Effects of non-negligible cable mass on the static behavior of large workspace cable-driven parallel mechanisms

Nicolas Riehl, Marc Gouttefarde, Sébastien Krut, Cédric Baradat and François Pierrot

Abstract—Cable-driven robots are currently extensively studied. Generally, for this type of manipulators, cables are considered to be massless and inextensible. But for large working volume applications, their mass cannot be neglected. Based on a well-known model which describes the profile of a cable under the action of its own weight, the inverse and forward kinematics of minimally constrained cable-driven manipulators can be numerically computed. This paper studies the effects of taking cable mass into account by comparison to classical massless cable model. It highlights the real effects of such a model on cable lengths to reach a given position. The effects on cable tensions are also studied.

I. INTRODUCTION

PARALLEL manipulators (PM) are well known to be more rigid and accurate than their serial counterpart. Cable-driven robots present most advantages of parallel mechanisms, but also prevent from a major drawback of usual PM: the poor ratio between working volume and machine size. Actually, cable-based parallel robots are a variant of Gough-Stewart platform in which rigid extensible legs are replaced by cables stored on spools.

This type of technology is known for times and several studies of cable-driven manipulators have been undertaken. There exist some works using the low inertia of such mechanisms for high speed pick and place manipulators [1]. Others have taken advantages of the large working volume that can be obtained. The NIST Robocrane [2] is an example of such studies. More examples exist: [3], [4], [5]. The fact that cables present the characteristic of being only able to act in tension, not in compression, leads to many difficulties with kinematic analysis and control of cable-driven manipulators. Thus, many works on cable-driven robots deal with kinematic study [6], [7]. This characteristic of cables also affects the workspace and the available operational wrench of such manipulators which turns out to be quite different than that of conventional parallel mechanism [8], [9]. As already mentioned, cables present the particularity of not being rigid contrary to conventional limbs in PM. This characteristic has lead to studies of the stiffness of cable-actuated mechanisms. Thus, Behzadipour [10] presented an equivalent stiffness model for a single cable with pretension using four springs. Lafourcade also proposed theoretical tools to study stiffness of wire-driven parallel manipulators applied to the SACSO [11].

In most studies of cable-driven mechanisms, cables are considered to be straight lines, as long as cable are massless and its tensions remain positive. Thereby, studies become much easier and, in most cases, the model obtained is quite close to reality. But, in some particular cases, more complex cable behaviors must be taken into account to fit well with the real manipulator. For instance, when the tensions in cables are large, axial elasticity must be included in the model. For the MARIONET robot, Merlet [12] has considered this aspect to derive the inverse kinematic model.

Another aspect that has to be taken into account is sagging. This phenomenon is due to the own mass of the cable. Sagging has mainly been studied in civil engineering applications like bridges. Irvine [13] presented a model to describe the shape of a cable under the effect of its own weight. This aspect has a great importance in large working volume applications in which the cables must withstand large tensions, and thus have a non negligible mass. Examples of such applications are a 1km² radio telescope [15], a system for contour crafting applications [5] and the FAST system [14]. For this last one, Kozak presented a methodology to compute numerically the inverse kinematics of the manipulator, and then studied the effects of sagging on the stiffness of the mechanism. Bouchard [15] has also used Irivnes model of a sagging cable to define a criterion, based on the stiffness of the cable, under which the cable mass can be considered to be negligible.

This paper focuses on this aspect of cable sagging. Based on a well known model of cable sagging, described in section II, the numerical computations of the inverse and forward kinematics are presented in section III. This paper studies the effects of sagging on different aspects. First, in section IV B it is studied how this model affects the length of the cables to reach a desired pose. The effects on the end-effector positioning are then presented in Section IV C. Section IV D deals with the study of how cable mass acts on cables tensions.

II. CABLE SAGGING MODEL

The model presented by Irvine [13] and used by Kozak [14] and Bouchard [15] describes the behavior of a cable fixed at one extremity, under the action of a force F tangent to its profile at the other extremity, and under the action of its own mass and gravity. This model allows the decrition of the whole static behavior (displacement) of all points of the cable taking into account sagging and axial elasticity.
as presented in Fig. 1. This figure shows different parameters useful in this model, $L_0$ being the length of the cable without any load, $\rho_0$, $E$ and $A_0$ being the mechanical characteristics of the cable (the linear density, the Young modulus and the unstrained section of the cable, respectively). $L$ denotes the real length of the cable (the unstrained length $L_0$ plus the length due to axial elasticity) under combined action of the weight of the cable and the force applied on the free end of the cable. In this model, $p$ corresponds to the strained length of a cable segment starting at the fixed end and $s$ to the corresponding unstrained length.

According to Irvine [13], this model exists for a long time, and was mainly used in civil engineering to build cable-stayed bridges. It was also presented by Kozak and thus its derivation will not be presented here. Equations (1) and (2) give the coordinates $(x, z)$ of any cable point defined by its curvilinear abscissa $s$, assuming the position of the fixed end of the cable, the cable characteristics, its unstrained length and the applied force on the other end of the cable to be known.

$$x(s) = \frac{F_s}{EA_0} + \frac{|F_s|}{\rho_0 g} \left[ \sinh^{-1} \left( \frac{F_z + \rho_0 g (s - L_0)}{F_s} \right) \right]$$

$$- \sinh^{-1} \left( \frac{F_z - \rho_0 g L_0}{F_s} \right) \right]$$

$$z(s) = \frac{F_z}{EA_0} + \frac{\rho_0 g (s^2 - L_0 s)}{2}$$

$$+ \frac{1}{\rho_0 g} \left[ \sqrt{F_x^2 + (F_z + \rho_0 g (s - L_0))^2} \right]$$

These equations describe the profile of an elastic cable under the action of gravity $g$ and forces $\mathbf{F} = [F_x; F_z]$ at its free end. In these equations, note that the axial elasticity (with $EA_0$ as denominator) can easily be identified.

### III. KINEMATICS MODELS

#### A. Inverse kinematics

1) Problem description: The problem of solving the inverse kinematics is quite difficult when taking into account the mass of the cables, contrary to computing the same model with massless inextensible cables. In this second case, the inverse kinematics simply consists in computing the norm of the vector linking the point of the fixed frame from which the cable extends, to the ending point of the cable where it is attached to the mobile platform. Thus, the inverse kinematics equation has this form

$$L_i = \| X + Q b_i - a_i \|$$

$L_i$ being the length of the $i^{th}$ cable, $X$ the position vector of the reference point on the end effector, $Q$ the rotation matrix from the reference fixed frame to the frame attached to the mobile platform, $b_i$ the position vector of the point at with the $i^{th}$ cable is attached to the platform expressed in the platform frame, and $a_i$ the position of the exit point of the $i^{th}$ cable on the fixed frame.

Solving the inverse kinematics of minimally constrained systems, that is to say of $n$ degrees of freedom (DOF) manipulator driven by $n$ cables, with non-negligible cable mass amounts to solving a non-linear system of $3n$ equations with $3n$ unknowns. The different cases which correspond to this model are depicted in Table I. Indeed the vector of unknowns in this problem is the following

$$[ F_{x1} \quad F_{z1} \quad L_{01} \quad \cdots \quad F_{xn} \quad F_{zn} \quad L_{0n} ]$$

where $F_{xi}$ and $F_{zi}$ are the forces applied by each cable on the platform expressed in the frame attached to the cable as defined in Fig. 1. The remaining unknowns ($L_{0i}$) are the lengths of the cables. It must be noted that this inverse kinematics model differs from the classical kinematics of cable-driven robots. First, as we can see with the number of unknowns, we must take into account both cable forces applied on the platform and cable lengths because, in this case, we cannot separate these both aspects of the kinematic modeling, the cable length depending directly on these forces according to the cable sagging model described in section II.

To solve this system, $3n$ equations are required. These equations are the static equilibrium equations corresponding to the d.o.f. of the manipulator, that is to say the equilibrium

### Table I

<table>
<thead>
<tr>
<th>Description of the Different Possible Configurations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of d.o.f.</td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>Planar</td>
</tr>
<tr>
<td>Spatial</td>
</tr>
<tr>
<td>Spatial</td>
</tr>
</tbody>
</table>
of forces and torques applied on the mobile platform. This will lead to \( n \) equations. The \( 2n \) equations missing come from the equations describing the behavior (profile) of each cable as defined in (1) and (2) expressed in each cable reference frame. For instance, for a 6 d.o.f. manipulator with 6 cables the set of static equilibrium equations are as follows

\[
\begin{align*}
\sum_{i=1}^{n} Q_i F_i - mg &= 0 \\
\sum_{i=1}^{n} b_i \times Q_i F_i &= 0
\end{align*}
\]  

\( Q_i \) being the rotation matrix from the frame attached to the cable \( i \) to the frame attached to the mobile platform, \( b_i \) the position vector of the end point of the \( i^{th} \) cable attached to the platform, and expressed in the frame attached to the center of gravity of the platform, and \( g \) the acceleration of gravity.

Replacing \( s \) by the value of \( L_{0i} \) in equations (1) and (2) allows us to obtain the \( 2n \) equations needed. Indeed, these equations describe the position of the ending point of the \( i^{th} \) cable given the force applied by cable \( i \) on the mobile platform \( (F_{xi} \) and \( F_{zi} \))

\[
x_{\text{end}} = \frac{E_{A_0}}{E_{A_0}} \left[ \frac{F_{zi}}{F_{xi}} \right] \left[ \sinh^{-1} \left( \frac{E_{xi}}{E_{zi}} \right) \right] - \sinh^{-1} \left( \frac{E_{xi}}{E_{zi}} \right)
\]

\[
z_{\text{end}} = \frac{E_{A_0}}{E_{A_0}} - \frac{L_{0i}}{2E_{A_0}} + \frac{1}{\rho g} \left[ \sqrt{F_{xi}^2 + F_{zi}^2} - \sqrt{F_{xi}^2 + (F_{zi} - \rho g L_{0i})^2} \right]
\]

where \( x_{\text{end}} \) and \( z_{\text{end}} \) are known since the pose of the mobile platform is known.

2) Computing the inverse kinematics: As mentioned before, solving the problem of inverse kinematics is more difficult when cable mass is not neglected. To solve the problem depicted in the previous subsection, which consists in solving a system of \( 3n \) non-linear equations with \( 3n \) unknowns, a numerical optimization solver ("lsqnonlin" under Matlab) has been used. The optimization with a 2.26GHz CPU lasts about 0.4s. In order to prevent from divergence and to obtain the minima in best delay, initial values of the unknowns have been introduced, these are the values of the massless inextensible model. In order to help the solver, boundary conditions have also been considered. These conditions correspond to physical and mechanical considerations in term of minimal cable tensions (to prevent from negative tensions), maximal cable tensions (linked with cable and motor characteristics), and maximal values for cable lengths.

B. Forward kinematics

The computation of the forward kinematics of parallel mechanisms has always been an issue compared to the inverse one. In our case, same issues are faced, i.e. the forward kinematics of the manipulator cannot be expressed analytically. But it can be computed numerically, as in the case of the inverse kinematics. This problem is very similar to the one presented before. That is to say, for the case with 6 d.o.f., the \( 3n \) unknowns are

\[
x, y, z, \psi, \theta, \phi, F_{xi}, F_{zi}, \ldots, F_{zn}
\]

\( x, y \) and \( z \) being the coordinates of the reference point of the frame attached to the mobile platform, and \( \psi, \theta \) and \( \phi \) being the Euler angles defining the orientation of the platform. As in the inverse kinematics model, \( F_{xi} \) and \( F_{zi} \) are the forces exerted by cable \( i \) on the platform expressed in the frame attached to the cable (Fig.1). The same equations (5), (6) and (7) are used, but now the unstrained lengths \( L_{0i} \) are given. The ranges assigned to the unknowns have also been changed, introducing the limit of the working volume to prevent from impossible postures. The initial values come again from the massless inextensible modeling of a cable.

IV. EFFECTS OF SAGGING CABLES

This section presents a study of the effects of sagging on cable lengths, on end-effector positioning and on cable tensions.

A. Studied model description

In order to fit well with the model described in the previous section, the mechanism studied here is a minimally constrained spatial one with 3 DOF. The 3 DOF are the 3 translations along the \( x, y \) and \( z \) axis. The 3 cable exit points are positioned at the 3 extremities of an equilateral triangle. In order to see the influence of the mass of the cables, the mechanism must have great dimensions. For realistic study, we have chosen the dimensions of the existing robot FAST described by Kozak [14]. Thus, the positions of the exit points of each cable, expressed in the fixed frame, are given in TABLE II.

The platform is assimilated to a unique point mass. Concerning the characteristics of the cables and the mass of the platform, the same characteristics as the FAST system are used. These parameters are described in TABLE III.

The resulting manipulator is schematically shown in Fig.2. This figure also shows the effects of the non-negligible cable mass on the profile of each cable.

B. Effect on the length of the cables

1) For a given pose: The introduction of non-negligible cable mass in the model leads to many changes in the

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>EXIT POINTS POSITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>x (m)</td>
<td>y (m)</td>
</tr>
<tr>
<td>a1</td>
<td>0</td>
</tr>
<tr>
<td>a2</td>
<td>0</td>
</tr>
<tr>
<td>a3</td>
<td>0</td>
</tr>
</tbody>
</table>
### TABLE III
**MODEL CHARACTERISTICS**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_0 )</td>
<td>Linear cable density</td>
</tr>
<tr>
<td>( E )</td>
<td>Young modulus</td>
</tr>
<tr>
<td>( A_0 )</td>
<td>Cable section</td>
</tr>
<tr>
<td>( m )</td>
<td>Platform mass</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravity acceleration</td>
</tr>
</tbody>
</table>

![Fig. 2. 3 DOF / 3 cables point mass cable-driven mechanism.](image)

...results. First, it will be shown how the cable lengths are impacted. Actually, cable length is one of the most important aspects because this is usually the parameter used to control a cable-driven robot. Generally, the cables are rolled on an actuated drum, and thus a direct relationship exists between the rotation of the drum and the length of the cable.

For large dimensions cable-driven robots, like the one described before, reaching a defined position could lead to substantially different results in terms of cable lengths when the cable mass is considered. Table IV shows the resulting cable lengths for the position \( x = 200 \) m, \( y = 120 \) m and \( z = -250 \) m, for the two different models, i.e. massless inextensible cables (Case A) and cables with non-neglected mass (Case B).

For this pose, it can be noted that the difference is not negligible. Indeed, the cable length needed to reach the desired position, is much larger in case B than in case A. Actually, the difference is 0.3%, 1.5% and 1.3% for cable 1, 2 and 3 respectively. These are not negligible values if the mechanism needs to be accurate, indeed the differences for this pose goes above 6m. This result also highlights that the error is quite different for the three cables. That is why that type of study should be done along a trajectory.

2) Along a given trajectory: The static study has been made on a trajectory created by choosing a set of positions (810 poses) in the working volume, and interpolating the trajectory between one position and the following one. These interpolations have been done by means of a 5th degree polynomial with null first and second derivatives at each intermediate position. The chosen trajectory is shown in Fig.3.

At each point of this trajectory, the inverse kinematics (with non-negligible cable mass) is computed to obtain the length of each cable. These cable lengths have been plotted (red curves) and compared to cable lengths obtained with the inverse kinematics of the massless cable model described in (3) (dashed curve). The results for cable 2 are presented in Fig.4.

It can also be noted that the cable lengths of the model described by case B are larger than those obtained in case A. The difference between the two results along the whole trajectory is presented in Fig.5. The mean difference between the lengths is above 6m. This represents a mean length difference of 1.75%. In large workspace applications, these differences could be really harmful concerning the accuracy of the robot if the sagging cable model is not used. Also it must be noted that this study only compares the length controlled by the actuator: we are comparing the unstrained length (in case B).

### C. Effects on end-effector position

The previous subsection has presented the effects of using a model taking into account the mass of the cables on the length of cables needed to reach a desired position. This subsection will describe the effects of these considerations on the position of the end-effector of the manipulator.

![Fig. 3. Trajectory description.](image)

### TABLE IV
**CABLE LENGTHS FOR A GIVEN POSE WITH OR WITHOUT CABLE MASS TAKEN INTO ACCOUNT.**

<table>
<thead>
<tr>
<th></th>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st cable length</td>
<td>341.9064 m</td>
<td>343.0108 m</td>
</tr>
<tr>
<td>2nd cable length</td>
<td>408.5340 m</td>
<td>414.5476 m</td>
</tr>
<tr>
<td>3rd cable length</td>
<td>403.7040 m</td>
<td>408.8957 m</td>
</tr>
</tbody>
</table>
1) Methodology: In this subsection, the computation of the forward kinematics is used. The objective of this part is to determine the difference that appears in the position of the manipulator end-effector when the effect of the mass of the cables is not included in the model whereas it should have been. To this end, a methodology based on the massless inextensible cable model will be used. It starts by choosing a position of the end-effector. The next step consists in determining the lengths of each cable corresponding to this position using the massless inextensible cable model. These cable lengths are then introduced into the forward kinematic model taking cable mass into account (cf. section III.B.). It allows us to compare the two positions obtained with the same cable lengths. The methodology is depicted in Fig.5.

2) Computations and results: This study has been realized for the trajectory used in section IV B. The results of the comparison between the two trajectories obtained with the two models are presented in Fig.7. The trajectories followed by the two models are quite different. Indeed, the figure shows that the mean differences along the trajectory are 4.62m, 3.62m and 5.09m along the x-axis, y-axis and z-axis, respectively. This corresponds to a mean difference along the trajectory of 7.77m. Because this is not a negligible value, cable masses must be taken into account for this type of applications in order to have a good positioning of the end-effector.

D. Effects on cable tensions

The previous sections have shown that cable mass has a significant effect on the length of the cables and on the positioning of the end-effector. This section presents the effects of sagging on cable tensions. Indeed, contrary to straight cables, tensions in cables with non-negligible masses is not constant along the profile. From the model described in the first section, we can obtain the equation of the tension of the cable at any point of the cable profile. It has the following form

\[ T(s) = \sqrt{F_x^2 + (F_z + \rho_0 g (s - L_0))^2}. \]

From (9), the maximal tension is obtained at the exit point of the cable on the frame (in our case \( F_z < 0 \)). This value is significantly different from the tension existing at the other end of the cable. In the present study, the point of maximal tension has been chosen because it corresponds to the cable tension seen by the actuator. The study of the evolution of maximal cable tensions along the example trajectory used in the previous section has been made in order to compare the two models. The results are presented in Fig.8.

According to these results, cable tensions seem to be the parameter that is mostly affected by the non-negligible
cable mass model. Indeed, it can be noticed that the mean difference between the tensions in the two models is about 100% more for the model including cable mass. Using this model turns out to be essential to an appropriate design of the robot, i.e. to the choice of the motors able to supply enough torques, but also to the dimensioning of the structure to prevent from plastic deformations, or worse, breakdown.

V. FUTURE WORKS

In the mean time, we are computing the sagging model on different types of cable driven manipulators. We will study the same effects on a 6 d.o.f. / 6 cables mechanism. This type of mechanism will allow us to study the effect on the orientation of the platform. It would also be interesting to learn more on the effects of such models on the operational applicable wrench because of the change of direction of the forces applied by the cables due to sagging, and because of the large difference of tension values along the profile of the cable. The same way, this model should have an important impact on the wrench feasible workspace.

At the present time, we are building a prototype of a large working volume cable manipulator which will allow us to test different configurations and the main problems we have highlighted in this paper. The CAD of this prototype is presented in Fig.9.

VI. CONCLUSION

This paper presents some results about the effects of taking into account the cable masses on the computation of the forward and inverse kinematics of parallel cable-driven robots. Based on a model described by Irvine, the inverse and forward kinematics of a 3 DOF / 3 cables spatial manipulator have been computed. The influence of such a model on the end-effector positioning, on the control lengths of the cables and on the tensions in the cables has been studied. This work shows that for large working volume manipulators, this aspect needs to be taken into account at the design level in order to achieve good positioning and accuracy.

VII. ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial contribution of Fatronik-Tecnalia.

REFERENCES