Problem session of SGT 2018

June 2018

Contact representation of toroidal graph

Communicated by D. Gonçalves.

A *toroidal graph* is a graph that can be embedded in the torus.

Conjecture 1 (Gonçalves)**.** If in a toroidal triangulation T every triangle bounds a face, then T has a contact representation with rectangles in a toric parallellogram (i.e. with opposite sides identified).

Even set

Communicated par J. Baste.

EVEN SET
Input : A set U, and $S_i \subseteq U$, $i \in \{1, \ldots, n\}$.
Question : Does there exists a non-empty set X such that $|X \cap S_i|$ is even for all $i \in \{1, \ldots, n\}$ and $|X| \leq k$.
Parameter : k.

Is the problem Even Set FPT ?

About the dichromatic number

Communicated by S. Bessy.

Let D be a digraph. A k*-dicolouring* is a partition into k sets inducing acyclic subdigtraphs. The *dichromatic number* of D, denoted by $\vec{\chi}(D)$, is the minimum k for which D admits a k-dicolouring.

Conjecture 2 (Neumann-Lara [1])**.** Every planar oriented graph has dichromatic number at most 2.

Conjecture 3 (Erdős)**.** Is there a function f such that every graph with chromatic number k admits an orientation with dichromatic number at least k ?

We know that $f(2) = 3$, but the existence of $f(3)$ is not known.

Question 4. Is is true that for every graph G with huge chromatic number k, there is an orientation D of G such that for every subgraph with chromatic number at least $k/2$ its induced orientation $D\langle V(H)\rangle$ contains a directed cycle.

Decomposing a graph into k-edge-connected subgraphs.

Communicated by J. Bang-Jensen.

Theorem 5 (Nash-Williams)**.** *If G is $2k$-edge-connected, then there are k edge-disjoint-spanning tree.*

Corollary 6. *If G is $4k$-edge-connected, then its edge set $E(G)$ has a partition (E_1, E_2) such that $(V(G), E_i)$ is k-edge-connected for all $i = 1, 2$.*

Conjecture 7 (Bang-Jensen)**.** If G is $2k$-edge-connected, then its edge set $E(G)$ has a partition (E_1, E_2) such that $(V(G), E_i)$ is k-edge-connected for all $i = 1, 2$.

Euclidean arrangement of pseudolines

Communicated by S. Felsner.

Think of a *pseudoline* as an x-monotone curve in the plane which is unbounded on both sides. An *arrangement* of pseudolines is a set of pseudolines such that any two of them intersect in exactly one point where they cross. An arrangement is *simple* if there is no point where more than 2 pseudolines intersect.

Conjecture 8. For every simple arrangement of at least 3 pseudolines, each of which is coloured either red or blue but not all of the same colour, there is a bi-chromatic triangle.

Eulerian trail in eulerian multigraphs

Communicated by A. Sebő.

Problème 9. What is the complexity of the following problem: Given an Eulerian multigraph G, does there exist an eulerian trail without two consecutive parallel edges.

Hamiltonicity of squares

Communicated by A. Sebő.

Theorem 10 (Fleischner)**.** *If G is 2-connected, then G^2 is Hamiltonian.*

Can you provide a simple proof of this theorem.

Problème 11. Can you characterize the graphs that can be turned into a Hamiltonian cycle repeated splitting operations.

Domination in tournament

Communicated by P. Aboulker.

A *tournament* is an orientation of a complete graph.

A *dominating set* in a digraph D is a set S such that every vertex in $V(D) \setminus S$ is dominated by a vertex of S. The *domination number*, denoted by $\gamma(D)$ is the minimum size of a dominating set of D. The *induced domination number* is $\gamma_H(D) = \max\{\gamma(D') \mid D' \text{ induced subdigraph of } D\}$.

Problème 12. Is there a function f such that every tournament T such that $\gamma_H(N^+(x)) \leq k$ for all $v \in V(T)$, satisfies $\gamma(T) \leq f(k)$?

Extending matchings to hamiltonian cycle in the hypercube.

Communicated by L. Beaudou.

Conjecture 13 (Ruskey and Savage [2])**.** In a hypercube, every matching can be extended to a Hamiltonian cycle.

Fink[3] proved Kreweras' conjecture [4] which asserts that every perfect matching of a hypercube extends to a Hamiltonian cycle.

Tiling a rectangle with a poliomino

Communicated by A. Talon.

A poliomino is *tiling* if there exists k such that a rectangle can be tiled with k copies of the poliomino (rotation and symmetries are allowed). The *order* of a tiling poliomino is the minimum k such that a rectangle can be tiled with k copies of the poliomino.

Conjecture 14. The order of a tiling poliomino is either even or 1.

We know that there is no tiling poliomino with order 3. The next step is to prove that there is no tiling poliomino with order 5.

Burning number of a graph

Communicated by A. Lahiri.

A *burning procedure* is the following [5]. It beging with a graph with all vertices unburned. At each step, you burn the neighbours of the already burnt vertices, and burn another unburned vertex in the graph according to your preference.

The *burning number* of a graph, denoted by $b(G)$, is the minimum number of steps for a burning procedure to burn all vertices of the graph.

Conjecture 15. $b(G) \leq \lceil \sqrt{|V(G)|} \rceil$ for all graph G.

A known upper bound is $2\lceil \sqrt{|V(G)|} \rceil - 1$ which is not difficult to observe [5].

References

[1] V. Neumann-Lara. Vertex colourings in digraphs. Some problems. Technical report, University of Waterloo (1985).

[2] F. Ruskey and C. D. Savage, SIAM Journal on Discrete Mathematics 6, No.1 (1993) 152-166.

[3] J. Fink. Perfect matchings extend to Hamilton cycles in hypercubes. J. Comb. Theory, Ser. B, 97(6):1074-1076, 2007.

[4] G. Kreweras, Matchings and Hamiltonian cycles on hypercubes, Bull. Inst. Combin. Appl. 16 (1996), 87–91.

[5] A. Bonato, J. Janssen, E. Roshanbin, Burning a Graph as a Model of Social Contagion, Lecture Notes in Computer Science, 8882, (2014) 13–22.