Tutorial 05 – Satellites

Exercise 1. *Back to school*

1. Show that the language $\textsc{Add} = \{\langle \overline{n}, \overline{m}, \overline{n+m} \rangle : n, m \in \mathbb{N}\}$ is in L, where $\overline{n}$ is the binary representation of n.
2. Show that the language $\textsc{Mult} = \{\langle \overline{n}, \overline{m}, \overline{n \cdot m} \rangle : n, m \in \mathbb{N}\}$ is in L.
3. Show that the language of well parenthesized words is in L.
4. And what if there are two hind of parentheses? For example, $([[]()]())[]$ is in the language but $([)]$ does not.

Exercise 2. *Baire and Banach*

1. Show that $\textsc{Space-TMSat} = \{\langle \alpha, x, 1^n \rangle : M_\alpha \text{ accepte } x \text{ en espace } n\}$ is PSPACE-complete.
2. You saw during the lecture that TQBF is PSPACE-hard. Show that it is in PSPACE.
3. Ameliorate the previous algorithm so that it runs in $O(n+m)$ space where n is the number of variables and m the input length.

Exercise 3. *Deterministic Space Hierarchy Theorem*

1. Show the existence of a DTM $\mathcal{U}$ which on input $\langle \alpha, x \rangle$ so that M_α runs in $s(n)$ space simulate $M_\alpha(x)$ using $c \cdot s(|x|)$ space (where c does not depend on $|x|$).
2. Show the following theorem, due to Stearns, Hartmanis and Lewis (1965) :

   Theorem. *Let s_1 and s_2 two space-constructible functions so that $s_1(n) = o(s_2(n))$. Then $\mathsf{DSPACE}(s_1(n)) \subsetneq \mathsf{DSPACE}(s_2(n))$.*

Exercise 4.
For $k \geq 0$, let w_k be the concatenation in lexicographic order of all length-k strings, delimited by $\sharp$ (*i.e.* $w_k = 0^{k-2}00\sharp 0^{k-2}01\sharp 0^{k-2}10\sharp \ldots \sharp 1^k$). Let $W = \{w_k : k \in \mathbb{N}\}$.

1. Show that W is nonregular.
2. Show that $W \in \mathsf{DSPACE}(\log\log n)$.
3. Show that for every $s(n) = o(\log\log(n))$, $\mathsf{DSPACE}(s(n)) = \mathsf{DSPACE}(1) = \mathsf{REG}$. The notion of *crossing sequence* shall be useful.