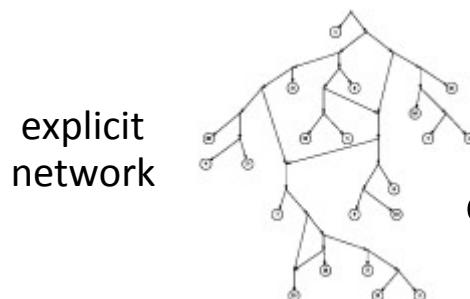
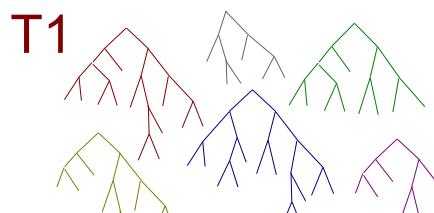
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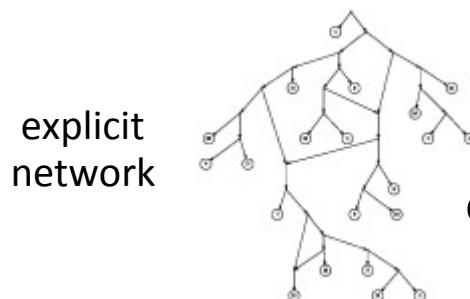
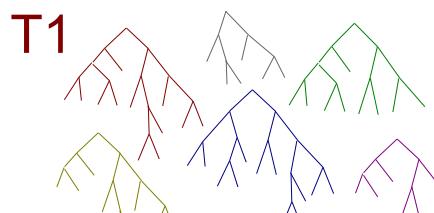
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Tree reconciliation or consensus

Tree Containment Problem

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The Tree Containment Problem (T.C.P.)

Input: A binary phylogenetic network N and a tree T over the same set of taxa.

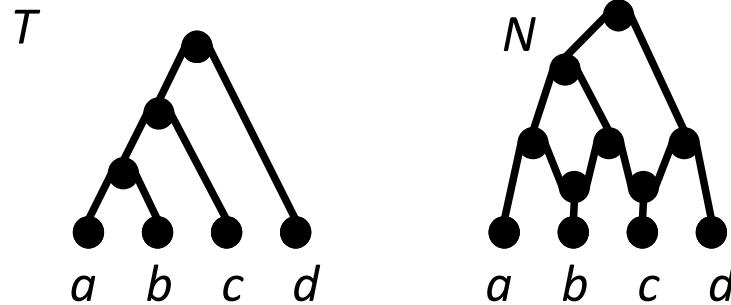
Question: Does N display T ?

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→ Can we remove one incoming arc, for each vertex with >1 parent in N , such that the obtained tree is equivalent to T ?

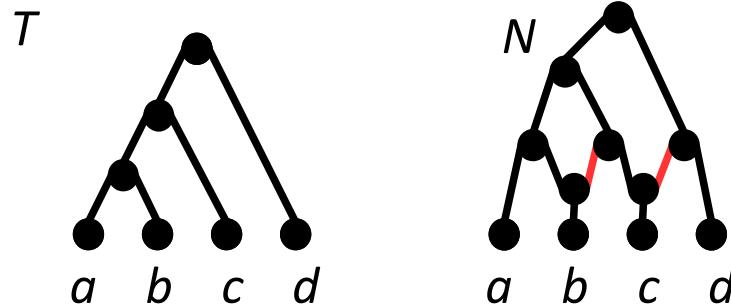


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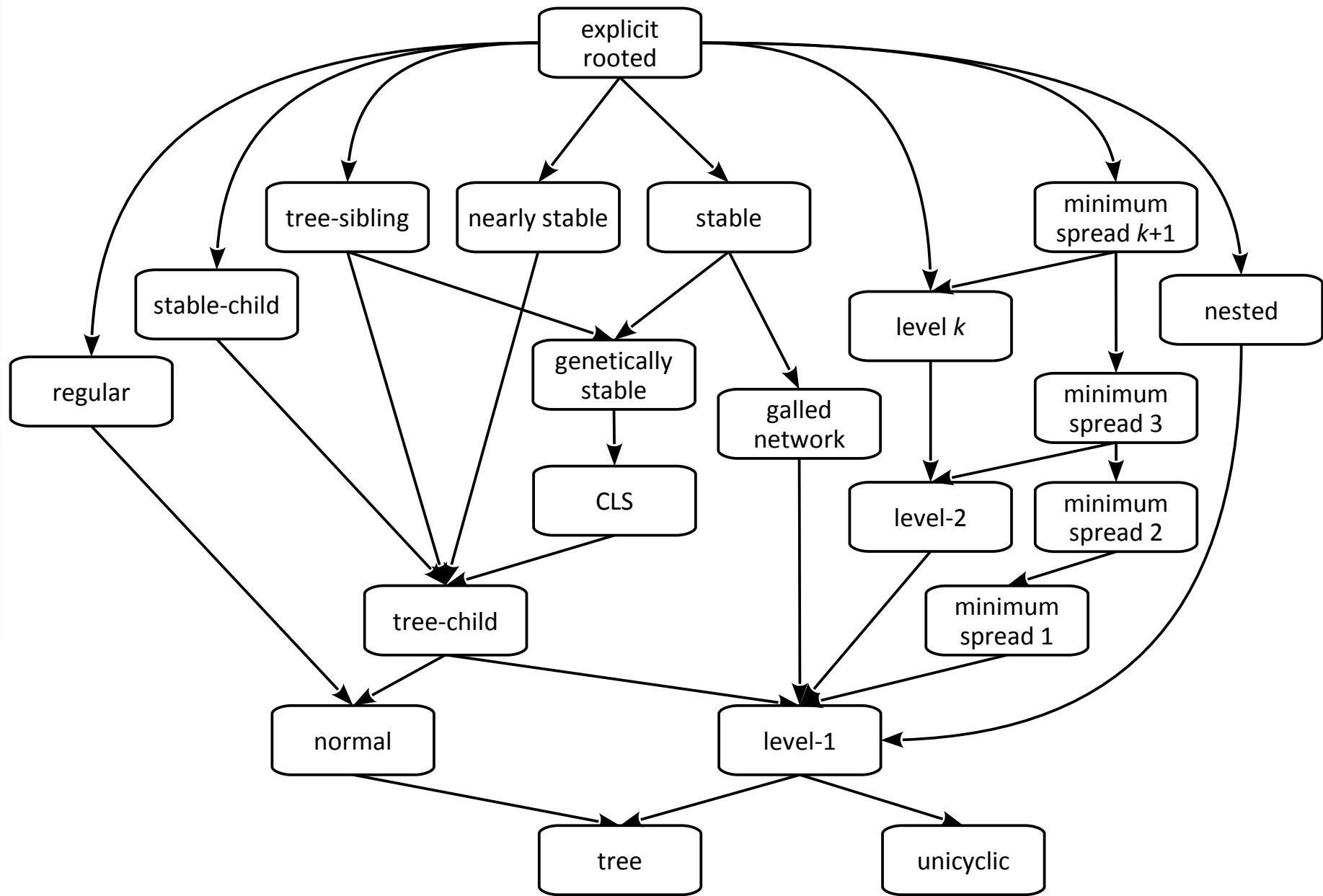
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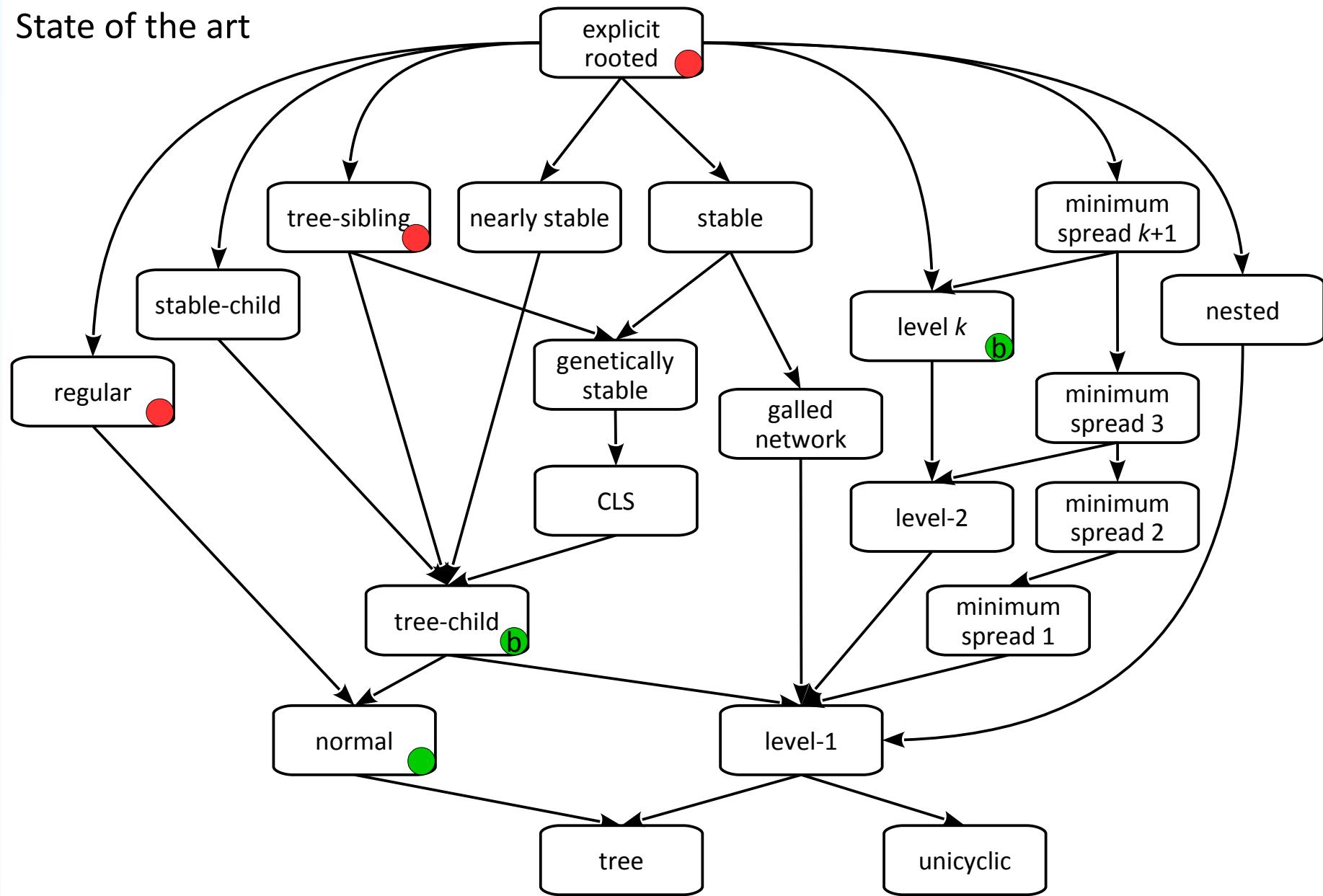
- **NP-complete** in general (Kanj, Nakhleh, Than & Xia, 2008)
- **NP-complete** for tree-sibling, time-consistent, regular networks (Iersel, Semple & Steel, 2010)
- **Polynomial-time solvable** for normal networks, for binary tree-child networks, and for level-k networks (Iersel, Semple & Steel, 2010)

Classes of Phylogenetic Networks and the T.C.P.



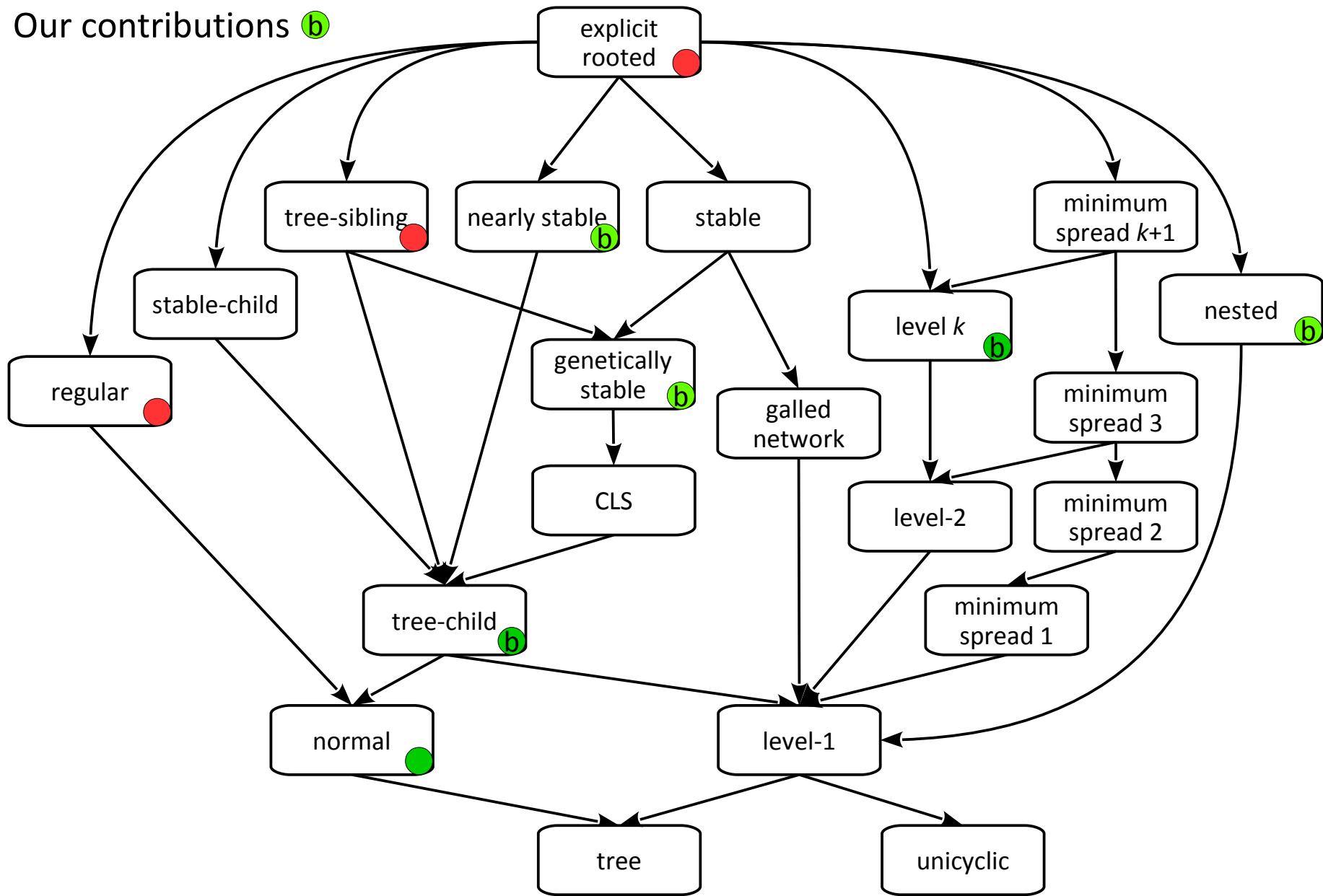
Classes of Phylogenetic Networks and the T.C.P.

State of the art



Classes of Phylogenetic Networks and the T.C.P.

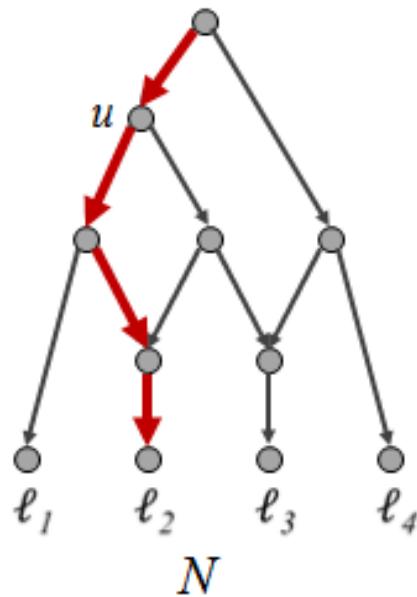
Our contributions 



Outline

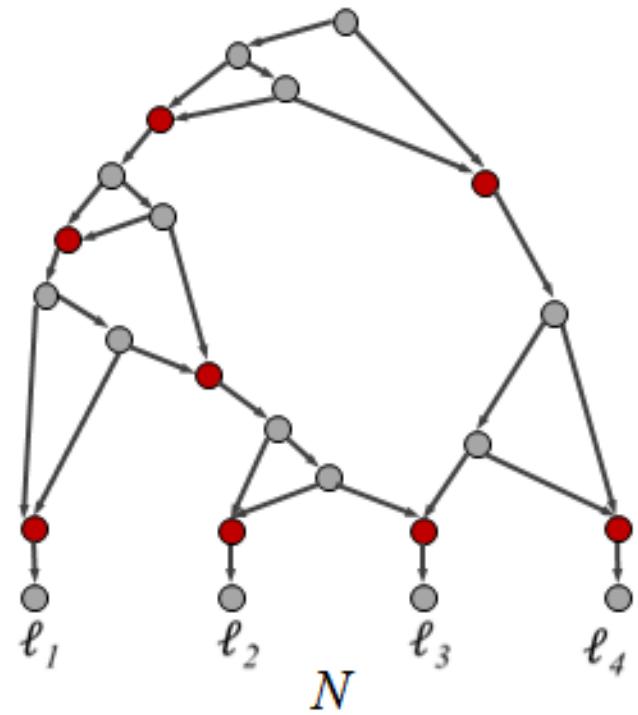
- Phylogenetic networks
- Who is who in phylogenetic networks?
- Classes of phylogenetic networks
- The Tree Containment Problem
- A quadratic-time algorithm on quasi-stable networks

Stable and nearly-stable networks



A vertex u is **stable** if there exists a leaf l such that all paths from the root to l go through u .

A phylogenetic network is **stable** if every reticulation vertex is stable.



A phylogenetic network is **nearly-stable** if for each vertex, either it is stable or its parents are.

Strategy to get a quadratic time algorithm for T.C.P.

Given N , a phylogenetic network with n leaves and the input tree T of the T.C.P.

Theorem 2: If N is nearly-stable then $\#\{\text{reticulation vertices of } N\} \leq 12(n-1)$

Theorem 3: Considering a longest path in N , and following a case analysis:

- either realize that T is not contained in N
- or build a network N' with less arcs than N such that T contained in N if and only if T contained in N'

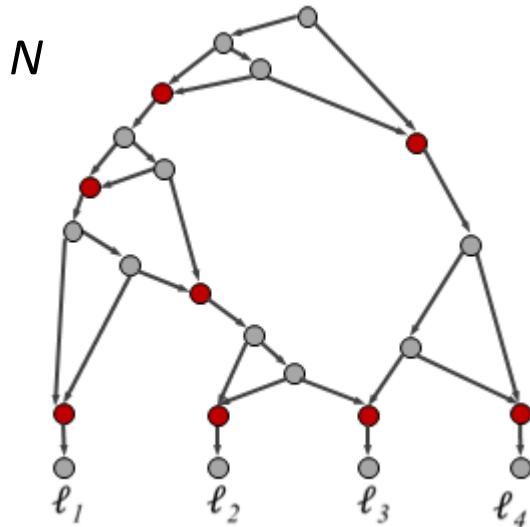
Number of reticulations of a stable network

Decompose N into **$2n-2$ paths**:

- remove one reticulation arc per reticulation, ensuring we get no « dummy leaf », to get a tree T with n leaves
- summarize T into a rooted binary tree T' with n leaves... and $2n-2$ arcs

We can prove (technical) that:

each path contains at most 2 reticulation vertices



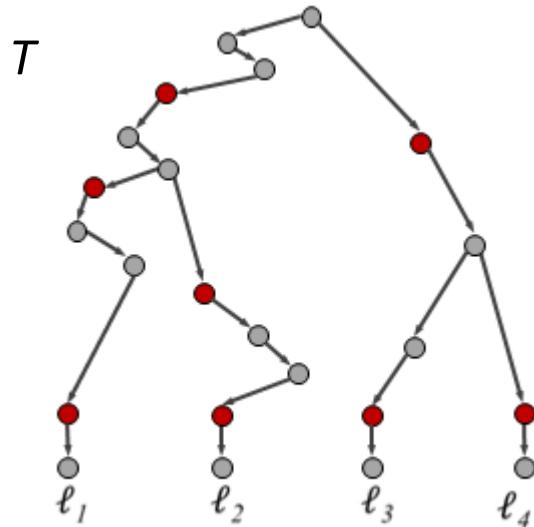
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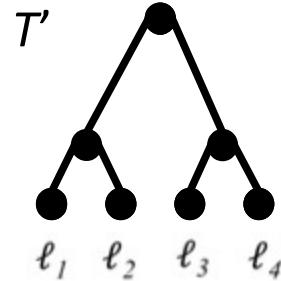
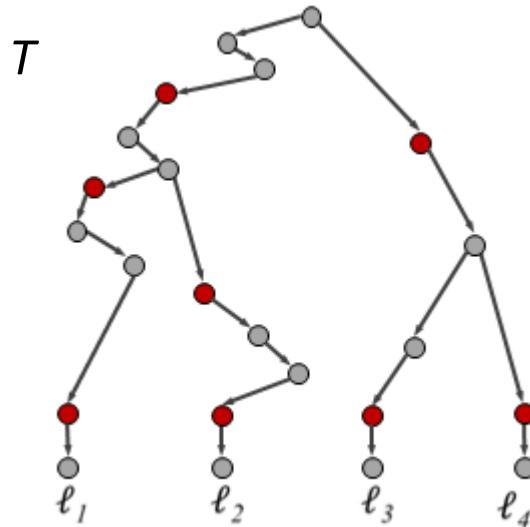
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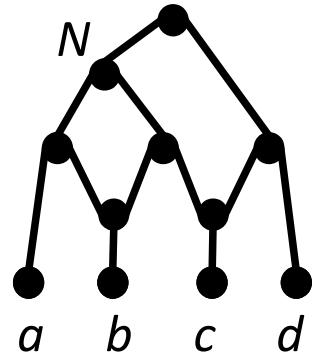
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→ N contains at most $4(n-1)$ reticulation vertices

Number of reticulations of a stable network

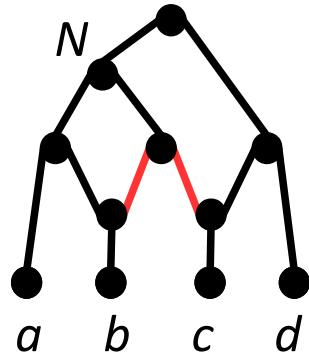
« Dummy leaves »?



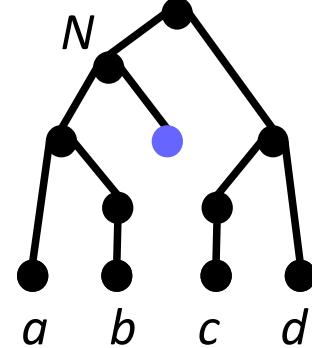
Deleting reticulation arcs can create « dummy leaves »

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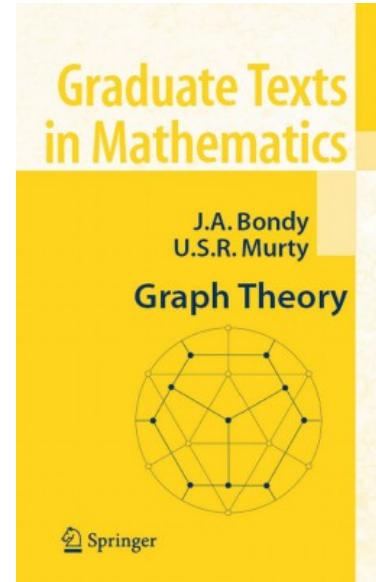
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Possible to avoid creating « dummy leaves »?

16.2 Matchings in Bipartite Graphs 423

16.2.13 Let $G := G[X, Y]$ be a bipartite graph such that $d(x) \geq 1$ for all $x \in X$ and $d(x) \geq d(y)$ for all $xy \in E$, where $x \in X$ and $y \in Y$. Show that G has a matching covering every vertex of X .
(N. ALON)

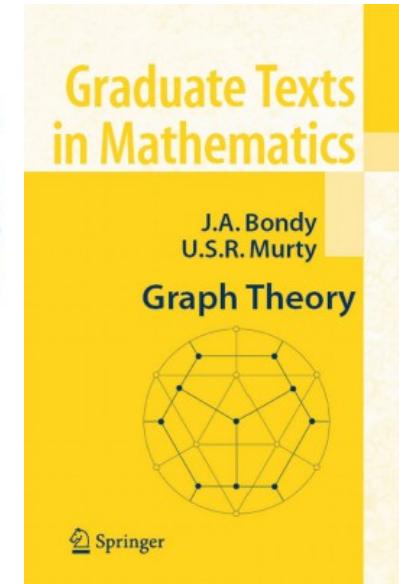


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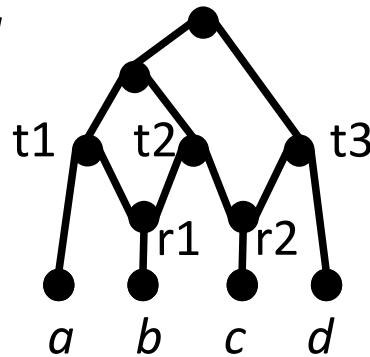
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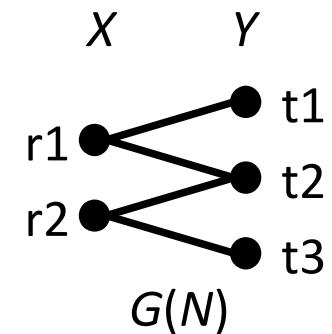


Given N



Build $G(N)$, bipartite graph such that:

- $X =$ reticulation vertices of N
→ all vertices in X have degree 2
- $Y =$ tree vertices of N with at least one reticulation child
→ all vertices in Y have degree 1 or 2
- edge between x and y iff x is a child of y

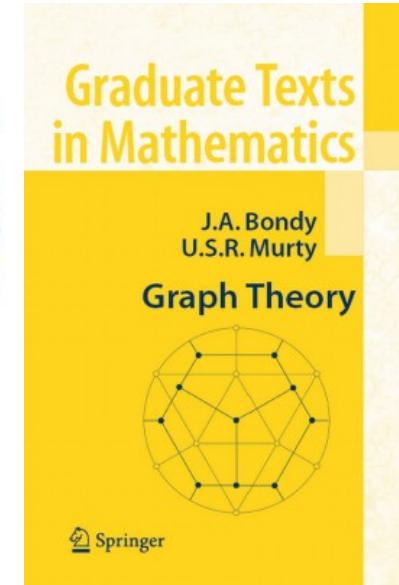


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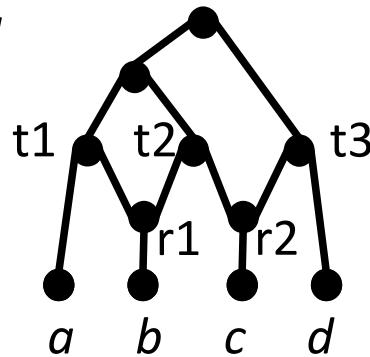
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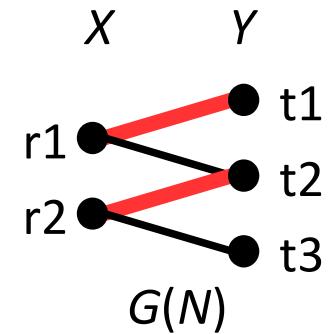
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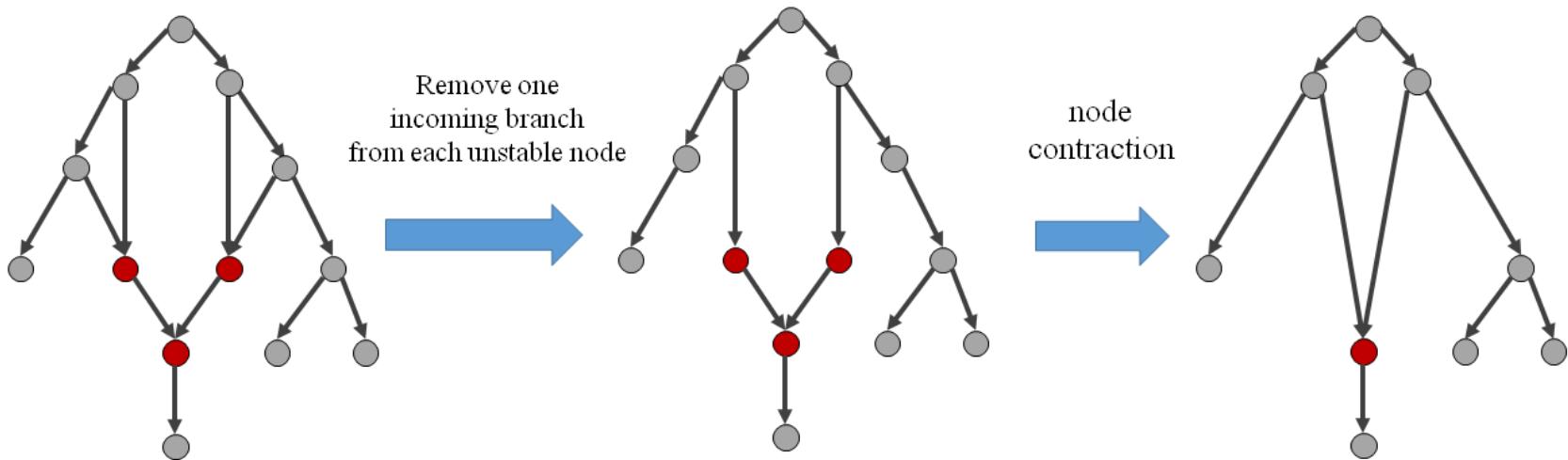
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→ matching covering every vertex of X → edges to remove from N



Number of reticulations of a nearly-stable network

Reduce nearly-stable networks to stable networks



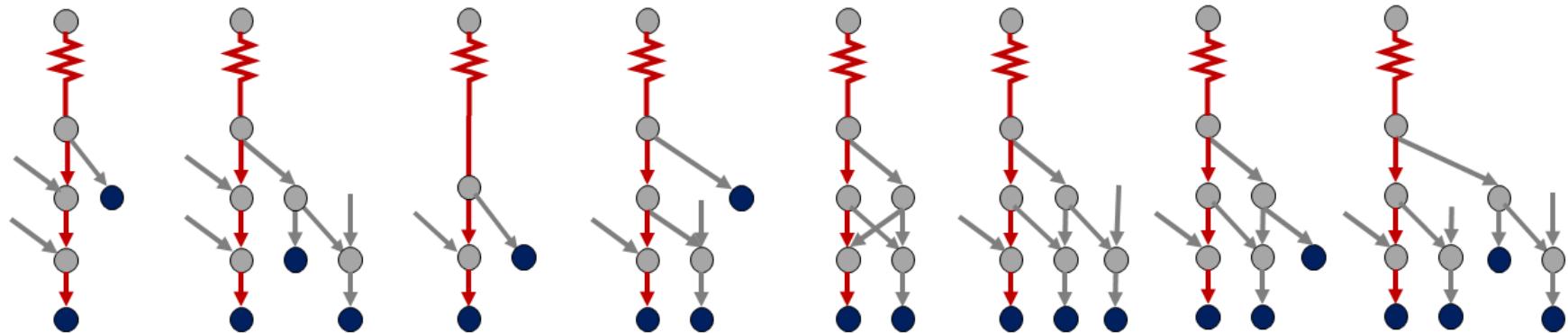
$$\#UnstableRet \leq 2 \quad \#StableRet \leq 8(n-1)$$

$$\#StableRet \leq 4(n-1)$$

Deleting reticulation arcs to simplify the question

Simplify N by removing an edge near the end of a **longest path P** .

Case analysis (8 cases):



Perspectives...

