A Control Law for Human Like Walking Biped Robot SHERPA Based on a Control and a Ballistic Phase - Application on the Cart-Table Model

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I. INTRODUCTION

The most popular control approaches to ensure dynamic balance during walking from now on are based on ZMP (Zero-Moment Point) [1] or on similar indicator related to the dynamic of the robot. Numerous famous works have been done in this way. Among them AIST [2] plans the trajectory of the CoM (Center of Mass) so that the ZMP lies always inside the support polygon. The design of the control law of the robot ASIMO from HONDA [3] is known to use the ZMP. Numerous indicators like FRI (Foot Rotation Indicator) and ZRAM (Zero Rate of change of Angular Momentum) [4] have been created and implemented in order to characterize dynamic balance.

Some other approaches try to achieve radically different goals. For instance the robot Rabbit [5] is an underactuated biped robot with adaptive speed. The robot Runbot [6] uses learning techniques in order to master a wide range of speed. Finally a control law for the robots from the MIT [7] were designed to ensure a strong adaptability to rough terrain, this work leading to the well-known Big Dog robot.

The manufacture of our robot SHERPA is in progress. Its purpose is the transport of a light load in an office-like environment with possible stairs (this issue will be discussed in a future paper). Two stereo omnidirectional cameras on top of its structure will enable it to follow somebody and to tune its speed to the one of its leader. SHERPA is a two-legged robot with six degrees of freedom on each leg but no trunk. The six degrees of freedom will be totally actuated thanks to linear motor along with cable transmission [8]. The characteristics of this technology is the small inertia and the full reversibility. SHERPA will be 1m tall and will weight 30 kg.

SHERPA has been designed to fit with two specificity. The first one is an energetically effective use of free dynamics. A better use of impacts on the ground and disturbances in the direction of the walking is the final goal. Thus the mechanical design was led with such ideas as complete reversibility of the actuators and compliant feet. The second specificity is a very smooth gait, close to the human one. It implies that we should consider phases of loss and then regain of balance in the direction of walking.

Firstly this paper presents the general approach for this control law. Then it focuses on its implementation on a simplified model of a biped robot, the cart-table, with details on trajectory planning and tracking. Some results of simulation are shown through two concrete examples. Lastly the numerous opportunities given by this type of control law are listed.

II. AN INNOVATIVE APPROACH FOR SHERPA

A. Presentation of a step of SHERPA

Impact is a critical phase of the walking cycle regarding energy losses. That is why we decided to design the control law around this phase, thus allowing us to get advantage of the compliance of the feet of SHERPA. For any given step length, the choice of an impact speed determines the walking speed of the robot. It becomes then critical to calculate this impact speed and to control it.

A step of SHERPA is shown on figure 1. There is no double support phase: the step is divided into an impact phase and a single support phase, which is itself divided into a free dynamics phase and an impact preparation phase. During the free dynamics phase SHERPA’s motion is due to the post-impact energy and the inertia of the robot. While preparing the impact, the foot move so that it will hit the ground with a model-based precalculated speed. This speed involve a precise position and speed of the CoM at the end of the free dynamics phase.

B. The Cart-table model

A two-dimensional simplified model has been chosen in order to work more rapidly on the control law. It is constituted of a cart moving on top of a table: it is the cart-table. By controlling the force applied on the cart, it is possible to make the table move. The two edges of the foot
of the table can be seen as the two feet of the biped robot and the CoM of the cart as its CoM since the mass of the table is considered ten times smaller than the mass of the cart. The cart-table has already been used as a linearized inverse pendulum to model a biped robot during the single support phase [2]. In our study it is used to simulate a complete step with a critical phase, the impact between the table and the ground.

How can we simulate an entire step of a biped robot with such a cart-table? Let’s consider a biped robot which walks with a fixed step length. The impact preparation phase seen in figure 1 is obtained with the cart-table when the table swing around its left edge. In this case the cart moves thanks to the motor. Then the instantaneous impact phase on the biped robot (figure 1) occurs with the cart-table when the right edge of the table hit the ground while swinging around the left edge. Finally the free dynamics of the biped robot is obtained with the cart-table when the table swing around its right edge. During this phase the control of the motor is off and the cart can move freely thanks to the reversibility of the motor. A complete step is thus realized.

In previous work the purpose of the cart-table model was to generate a simplified equation of the dynamics of walking. We have worked further by creating a complete dynamic simulator with matlab and building a mechanical prototype in order to test our control law until SHERP A is available. The simulator is already fully operational and numerous tuning parameters are available (mass, length, friction, initial conditions...).

The planar cart-table model shown on figure 2 is an underactuated robot with only one actuator and four degrees of freedom if we consider the full hybrid model (the horizontal and vertical position of the CoM of body 1, $x$ and $y$, the angle between the ground and the table, $\theta$ and the position of the cart on the table, $l$). Its characteristics are listed in table I.

This model is hybrid because there are different possible contact conditions between the table and the ground: one full contact conditions (equivalent to the double support phase for SHERPA), two symmetric one-point contact conditions (equivalent to the single support phase for SHERPA) and two symmetric impact conditions. These different cases will be noted with an indice $c$ varying from 1 to 3 for the contact conditions (1 is the full-contact condition, 2 and 3 are the one-point contact condition respectively on the left edge and on the right edge) and an indice $i$ varying from 1 to 2 (1 is the impact on the left edge, 2 is the impact on the right edge). If we consider one of these contact conditions alone, then the model is no more hybrid and the number of degrees of freedom can be reduced to two: $\theta$ and $l$ which is the same as for the classical inverse pendulum model. A state machine has been created in order to modify these contact conditions.
conditions when the cart-table move. The model for the full and one-point contact is classically written [9]:

\[
\begin{align*}
\mathbf{M}(q)\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) &= \mathbf{J}_\mathbf{c}(\mathbf{q})\dot{\lambda} + s\mathbf{u} \\
\mathbf{J}_\mathbf{c}(\mathbf{q})\dot{\mathbf{q}} + \Pi_\mathbf{c}(\mathbf{q}, \ddot{\mathbf{q}}) &= 0
\end{align*}
\]

(1)

\( \mathbf{q} = [x, y, \theta]^T \) is the vector of generalized coordinates, \( \mathbf{M} \) is the inertia matrix, \( \mathbf{N} \) is the Coriolis and centrifugal matrix, \( \mathbf{g} \) is the potential energy vector, \( \dot{\lambda} \) is the vector of the forces of the contact points, \( s\mathbf{u} \) is the command vector, \( \mathbf{J}_\mathbf{c} \) and \( \Pi_\mathbf{c} \) are respectively the jacobian and the hessian of the equations of contact when the cart-table is in the contact situation \( c \). The evolution of \( \mathbf{q} \) is calculated by integrating this equation.

Newton’s model is used as the impact model. It describes the transmission of the energy during the impact with a coefficient of restitution. Some more complex impact law can be studied like Moreau’s law. Additionally to this law, the integration of equation (1) over an infinitesimal time interval when considering acceleration and forces as impulsive gives the system of equations:

\[
\begin{align*}
\mathbf{M}(\mathbf{q})(\dot{\mathbf{q}}_+ - \dot{\mathbf{q}}_-) &= \mathbf{J}_\mathbf{c}(\mathbf{q})\dot{\lambda}_i \\
\mathbf{J}_\mathbf{c}(\mathbf{q})\dot{\mathbf{q}}_+ &= \mathbf{E}_\mathbf{v}\mathbf{J}_\mathbf{c}(\mathbf{q})\dot{\mathbf{q}}_-
\end{align*}
\]

(2)

\( \dot{\mathbf{q}}_- \) and \( \dot{\mathbf{q}}_+ \) are respectively the speed of the generalized coordinates before and after the impact, \( \lambda_i \) is the vector of the integrated impulsive forces, \( \mathbf{J}_\mathbf{c} \) is the jacobian of the impacts, \( \mathbf{E}_\mathbf{v} \) is the matrix of restitution from Newton’s model. Thanks to this equation it is possible to obtain directly the speed of the generalized coordinates after impact from those before impact. Indeed equation (2) is equivalent to equation (3):

\[
\begin{align*}
\dot{\mathbf{q}}_+ &= \mathbf{I}_k - \mathbf{M}^{-1}\mathbf{J}_\mathbf{c}^T (\mathbf{J}_\mathbf{c}\mathbf{M}^{-1}\mathbf{J}_\mathbf{c}^T)^{-1}(\mathbf{I}_k + \mathbf{E}_\mathbf{v}\mathbf{J}_\mathbf{c})\dot{\mathbf{q}}_- \\
\lambda_i &= -((\mathbf{J}_\mathbf{c}\mathbf{M}^{-1}\mathbf{J}_\mathbf{c}^T)^{-1}(\mathbf{I}_k + \mathbf{E}_\mathbf{v}\mathbf{J}_\mathbf{c}))\dot{\mathbf{q}}_-
\end{align*}
\]

(3)

\( \mathbf{I}_k \) is the unit matrix of size \( n \), \( i \) is the number of contact constraints considered in the impact case \( i \). The modified matrix of restitution \( \mathbf{E}_\mathbf{q} \) is the direct link between \( \dot{\mathbf{q}}_+ \) and \( \dot{\mathbf{q}}_- \).

This hybrid model works around a state machine that can be partially seen in figure 3. This state machine indicates which model should be applied to calculate the dynamics, depending on the values of the state variables.

C. Calculating the impact speed

The free dynamics of phase 6 is entirely determined by the dynamic equation (1) and the values of the state variable just after the impact \( \zeta_- = [\theta_-, \dot{\theta}_-, l_-, \dot{l}_-] \). So the desired final position and speed at the end of the free dynamics \( \zeta_f = [\theta_f, \dot{\theta}_f, l_f, \dot{l}_f] \) is obtained by choosing the corresponding \( \zeta_- \). Finally, considering the matrix \( \mathbf{E}_\mathbf{q} \) and the impact at \( \theta = 0 \), it is possible to linked directly \( \zeta_\theta \) to the values of the state variable before the impact \( \zeta_- \). The key is to find a method to determine the initial condition of a system from its final condition and its dynamic equation.

\[
\begin{align*}
\zeta_f - \zeta_- &= \int_{t=0}^{t_f} \dot{\zeta}(t)dt^2 \\
\zeta_f - \zeta_- &= \int_{t=0}^{t_f} \dot{\zeta}(t)dt
\end{align*}
\]

with the initial condition \( \zeta_- \). By replacing the variable \( t' = \)
\( t_f - t \) (backward variable), it gives:
\[
\begin{align*}
\zeta^+ - \zeta_f &= \int_0^{t_f} \dot{\zeta}(t_f - t') \, dt' \\
\dot{\zeta}^+ - (\dot{\zeta}_f) &= \int_0^{t_f} \ddot{\zeta}(t_f - t') \, dt'
\end{align*}
\] (4)

The system (5) shows that the reverse dynamics is obtained by interchanging initial and final position (from the first equation) and by interchanging initial and final velocity and taking their opposite (from the second equation). The values before impact can then be determined by reversing the impact model as well. In our case it means the computation of the inverse of \( \mathbf{E}_q \) since \( \dot{\theta}_- = \mathbf{E}_q^{-1} \dot{\theta}_+ \) gives the values of the state variables before impact. A control law has now to be designed to impact the ground with these precise values.

III. PARTIAL LINEARIZATION BY STATE FEEDBACK

A. Linearizing the dynamic equation

According to the notations given in figure 2 the nonlinear model is:
\[
\begin{align*}
m_{11} \dot{\theta} - M_2 h_2 \dot{\theta} + 2 M_2 (P_{max} + l) \ddot{\theta} + g_1 &= 0 \\
-M_2 h_2 \ddot{\theta} + M_2 \dot{\theta} - M_2 (P_{max} + l) \dot{\theta} &= m_{11} \ddot{\theta} + M_2 \ddot{\theta} - g_1 + M_2 \sin \theta = u
\end{align*}
\] (5)
\[m_{11} \] is the first-column first-row term of the inertia matrix \( M \) and \( g_1 \) is the first term of the potential energy vector \( g \):
\[
\begin{align*}
m_{11} &= I_T + M_1 (P_{max}^2 + h_2^2) \\
g_1 &= g_1 M_1 (P_{max} \cos \theta - h_2 \sin \theta) \\
&\quad + M_2 (P_{max} + l) \cos \theta - h_2 \sin \theta
\end{align*}
\]

In order to linearize this system relatively to the variable \( \theta \), we must choose \( u \) such that:
\[
u = 2 M_2 \frac{P_{max}^2}{h_2^2} \dot{\theta} - M_2 (P_{max} + l) \dot{\theta} + \frac{\theta}{\max} + M_2 \sin \theta + \frac{m_{11} - m_{11} \ddot{\theta}}{h_2^2} (\ddot{\theta} + K_\nu (\ddot{\theta} - \dot{\theta}) + K_p (\theta - \theta))
\] (6)

\( K_p \) and \( K_\nu \) are coefficients whose tuning will give the characteristics of the trajectory tracking (damping and speed).

The linearized system is then:
\[
\begin{align*}
\dot{\theta} &= \ddot{\theta} + K_p (\ddot{\theta} - \dot{\theta}) + K_\nu (\theta - \dot{\theta}) \\
\dot{\theta} &= \frac{\theta}{M_2} + h_2 \ddot{\theta} + (P_{max} + l) \dot{\theta} - g \sin \theta
\end{align*}
\] (7)

Two choices are necessary to determine completely \( u \). The first one is the reference trajectories \( \ddot{\theta} = \ddot{\theta}_d \), \( \dot{\theta} = \dot{\theta}_d \) and \( \theta = \theta_d \) and the second one is the coefficient \( K_p \) and \( K_\nu \) in order to tune the reactivity of the trajectory tracking after a disturbance.

B. Calculating the reference trajectory

A 6 degrees polynomial has been chosen to define the reference trajectory for \( \theta \). Two others are determined by the initial conditions \( [\theta_0, \dot{\theta}_0] \) and two other degrees by the desired conditions before impact \( [\theta_-, \dot{\theta}_-] \). Lastly \( [\mu_1 = \dot{\theta}_1, \mu_2 = \ddot{\theta}] \) are used as parameters of an optimization problem. This problem forces the values before impact to our method, the impact speed of the state variables must then be zeroed: \( \dot{\theta}_- = 0 \) and \( \dot{\theta}_- = 0 \). Thus, from equation

IV. TWO CONCRETE EXAMPLES AND SIMULATIONS

A. Example 1: Stopping the walking with minimization of the command energy

In this example the goal is to stop the cart. According to our method, the impact speed of the state variables must then be zeroed: \( \dot{\theta}_- = 0 \) and \( \dot{\theta}_- = 0 \). Thus, from equation
Fig. 6. Reference trajectories (dotted line) and real trajectory (plain line) of the four state variables with $K_p = 400$ and $K_v = 40$

(2), the robot will stop whatever the matrix restitution $E_{st}$ is. Moreover, we must add the static balance equation to ensure that the robot does not move after the impact. It means that the projection of the CoM on the ground must lie inside the support polygon. The best way of enforcing this criterion is to force $l$ to be as close as possible from $0$ at the end of the free dynamic phase. Let’s choose:

$$\begin{align*}
\theta_0 &= 0 \\
\dot{\theta}_0 &= \dot{\theta}_f = 0 \\
l_0 &= 0 \\
\dot{l}_0 &= \dot{l}_f = 0
\end{align*}$$

In this case, we choose $k_2 \gg k_1, k_3$ so that the main condition $l_\sim = 0$ weights bigger than the secondary condition. By choosing $k_1, k_3 = 1$ and $k_2 = 10^5$, with the initial conditions $\theta_0 = \pi$, $\dot{\theta}_0 = 0$, and $l_0 = 0$, the reference trajectories $\dot{\theta}_d, \dot{\theta}_d$ and $\dot{l}_d$ are shown in figure 5.

In order to study the trajectory tracking, a significant disturbance has been introduced at the beginning of the simulation. Indeed the initial condition on $\theta$ have been set to $\theta_0 = \pi$, whereas the value used to generate the trajectory was $\theta_0 = \pi$. The results of this simulation are presented in figure 6 and 7 with different values for $K_p$ and $K_v$.

These curves give two important leads to choose the coefficient $K_p$ and $K_v$. Regarding the two first curves of each graph concerning $\theta$ and $\dot{\theta}$, we observe some considerable oscillations when $m = 0.1$. This behavior may cause undesired impact between the table and the ground. So selecting $K_p$ and $K_v$ such that $m = 1$ avoid this issue. Furthermore regarding the two last curves of each graph concerning $l$ and $\dot{l}$, we do not obtain exactly the values before impact which was stated by the optimization problem. Indeed the system is underactuated and the tracking is done to fit the trajectory of $\theta$. Thus any disturbance on $\theta$ induces significant variations on $u$ and so on $l$. That is why a fast non-oscillatory behavior leads to values of $l$ and $\dot{l}$

closer to the ones desired. In our simulation we then choose $K_p = 400$ and $K_v = 40$ which leads to such a behavior.

B. Example 2: Generation of a full step

In this example, the complete methodology to generate a full step is exposed. Firstly we must decide the values of the state variables we wish to reach at the end of the free dynamic phase. Let’s choose:

$$\begin{align*}
\theta_f &= -\frac{\pi}{4} \\
\dot{\theta}_f &= -\pi s^{-1} \\
l_f &= 0.2m \\
\dot{l}_f &= 0.5m.s^{-1}
\end{align*}$$

Secondly, using the reverse dynamic method and feeding it with the initial conditions $\theta_{ini} = \theta_f$, $\dot{\theta}_{ini} = -\dot{\theta}_f$, $l_{ini} = l_f$ and $\dot{l}_{ini} = \dot{l}_f$, we can determined the values of the state variables just after the impact. These values are read at the end of the simulation shown in figure 8:

$$\begin{align*}
\theta_+ &= 0 \\
\dot{\theta}_+ &= 1.8 s^{-1} \\
l_+ &= 0.22m \\
\dot{l}_+ &= 0m.s^{-1}
\end{align*}$$
As a function of $\dot{\theta}_+$ and $\dot{l}_+$

<table>
<thead>
<tr>
<th>inelastic impact (e = 1)</th>
<th>$\dot{\theta}<em>+ = \dot{\theta}</em>-$</th>
<th>$\dot{l}<em>+ = \dot{l}</em>-$</th>
<th>$\dot{\theta}_- = -1.8s^{-1}$</th>
<th>$\dot{l}_- = 0m.s^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>intermediary impact (e = 0.5)</td>
<td>$\dot{\theta}_+ = \frac{\theta}{\ell}$</td>
<td>$\dot{l}_+ = \ell - 0.45\theta$</td>
<td>$\dot{\theta}_- = 3.6s^{-1}$</td>
<td>$\dot{l}_- = 1.62m.s^{-1}$</td>
</tr>
<tr>
<td>elastic impact (e = 0)</td>
<td>$\dot{\theta}_+ = 0$</td>
<td>$\dot{l}_+ = \ell - 0.96\theta$</td>
<td>No solution</td>
<td></td>
</tr>
</tbody>
</table>

TABLE III

RELATIONSHIP BETWEEN SPEED BEFORE AND AFTER IMPACT

![Graph](image)

Fig. 9. Reference trajectories to ensure an impact with $\dot{\theta}_- \approx 1.8s^{-1}$, $\dot{l}_- \approx 0.22m$ and $\dot{l}_- \approx 0m.s^{-1}$. The duration has been arbitrarily fixed to 1s.

Thirdly equation (3) enables the calculation of the values of the state variables just before the impact depending on the restitution matrix $E_e$. If we consider this matrix of the form:

$$E_e = \begin{bmatrix} 0 & e \\ e & 0 \end{bmatrix}$$

with $e$ the restitution coefficient, table III sums up the effect of the choice of this coefficient on the values of the state variables before impact. For this example we consider the impact as completely inelastic.

Lastly we must generate the control law that lead our robot to this pre-impact state. The reference trajectory is defined by the solution of the optimization problem:

$$\min_{\mu_1, \mu_2} J = k_1 (\ell - 0.22)^2 + k_2 \ell^2$$

$$\dot{\theta}_d = P(\mu_1, \mu_2) \text{ and } \theta_0^* \text{ (6)}$$

$$\text{(7)}$$

As velocity has no reason to be more important than the position in this case, we choose $k_1 = k_2 = 1$. The reference trajectory obtained are shown in figure 9.

The full step is completely defined. The system must now calculate $u$ at each time step from the values of the current state variables using equation (7).

V. CONCLUSION AND FUTURE WORKS

This paper proposes a new method to implement a control law for biped robot based on a two-phase step and control of the velocity when the foot of the robot impact the ground. It can be summed up in this way:

- Decomposing the step in two phase: the free dynamic phase and the impact preparation phase.
- Choosing values at the end of the free dynamic for the state variables.
- Determining the corresponding values of the state variables just after the impact by using the reverse dynamic method.
- Determining the corresponding values of the state variables just before the impact by using the restitution matrix.
- Calculating an optimal reference trajectory that enables to reach the desired pre-impact values of the state variables.
- Calculating the command at each time step from the error between the reference and the real trajectory.

Our future works will concentrate on different tracks. The priority one is to design a control law for the impact preparation phase that is much consistent with an under-actuated system like the cart-table. Afterwards more subtle improvements could be made, especially about the impact model.

The final goal is to transcribe this method on the prototype SHERPA in consideration of its desired walking speed. Moreover this speed must be adaptive to the speed of the person leading SHERPA. Thus an important work on the definition of the value of the state variables at the end of the free dynamic should be done along with the definition of the step length.

REFERENCES