

Repetition-free binary words of minimal density

Pascal Ochem
LRI, CNRS
ochem@lri.fr

December 9, 2007

The latest version of this file is at: <http://www.lri.fr/~ochem/morphisms/conj.pdf>

Let $\rho(x)$ (resp. $\rho(x^+)$) be the minimal density of a letter in an infinite binary word with no repetition of exponent $\geq x$ (resp. $> x$). The function ρ has been defined in [1] and also studied in [2].

Conjecture 1 For every integer $n \geq 4$,

1. $\rho([n-1, \overline{1, n-3}]) = \rho(n) = [0, n-1, \overline{1, n-3}]$,
2. For $k \in \mathbb{N}$, $\rho(U_{n,k}^+) = \rho(U_{n,k+1}) = [0, n, (1, n-2)^k, \overline{1, n-3}]$.

where:

$U_{n,k} = n+1 - \frac{D_{n,k-1}+2}{D_{n,k}}$, $D_{n,-1} = -1$, $D_{n,0} = 1$, $D_{n,k+1} = nD_{n,k} - D_{n,k-1}$.
The values of $\rho(x)$ are given by the sturmian word of density (or slope) $\rho(x)$.

References

- [1] R. Kolpakov, G. Kucherov, and Y. Tarannikov. On repetition-free binary words of minimal density, *Theoret. Comput. Sci.* **218** (1999), 161–175.
- [2] P. Ochem. Letter frequency in infinite repetition-free words, *Theoret. Comput. Sci.* **380(3)** (2007), 388–392.