Supplier Selection Using Interpolative Boolean Algebra and Logic Aggregation

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Abstract. The interest of the decision makers in the selection process of suppliers is constantly growing as a reliable supplier reduces costs and improves the quality of products/services. This process is essentially reducible to the problem of multi-attribute decision-making. Namely, the large number of quantitative and qualitative attributes is considered. This paper presents a model of supplier selection. Weighted approach for solving this model was used combined with logical interactions between attributes. Setting logical conditions between attributes was carried out by using the Boolean Interpolative Algebra. Then the logical conditions are transformed into generalized Boolean polynomial that is through logical aggregation translated into a single value. In this way, the ranking of the suppliers is provided. Using this model managers will be able to clearly express their demands through logical conditions, i.e. will be able to conduct a comprehensive analysis of the problem and to make an informed decision.

Keywords: Fuzzy logic, Interpolative Boolean algebra, Generalized Boolean polynomial, Logical aggregation, Supplier selection problem.

1 Introduction

Selection of the most favorable supplier is a strategic decision that ensures profitability and long-term survival of the company. The company's goal is to carefully choose the right suppliers who will provide the requested product at the right time. In most cases, the strengths and weaknesses of suppliers vary over time, so that managers are in a position to have to make complex decisions in the selection.

In real situations, managers often want to set up mutual relationships between the attributes in order to bring the best possible decision. As conventional fuzzy methods of multi-attribute decision-making do not allow setting of logical interactions between attributes, i.e. they are not in the Boolean frame, the consistent fuzzy logic is introduced. The basis of this approach is interpolative realization of Boolean algebra that transforms logical conditions between attributes into a generalized Boolean polynomial, then the set logical conditions merge into a single value by using the logic

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aggregation. In this way, a suitable tool is developed for mapping linguistic requirements of decision-makers in the appropriate Boolean polynomial.

The paper is organized as follows: Section 2 provides an introduction to Boolean consistent fuzzy logic and transformation of logic functions in generalized Boolean polynomial and applying Boolean aggregation. Section 3 analyzes the problem of selecting suppliers and using Boolean consistent fuzzy logic is presented. The paper concludes with Section 4 where the conclusive considerations are presented.

2 Boolean Consistent Fuzzy Logic

Classical Boolean algebra [1] is based on the statements that are true/false, yes/no, white/black. However, there are situations in which classical two-valued realization of Boolean algebra is not adequate. Often it is impossible to express in the absolutely precise way, but we are forced to use vague constellations. In this regard, the necessity of gradation in relations is recognized, so that fuzzy logic is introduced [2], which in its implementation uses the principle of Many-valued logic [3]. The main advantage of fuzzy logic is that it is very close to human perception and does not require completely exact data. Indicating that it is not precisely defined by an element belonging to a certain set, but elements can take values from the interval [0,1]. However, the main disadvantage of fuzzy logic is that it is not in the Boolean frame.

Extension of fuzzy logic by introducing logical interactions is enabled by using Interpolative Boolean Algebra - IBA [4,5], which is a consistent generalization of fuzzy logic. IBA is a real valued, and/or, [0,1] value realization of Boolean algebra [6]. Under the IBA all Boolean axioms and theorems apply.

IBA consists of two levels: a) symbolic or qualitative - at this level the elements structure is defined, and is the matter of final Boolean framework, b) semantic or value - at this level the values are introduced in this way to preserve all the laws set symbolically, in the general case it is a matter of interpolation [7,8].

In fact, the IBA represents an atomic algebra (as a result of a finite number of elements) and is substantially a different gradation approach in comparison to fuzzy approach. Atoms as the simplest elements of algebra play a fundamental role. One of the basic concepts of symbolic levels is the structure of IBA elements. The structure of any IBA element and/or the principle of structural functionality [9] is a bridge between these two levels and basis of generalization, as long as the elements are value-independent. The structure of the analyzed elements determines which atom is (of the final set of elements IBA) included and/or not included. The principle of structural functionality indicates that the structure of any element of IBA may be directly calculated based on the structure of its components. The structure is an independent value and that is the key to preserving Boolean laws both at the symbolic and at the level of values [10]. This principle requires that the IBA transformations are performed at the symbolic level before the introduction of value. Indicating that the negation is treated in a different way, at a structural level, rather than negated variable immediately transforms into value. Thus the observation of negation allows preservation of all Boolean laws. Also, within the IBA applies the law of excluded middle, the axiom of

Boolean logic, where $a \vee \neg a = 1$, which is not respected in the conventional fuzzy logic [11]. Based on all the foregoing, we conclude that fuzzy logic is not in the Boolean frame.

2.1 Generalized Boolean Polynomial and Logical Aggregation

IBA is an algebraic structure with elements which is represented by the Eq. (1) [12]:

$$\langle BA, \wedge, \vee, \neg \rangle \tag{1}$$

where BA is the set of finite elements, binary operators of conjunction $\wedge$ and disjunction $\vee$ and unary negation operator $\neg$, for which all Boolean axioms and theorems are valid.

Under the IBA every Boolean function can be uniquely transformed into the corresponding generalized Boolean polynomial (GBP) [8]. Technically, if any element of Boolean algebra can be represented in a canonical disjunction way, it can be represented also by appropriate GBP. And thus, it allows for the processing of the corresponding element of Boolean algebra into the value on the real interval [0, 1] using operators such as classical (+), classical (-) and generalized product ($\otimes$) [13]. Generalized product (GP) can be any function ($\otimes$): $[0,1] \times [0,1] \rightarrow [0,1]$ which meets all the requirements that one function be a t-norm (commutativity, associativity, monotonicity and limitation Eq. (2,3,4,5)), as well as additional non negativity condition, which is defined as Eq. (6) [7]:

1. $$A_i \otimes A_j = A_j \otimes A_i \tag{2}$$
2. $$A_i \otimes (A_j \otimes A_k) = (A_i \otimes A_j) \otimes A_k \tag{3}$$
3. $$A_i \leq A_j \Rightarrow A_i \otimes A_k \leq A_j \otimes A_k \tag{4}$$
4. $$A_i \otimes 1 = A_j \tag{5}$$
5. $$\sum_{K \in P(\Omega/S)} (-1)^{|K|} \otimes A_i(x) \geq 0,\ S \in P(\Omega),\ A_i(x) \in [0,1],\ A_i \in \Omega \tag{6}$$

Within the IBA, the method enabling unification of factors is referred to as Logical Aggregation (LA). The main task of LA is a merger of the primary attributes $\Omega = \{a_1, \ldots, a_n\}$ into a single value, which represents a given set, by using the logical/pseudological function. If we consider the problem of multi-attribute decision making, which is the subject of this paper, LA can be realized in two steps [14]:

1. The normalization of attributes values, which is represented by the Eq. (7):

$$||\cdot||: \Omega \rightarrow [0,1] \tag{7}$$

2. Aggregation of normalized attributes values into one, by using a logical aggregation or pseudological functions as LA operator, defined by Eq. (8) [15]:

$$Aggr\ [0,1]^n \rightarrow [0,1] \tag{8}$$

Boolean function enables the aggregation of factors, i.e. it is an expression that transforms into GBP. Pseudological function is a convex combination of GBP. LA is a technique that gives the user the most options in modeling and treating negation in the right way.

3 The Method of Solving the Problem of Supplier Selection by Using IBA

The problem which is analyzed in the paper is the selection of the best suppliers within a telecommunications company. The company specializes in the manufacture of the equipment necessary for building, monitoring and maintenance of telecommunication systems and wants to choose the best company for the supply of repeater transmission frequencies that allow coverage area without GSM signal or a very weak signal. Three suppliers' companies were considered that are ranked based on four basic attributes and nine sub-attributes (Table 1).

Table 1. Presentation of attributes and sub-attributes

Attributes	Sub-attributes	Attribute type	Unit	Max/Min
Production characteristics (K_1)	Technical performances (k_{11})	Quantitative	Excellent, Very good, Good, Satisfactory, Unsatisfactory	Max
	Product quality (k_{12})	Qualitative	Excellent, Very good, Good, Satisfactory, Unsatisfactory	Max
	Delivery time (k_{13})	Quantitative	Day	Min
Supplier profile (K_2)	Reference (k_{21})	Qualitative	Excellent, Very good, Good, Satisfactory, Unsatisfactory	Max
	Brand position (k_{22})	Qualitative	Excellent, Very good, Good, Satisfactory, Unsatisfactory	Max
Financial aspect (K_3)	Product price (k_{31})	Quantitative	Eur	Min
	Product costs (k_{32})	Quantitative	Eur	Min
Support and services (K_4)	Service (k_{41})	Qualitative	Excellent, Very good, Good, Satisfactory, Unsatisfactory	Max
	Technical support (k_{42})	Qualitative	Excellent, Very good, Good, Satisfactory, Unsatisfactory	Max

Within this paper it will be displayed how IBA can help managers include their preferences in a more sophisticated way compared to weights approach. In many techniques of decision making (conventional fuzzy) a weighted approach is used which allows exclusively linear relationship between the attributes. However, when solving problems with multi-attribute decision-making method, such as the problem of selection of suppliers, often attributes are interdependent and it is needed to establish between them the logical interactions. Logical interactions are based on the introduction of Boolean algebra operators $\wedge, \vee, \neg$, by which managers can more clearly show dependence and comparisons between attributes. In this way, a large number of real problems can be expressed by the Boolean algebra.

As part of Table 2 the quantitative and qualitative values of the sub-attributes are presented.

Table 2. The values of sub-attributes

	Production characteristics			Supplier profile		Financial aspect		Support and services	
	Tech perform.	Quality	Delivery Time	References	Brand	Price	Costs	Service	Tech. support
S_1	good	very good	30	good	satisfactory	250	120	excellent	excellent
S_2	very good	very good	45	excellent	very good	345	85	excellent	very good
S_3	satisfactory	good	30	good	good	275	110	good	excellent

The problem was analyzed in the initial interval $[1,5]$, the values of Table 2 are converted into the quantitative values presented in Table 3.

Table 3. Quantitative values of the sub-attributes

	Production characteristics			Supplier profile		Financial aspect		Support and services	
	Technical perform.	Quality	Delivery Time	References	Brand	Price	Costs	Service	Technical support
S_1	3	4	3	3	2	3	4	5	5
S_2	4	4	2	5	4	2	5	5	4
S_3	2	3	3	3	3	3	4	3	5

As mentioned above, fuzzy logic takes values from the interval $[0,1]$, it indicates that it is necessary to convert the value of the sub-attributes from the initial val $[1,5]$ to interval $[0,1]$, i.e. it is necessary to perform a normalization. After the normalization, the values of the sub-attributes are presented in Table 4.

Table 4. Normalized values of sub-attributes

	Production characteristics			Supplier profile		Financial aspect		Support and services	
	Technical perform.	Quality	Delivery Time	References	Brand	Price	Costs	Service	Technical support
S_1	0,6	0,8	0,6	0,6	0,4	0,6	0,8	1	1
S_2	0,8	0,8	0,4	1	0,8	0,4	1	1	0,8
S_3	0,4	0,6	0,6	0,6	0,6	0,6	0,8	0,6	1

In order to select the best supplier it is necessary to introduce a weighted sum of the attributes/sub-attributes, which is represented by Eq. (9):

$$w_1 * k_1 + w_2 * k_2 = p \, , \quad (9)$$

where w_1 and w_2 represent the weight in this model, k_1 and k_2 are the values of attributes/sub-attributes and p represents supplier's total point in interval $[0,1]$.

Managers believe that for the selection of suppliers, in this case, it is important to take into consideration sub-attributes Technical performances and Quality, as an sub-attribute does not exclude other. Hence, the logical relation was established by using the Boolean operator $\wedge$ and the sub-attribute function have the following form Eq. (10):

$$0{,}7 * (k_{11} \wedge k_{12}) + 0{,}3 * k_{13} = 0{,}7 * (k_{11} \otimes k_{12}) + 0{,}3 * k_{13} = p, \quad (10)$$

where weights w_1 and w_2 have following values 0.7 and 0.3. Also, within the equation was used standard product as appropriate operator of GP.

Weight sum for sub-attributes Reference and Brand position have values 0.6 and 0.4 respectively, shown in Eq. (11):

$$0{,}6 * k_{21} + 0{,}4 * k_{22} = p, \quad (11)$$

for sub-attributes Price and Costs weight sum are 0.7 and 0.3 Eq. (12):

$$0{,}7 * k_{31} + 0{,}3 * k_{32} = p \, , \quad (12)$$

and for sub-attributes Service and Technical support are 0.6 and 0.4 Eq. (13):

$$0{,}6 * k_{41} + 0{,}4 * k_{42} = p \, , \quad (13)$$

By the inclusion of normalized k-values from Table 4 sub-attributes functions were set and by the application of LA we obtain the values of alternatives (suppliers) for the four basic attributes as shown in the Table 5.

Table 5. Values of suppliers for the four basic attributes

	Production characteristics (K_1)	Supplier Profile (K_2)	Financial aspect (K_3)	Support and Services (K_4)
S_1	0,516	0,52	0,66	1
S_2	0,568	0,92	0,58	0,92
S_3	0,348	0,6	0,66	0,76

However, in real situations, managers often want to set the mutual relationships between the attributes in order to bring the best possible decision. This was enabled by using the logical conditions, presented hereinafter:

Condition 1: "If the production characteristics are at a high level, then the product is acceptable, if it is not at high level pay attention to the profile of the supplier, the financial aspect and the support and services." (Eq. (14))

$$k_1 \lor (\neg k_1 \land k_2 \land k_3 \land k_4) \quad (14)$$

Condition 2: "If a supplier profile is satisfying he should also have good production characteristics, if the profile of the supplier is not satisfactory attention should be paid to the financial aspect and the support and services." (Eq. (15))

$$(k_2 \land k_1) \lor (\neg k_2 \land k_3 \land k_4) \quad (15)$$

Condition 3: "If the financial aspect is high, attention should be paid to the manufacturing characteristics, if not high, attention should be paid to profile of supplier." (Eq. (16))

$$(k_3 \land k_1) \lor (\neg k_3 \land k_2) \quad (16)$$

Each of these logical conditions is transformed to the GBP, by using standard product as appropriate operator of GP. Transformation is given in the following steps Eq. (17):

$$\begin{aligned} k_1 \lor (\neg k_1 \land k_2 \land k_3 \land k_4) \\ &= k_1 + (\neg k_1 \land k_2 \land k_3 \land k_4) - k_1 \otimes (\neg k_1 \land k_2 \land k_3 \land k_4) \\ &= k_1 + \big((1 - k_1) \otimes k_2 \otimes k_3 \otimes k_4\big) - k_1 \\ &\otimes \big((1 - k_1) \otimes k_2 \otimes k_3 \otimes k_4\big) \\ &= k_1 + k_2 \otimes k_3 \otimes k_4 - k_1 \otimes k_2 \otimes k_3 \\ &\otimes k_4 \end{aligned} \quad (17)$$

In the same way the remaining two logical conditions are transformed, which is represented by the Eq. (18,19):

$$(k_2 \land k_1) \lor (\neg k_2 \land k_3 \land k_4) = k_2 \otimes k_1 + k_3 \otimes k_4 - k_2 \otimes k_3 \otimes k_4 \quad (18)$$

$$(k_3 \land k_1) \lor (\neg k_3 \land k_2) = k_2 - k_2 \otimes k_3 + k_3 \otimes k_1 \quad (19)$$

In the presented GBP equations we will introduce the attributes values from Table 5 based on which by using LA we obtain the values in Table 6.

Table 6. The values of logical conditions for three suppliers (S_1, S_2, S_3)

	Condition 1	Condition 2	Condition 3
S_1	0,682	0,585	0,517
S_2	0,78	0,565	0,715
S_3	0,544	0,409	0,433

The final ranking of suppliers is obtained by placing another of the weighted function, where instead of individual values of sub-attributes we introduce previously mentioned logical conditions, shown in Eq. (20):

$$0{,}5 * (k_1 \vee (\neg k_1 \wedge k_2 \wedge k_3 \wedge k_4)) + 0{,}2 * ((k_2 \wedge k_1) \vee (\neg k_2 \wedge k_3 \wedge k_4) + 0{,}3 * (k_3 \wedge k_1) \vee (\neg k_3 \wedge k_2) = p \quad (20)$$

Weights for conditions are 0.5, 0.2 and 0.3 respectively, and p represents the final supplier rank.

By entering the obtained values of logical conditions of Table 6 in the expression and by application of pseudo logical aggregation we obtain values of Table 7.

Table 7. The final ranking of suppliers

S_1	0,613
S_2	0,717
S_3	0,484

From Table 7 we can see that the order of suppliers is as follows: $S2>S1>S3$.

4 Conclusion

The reason of analysis of the presented model is primarily to provide practical support to decision-makers i.e. managers when choosing suppliers in the telecommunications sector. Also, with the use of logical conditions managers are able to present more clearly their requirements. In this way, they can make a more comprehensive and better decision than would be the case with conventional fuzzy methods which are not in the Boolean framework and that in a different way treat negation.

In addition to solving the observed problems in this paper is used the weighted approach combined with the Boolean consistent fuzzy logic. IBA logic enabled the transformation of logic functions to a generalized Boolean polynomial. While by the use of Logical/pseudological aggregation GBP is reduced to values. In this way we achieved the ranking of suppliers ($S2>S1>S3$). What makes this logic more suitable way to solve these types of problems compared to conventional fuzzy logic is that the structural transformations are performed before the introduction of values.

Further research will be directed towards the inclusion of logical conditions into the multi-attribute decision-making method AHP, TOPSIS, ELECTRE and comparison of the obtained results.

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