Complexity dichotomy for oriented homomorphism of planar graphs with large girth☆

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Abstract

We consider the complexity of oriented homomorphism and two of its variants, namely strong oriented homomorphism and pushable homomorphism, for planar graphs with large girth. In each case, we consider the smallest target graph such that the corresponding homomorphism is NP-complete. These target graphs T_4, T_5, and T_6 have 4, 5, and 6 vertices, respectively. For $i \in \{4, 5, 6\}$ and for every g, we prove that if there exists a (bipartite) planar oriented graph with girth at least g that does not map to T_i, then deciding homomorphism to T_i is NP-complete for (bipartite) planar oriented graphs with girth at least g.

Keywords: Oriented homomorphism, Planar, NP-completeness.

1. Introduction

Esperet, Montassier, Ochem, and Pinlou [7] have proved that for many types of coloring, there exists an integer g such that deciding whether a planar graph with girth g is colorable is NP-complete, whereas every planar graph with girth at least $g + 1$ is colorable. In this paper, we obtain similar results for homomorphism to three interesting oriented graphs.

An oriented graph is a directed graph without loops, opposite arcs, nor multiple arcs. Equivalently, an oriented graph is obtained by orienting every edge of a simple graph. We denote by $V(G)$ the vertex set and by $A(G)$ the arc set of the oriented graph G. A homomorphism from an oriented graph G to an oriented graph T is a mapping $m : V(G) \to V(T)$ such that for every $\overrightarrow{uv} \in A(G)$, we have $\overrightarrow{m(u)m(v)} \in A(T)$. If G admits a homomorphism to T, then we say for short that G maps to T, or that G admits a T-coloring.

A series of recent papers [11, 4, 8, 5] considers the complexity of deciding homomorphism of an oriented graph to the tournament T_4 depicted in Figure 1(a). Homomorphism is decidable in polynomial time for every tournament with at most 4 vertices other than T_4. Each new paper shows that the problem is NP-complete on a smaller graph class. In Section 2, we prove Theorem 1 which improves these results.

A k-vertex is vertex of degree k. Let $\mathcal{P}_g$ denote the class of planar graphs with girth at least g. Therefore, $\mathcal{P}_{g+1}$ is a proper subclass of $\mathcal{P}_g$. We will use the well-known fact that graphs in $\mathcal{P}_6$ are 2-degenerate (see e.g [2]).

We also recall the definition of DAG-depth from [9]. For an oriented graph G and a vertex $v \in V(G)$, let $R(v)$ denote the subgraph of G induced by the vertices reachable from v. The reachable fragments of G are the graphs in the set $\{R(v) : v \in V(G)\}$ that are maximal with respect to the subgraph order. The DAG-depth $ddp(G)$ of an oriented graph G is defined inductively as follows: if $|V(G)| = 1$, then $ddp(G) = 1$. If G has a single reachable fragment, then $ddp(G) = 1 + \min\{ddp(G \setminus v) : v \in V(G)\}$. Otherwise, $ddp(G)$ equals the maximum over the DAG-depths of the reachable fragments of G.

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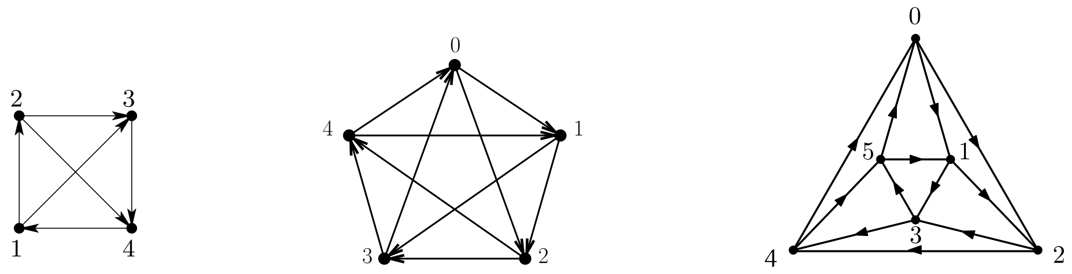

(a) (b) (c)

Figure 1: The tournaments T_4, T_5, and the oriented graph T_6.

Theorem 1. *For any fixed $g \geqslant 3$, deciding whether an oriented graph G maps to T_4 is NP-complete, even if G is restricted to be in $\mathcal{P}_g$, bipartite, subcubic, with DAG-depth 3, with maximum outdegree 2 and maximum indegree 2, and such that one part of the bipartition contains every 3-vertex.*

Borodin, Kostochka, and Ivanova [1] have considered homomorphism to the regular tournament T_5 depicted in Figure 1(b) and obtained the following.

Theorem 2. *[1] Every oriented graph in $\mathcal{P}_{12}$ maps to T_5.*

Notice that T_5 is not the only tournament on 5 vertices that can color planar graphs with large enough girth [13]. However, the *strong oriented chromatic number* introduced by Nešetřil and Raspaud [12] of an oriented graph is at most 5 if and only if it maps to T_5. We prove the following result in Section 4.

Theorem 3. *Let g be a fixed integer. Either every oriented graph in $\mathcal{P}_g$ maps to T_5 or it is NP-complete to decide whether a graph in $\mathcal{P}_g$ maps to T_5. Either every oriented bipartite graph in $\mathcal{P}_g$ maps to T_5 or it is NP-complete to decide whether a bipartite graph in $\mathcal{P}_g$ maps to T_5.*

Klostermeyer and MacGillivray [11] have considered the following variation of oriented homomorphism. Given an oriented graph G and a subset X of vertices of G, the graph obtained from G by reversing the direction of the arcs in the cut $(X, G \setminus X)$ is said to be *push equivalent* to G. The oriented graph G admits a *push homomorphism* to an oriented graph T if there exists a graph G' such that G' is push equivalent to G and G' maps to T. The pushable chromatic number of G is then defined as the minimum number of vertices of a graph T such that G has a push homomorphism to T.

Let T_6 be the oriented graph depicted in Figure 1(c) with vertex set $\{0, 1, \ldots, 5\}$ such that $\overrightarrow{ij}$ is an arc if and only if $j = i + 1 \pmod 6$ or $j = i + 2 \pmod 6$. Klostermeyer and MacGillivray [11] have shown in particular that the following statements are equivalent (see Corollary 4 and Lemma 16 in [11]).

- G maps to T_6.
- G admits a push homomorphism ot T_6.
- The pushable chromatic number of G is at most 3.

They also obtain that push homomorphism to T is polynomial time solvable if T maps to the circuit of length 4 and is NP-complete otherwise. This implies that deciding whether G has pushable chromatic number at most k is polynomial time solvable if $k \leqslant 2$ and is NP-complete otherwise. So, T_6-coloring is NP-complete.

Borodin, Kostochka, Nešetřil, Raspaud, and Sopena [3] have considered homomorphism to T_6 and obtained the following.

Theorem 4. *[3] Every oriented graph in $\mathcal{P}_{16}$ maps to T_6.*

Their motivation was that T_6 is itself planar. Then, Borodin, Kim, Kostochka, and West [2] obtained as a consequence of their main result (Theorem 2.5) that every oriented graph with girth at least 13 and maximum average degree strictly smaller than $\frac{16}{7}$ maps to T_6. They mistakenly conclude (Corollary 3.4) that every oriented graph in $\mathcal{P}_{13}$ maps to T_6. The correct corollary is again that every oriented graph in $\mathcal{P}_{16}$ maps to T_6 and thus does not improve Theorem 4.

We prove the following result in Section 5.

Theorem 5. *Let g be a fixed integer. Either every oriented graph in $\mathcal{P}_g$ maps to T_6 or it is NP-complete to decide whether a graph in $\mathcal{P}_g$ maps to T_6. Either every oriented bipartite graph in $\mathcal{P}_g$ maps to T_6 or it is NP-complete to decide whether a bipartite graph in $\mathcal{P}_g$ maps to T_6.*

Section 3 describes graphs that do not map to T_5 or T_6. Using these graphs, we obtain the following corollary of Theorems 3 and 5.

Corollary 6.

- *Deciding whether an oriented planar graph G maps to T_5 is NP-complete, even if G has girth 7, or if G is bipartite with girth 6.*
- *Deciding whether an oriented planar graph G maps to T_6 is NP-complete, even if G has girth 9, or if G is bipartite with girth 8.*

2. Proof of Theorem 1

Oriented homomorphism in general is clearly in NP. We reduce the NP-complete problem [6] RESTRICTED PLANAR 3-SAT. This variant of SAT is such that:

- every clause has size 2 or 3,
- every variable appears exactly twice positively and once negatively,
- the variable-clause incidence graph is planar.

Let us call a k-clause a clause of size k. An *alternating path* is an oriented path such that the length is even and at least 4, every vertex is a source or a sink, and the extremities are sinks. Alternating paths are represented Figure 2 by a dashed segment with an arrow at both ends.

Given an instance I of RESTRICTED PLANAR 3-SAT, we construct a corresponding oriented graph G. We take one copy of the variable gadget depicted in Figure 2 per variable of I, one copy of the 2-clause gadget depicted on the left of Figure 3 per 2-clause of I, and one copy of the 3-clause gadget depicted on the left of Figure 4 per 3-clause of I. In the gadget of the variable v, the vertices v_1' and v_2' correspond to the two occurrences of the positive literal of v and $\overline{v}'$ corresponds to the occurrence of the negative literal of v. In the gadget of the clause c, the vertex ℓ_i corresponds to the i^{th} literal in c. For every occurrence of a literal of a variable v in a clause c, we identify the vertex corresponding to this occurrence in the gadget of v with the vertex corresponding to this literal in the gadget of c.

Let us describe the variable gadget. Every T_4-coloring of the alternating path is such that if one extremity is colored 1, then the other extremity is colored 1, and if one extremity has any color distinct from 1, then the other extremity can be colored 2, 3, or 4. We consider first the possible T_4-colorings of the graph at the top of Figure 2. The vertex u_3 cannot be colored 1 since such a coloring cannot be extended to the vertices on the left of u_3. Similarly, u_3 cannot be colored 4 since such a coloring cannot be extended to the vertices on the right of u_3. Thus, u_3 can be colored 2 or 3, u_2 can be colored 1 or 2, and u_1 can be colored 1 or 4. This graph is used in the construction of the variable gadget depicted at the bottom of Figure 2. The variable gadget is such that if $\overline{v}$ is colored 1 (resp. 4), then v_1 and v_2 are colored 4 (resp. 1). Again, by the properties of the alternating path, if a vertex $\ell \in \{v_1, v_2, \overline{v}\}$ is colored 1, then the vertex ℓ' is colored 1, and if ℓ is colored 4, then ℓ' can be colored 2, 3, or 4.

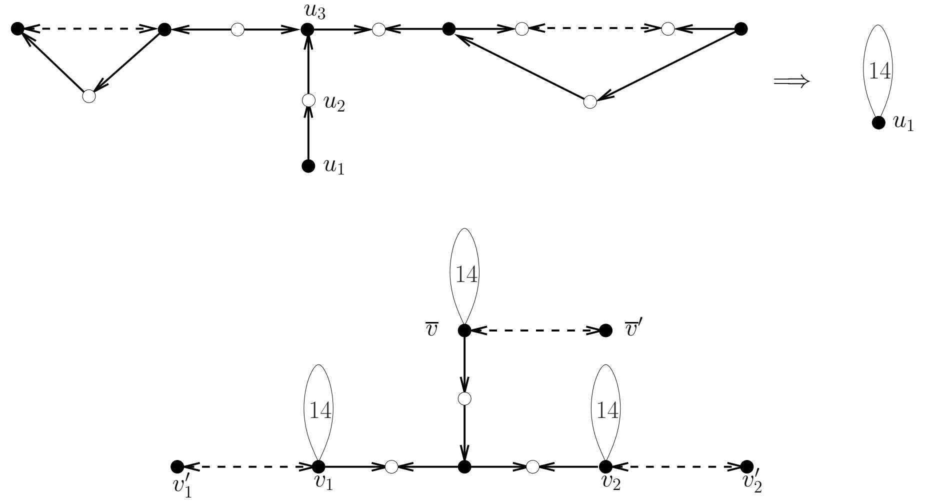

Figure 2: The variable gadget.

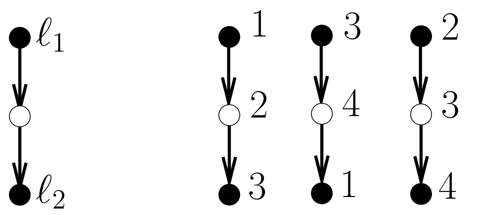

Figure 3: The 2-clause gadget.

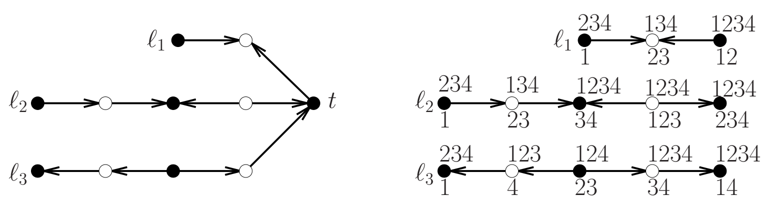

Figure 4: The 3-clause gadget.

Thus, the set of colors $\{2, 3, 4\}$ is associated to the boolean value *true* and $\{1\}$ is associated to the boolean value *false*.

Now let us assume that the vertices ℓ_i of the clause gadgets are precolored according to their corresponding literal. On the right of Figure 3, we give coloring extensions for a satisfied 2-clause (FT, TF and TT, respectively). If a 2-clause is not satisfied then both ℓ_1 and ℓ_2 are precolored 1 and this precoloring cannot be extended. On the right of Figure 4, we give the possible color extensions of the three paths of the 3-clause gadget, both in the case of a true literal (above the path) and in the case of a false literal (below the path). If a 3-clause is satisfied, then at least one of its literal is true and the precoloring can be extended to a T_4-coloring of the clause gadget. Indeed, if the literal corresponding to ℓ_1 (resp. ℓ_2, ℓ_3) is true, then the precoloring can be extended such that t is colored 4 (resp. 1, 2).

If a 3-clause is not satisfied, then the precoloring cannot be extended to a T_4-coloring of the clause gadget. Indeed, we have $c(\ell_1) = c(\ell_2) = c(\ell_3) = 1$, thus $c(t) \notin \{3, 4\}$ because $c(\ell_1) = 1$, $c(t) \neq 1$ because $c(\ell_2) = 1$, and $c(t) \notin \{2, 3\}$ because $c(\ell_3) = 1$, so $c(t) \notin \{1, 2, 3, 4\}$ and the clause gadget does not map to T_4.

Thus, G maps to T_4 if and only if I is satisfiable.

Let us show that G satisfies the conditions of Theorem 1. It is easy to check that G is planar, bipartite, subcubic, with maximum outdegree 2 and maximum indegree 2, and that every 3-vertex is in the "black" part of the bipartition. We can also assume that the girth is large, since every cycle contains an alternating path whose length can be as large as needed. The variable gadget contains the unique maximal reachable fragment of G. It is rooted in the in-neighbor of $\overline{v}$. This reachable fragment, and thus G, has DAG-depth 3. The girth of G is large and the length of a directed path is at most 4. This implies that G has K-width 1, i.e., there exists at most one directed path between two vertices, and that G is acyclic, i.e., G has no circuit.

3. Planar graphs that do not map to T_5 or T_6

Figure 5 shows an oriented bipartite graph G_6 in $\mathcal{P}_6$ that does not map to T_5. Suppose for contradiction that G_6 admits a homomorphism h to T_5. Without loss of generality, $h(t) = 0$. So $h(b) = c \in \{2, 3, 4\}$. Let us set $S = \{2, 3, 4\} \cap \{c + 2, c + 3, c + 4\}$ where additions are done modulo 5. Notice that $\{h(x_1), h(x_2), h(x_3)\} \subset S$. Since $c \neq 0$, S consists in a set of at most two consecutive values. We have a contradiction if $|S| = 1$ because $h(x_1) \neq h(x_2)$. If $S = \{t, t + 1\}$, then we must have $h(x_i) = t$ and $h(x_{i+1}) = t + 1$ for some $i \in \{1, 2\}$, which is impossible.

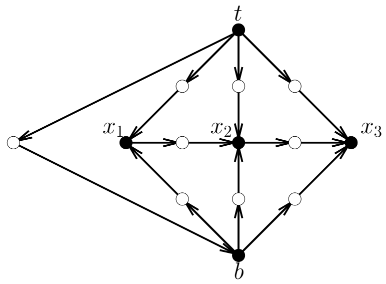

Figure 5: The bipartite graph G_6 in $\mathcal{P}_6$ that does not map to T_5.

Nešetřil, Raspaud, and Sopena [13] have shown that the oriented chromatic number of the class of planar graphs with girth at least 7 is at least 6. This implies that there exists an oriented graph G_7 in $\mathcal{P}_7$ that does not map to T_5.

Figure 6 shows an oriented graph G_9 in $\mathcal{P}_9$ that does not map to T_6 and Figure 7 shows an oriented bipartite graph G_8 in $\mathcal{P}_8$ that does not map to T_6.

To see that G_9 does not map to T_6, consider first the subgraph depicted on the left of Figure 6. Suppose that this subgraph has a T_6-coloring such that t is colored 0, without loss of generality.

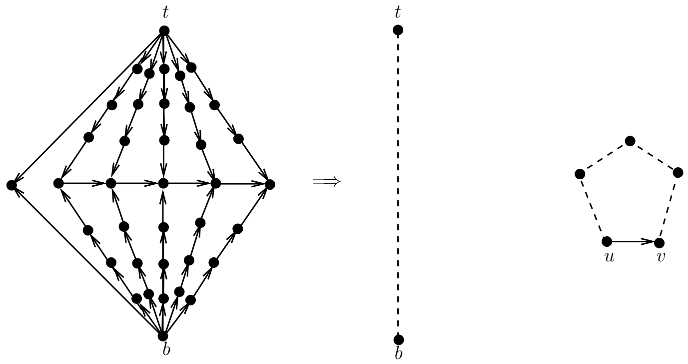

Figure 6: The graph G_9 in $\mathcal{P}_9$ that does not map to T_6.

Since t and b have a common out-neighbor, b must be colored 5, 0, or 1. Suppose that b is colored 1. Then the directed paths starting from t (resp. b) forbid that a vertex in the horizontal directed path is colored 3 (resp. 4). This is a contradiction since the directed path on 5 vertices does not map to $T_6 \setminus \{3,4\}$. Hence, b cannot be colored 1. By symmetry, b cannot be colored 5 and thus b is colored 0. So, t and b get the same color in any T_6-coloring. This implies that the whole graph G_9 depicted on the right of Figure 6 does not map to T_6.

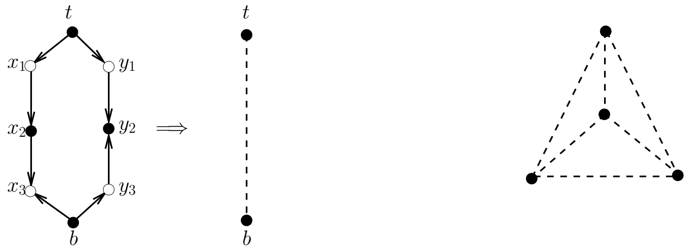

Figure 7: The bipartite graph G_8 in $\mathcal{P}_8$ that does not map to T_6.

To see that G_8 does not map to T_6, consider first the subgraph depicted on the left of Figure 7. Suppose that this subgraph has a T_6-coloring such that t is colored 0, without loss of generality. The path $tx_1x_2x_3b$ (resp. $ty_1y_2y_3b$) implies that b is not colored 0 (resp. 3). We associate to each vertex i of T_6 its *anti-twin* $i+3 \pmod 6$. Thus t and b cannot have the same color nor anti-twin colors. The graph G_8 depicted on the left of Figure 6 contains 4 vertices that are pairwise linked by the mentioned subgraph. No two of them can have the same color nor anti-twin colors. Since T_6 consists in only 3 pairs of anti-twins, G_8 does not map to T_6.

4. Proof of Theorem 3

We suppose that there exists a graph $H \in \mathcal{P}_g$ that does not map to T_5 and is minimal for the subgraph order. Notice that T_5 is a circulant tournament, so H must be 2-connected since otherwise we can obtain a T_5-coloring of H from the T_5-colorings of the 2-connected components of H. Thanks to Theorem 2 and the graph G_6 in Section 3, we can assume that $6 \leqslant g \leqslant 11$. Since $H \in \mathcal{P}_6$, we have that H is 2-degenerate and thus $\delta(H) = 2$. Let v be a 2-vertex of H and let x_1 and x_2 be the neighbors of v. Notice that the tournament obtained by reversing every arc of T_5 is isomorphic to T_5. So, by possibly reversing every arc of H, we assume without loss of generality that H contains $\overrightarrow{x_1v}$. The graph $H' = H \setminus v$ is a subgraph of H and thus admits at least one T_5-coloring. Let M be the set of T_5-colorings m of H' such that $m(x_1) = 0$. Let S be the set $\{m(x_2) \mid m \in M\}$.

So S is non-empty. Moreover, $S \subset \{0,1\}$ if $\overrightarrow{vx_2} \in A(H)$ and $S \subset \{2,3\}$ if $\overrightarrow{x_2v} \in A(H)$, since otherwise H would map to T_5.

Now we use H' to construct a series of gadgets U_X with two specified vertices z and z' on its outerface such that there exists a T_5-coloring of U_X such that z is colored 0 and z' is colored c if and only if $c \in X$. Our goal is to obtain $U_{2,3}$. By setting $z = x_1$ and $z' = x_2$, we obtain $U_S = H'$. So, if $S = \{2,3\}$, then we are done. If $|S| = 1$, then we obtain U_0 by identifying the vertices z' of two copies of U_S, such that the vertices z and z' of U_0 correspond to the vertices z of both copies of U_S. If $S = \{0,1\}$, then we obtain U_0 from two copies U_S^1 and U_S^2 of U_S by identifying z of U_S^1 with z' of U_S^2, and z' of U_S^1 with z of U_S^2, and choosing z and z' of U_S^1 as z and z' of U_0, respectively. Now, we obtain $U_{2,3}$ by adding a directed path $zy_1y_2y_3z'$ to U_0 such that the vertex z of $U_{2,3}$ is z and the vertex z' of $U_{2,3}$ is y_2. It is indeed easy to check that if z is colored 0 then the vertex z' of U_0 is colored 0 and y_2 can be colored 2 or 3.

The reduction is from PLANAR C_5-COLORABILITY, which is known to be NP-complete for planar graphs with girth at least 7 [7]. Let I be an instance of PLANAR C_5-COLORABILITY with girth at least 7. We obtain the oriented graph G from I by replacing every edge ab of I by a copy of $U_{2,3}$ such that $a = z$ and $b = z'$. Then I admits a C_5-coloring if and only if G maps to T_5 (a vertex colored $c \in \{0, \dots, 4\}$ in I is colored $2c \pmod 5$ in G).

Let us show that G satisfies the conditions of Theorem 3. Notice that in every case, the girth of $U_{2,3}$ is at least g. The distance between z and z' in $U_{2,3}$ is at least 2, so the distance between old vertices in G is at least 2 and the cycles of G that are not contained in a copy of $U_{2,3}$ have length at least $2 \times 7 = 14 > g$. So G contains no cycle of length strictly smaller than g. Finally, suppose that H is bipartite. Then $U_{2,3}$ is bipartite and the distance between z and z' is even. Thus G is bipartite too.

5. Proof of Theorem 5

We suppose that there exists a graph $H \in \mathcal{P}_g$ that does not map to T_6 and is minimal for the subgraph order. Notice that T_6 is a circulant graph, so H must be 2-connected since otherwise we can obtain a T_6-coloring of H from the T_6-colorings of the 2-connected components of H. Thanks to Theorem 4 and the graph G_8 in Section 3, we can assume that $8 \leqslant g \leqslant 15$. Since $H \in \mathcal{P}_6$, we have that H is 2-degenerate and thus $\delta(H) = 2$. Let v be a 2-vertex of H and let x_1 and x_2 be the neighbors of v. A graph that is push equivalent to H has a T_6-coloring if and only if H has a T_6-coloring, because if a vertex v that mapped to a vertex i is pushed, then v maps to the anti-twin of i. So, by possibly replacing H by a graph that is push equivalent to H, we can assume that H contains the arcs $\overrightarrow{x_1v}$ and $\overrightarrow{x_2v}$. The graph $H' = H \setminus v$ is a subgraph of H and thus admits at least one T_6-coloring. Let M be the set of T_6-colorings m of H' such that $m(x_1) = 0$. Let S be the set $\{m(x_2) \mid m \in M\}$. Notice that we cannot have $m(x_1) = 0$ and $m(x_2) \in \{0,1,5\}$, since otherwise it would be possible to extend m to H. So S is a non-empty subset of $\{2,3,4\}$.

Now we use H' to construct a duplicator gadget D with two specified vertices z and z' on its outerface such that D maps to T_6 and every T_6-coloring of D is such that z and z' have the same color. We consider two cases depending on S:

- If $S = \{3\}$, then we obtain D from H' by pushing the vertex x_2 and by setting $z = x_1$ and $z' = x_2$. Thus, z and z' have the same color.

- If $S \cap \{2,4\} \neq \emptyset$, then we obtain D from two copies of H' as follows (see Figure 8). We identify the vertices x_2 of both copies of H'. We rename the two vertices x_1 in z and z'. Finally, we add the directed paths $x_2x_3x_4x_5z$ and $x_2x_3x_4'x_5'z'$. Suppose that z is colored 0. Because of the copy of H' between z and x_2, x_2 cannot be colored 0, 1, or 5. Because of the directed 4-path $x_2x_3x_4x_5z$, x_2 cannot be colored 3.

  Suppose that x_2 is colored 2. So S contains 2 and the only possible coloring of the path $x_2x_3x_4x_5z$ is such that the color of x_i is i, for $2 \leqslant i \leqslant 5$. So x_3 is colored 3 and the path $x_3x_4'x_5'z'$ forbids that z' is colored 4 or 5. The copy of H' between z' and x_2 forbids that z'

is colored 1, 2, or 3. The only remaining possibility is that z' is colored 0, which is possible since S contains 2 and the vertices x'_i can be colored i, for $4 \leqslant i \leqslant 5$.

Suppose that x_2 is colored 4. So S contains 4 and the only possible coloring of the path $x_2x_3x_4x_5z$ is such that the color of x_i is $2i \pmod 6$, for $2 \leqslant i \leqslant 5$. So x_3 is colored 0 and the path $x_3x'_4x'_5z'$ forbids that z' is colored 1 or 2. The copy of H' between z' and x_2 forbids that z' is colored 3, 4, or 5. The only remaining possibility is that z' is colored 0, which is possible since S contains 4 and the vertices x'_i can be colored $2i \pmod 6$, for $4 \leqslant i \leqslant 5$.

Thus, z and z' have the same color.

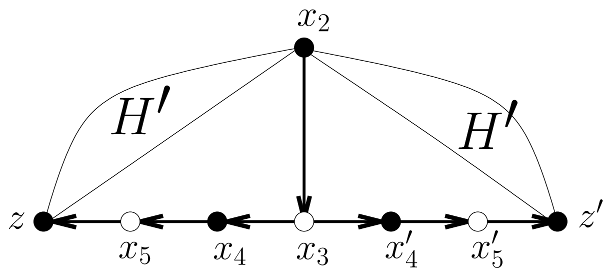

Figure 8: Construction of D.

The reduction is from PLANAR 3-COLORABILITY, which is known to be NP-complete [10] for planar graphs with maximum degree 4. Let I be an instance of PLANAR 3-COLORABILITY. We obtain the oriented graph G from I by replacing every edge of I by the edge gadget depicted in Figure 9. A vertex in G that corresponds to a vertex in I is said *old*. Consider a T_6-coloring of the edge gadget such that a is colored 0. By the property of D, the vertices a' and a'' are also colored 0. The directed 4-path between a' and b forbids color 3 for b. The directed 2-path between a'' and b forbids colors 0, 1, and 5 for b. On the other hand, b can be colored 2 or 4. This shows that I admits a 3-coloring if and only if G maps to T_6 (a vertex colored $c \in \{0, 1, 2\}$ in I is colored $2c$ in G).

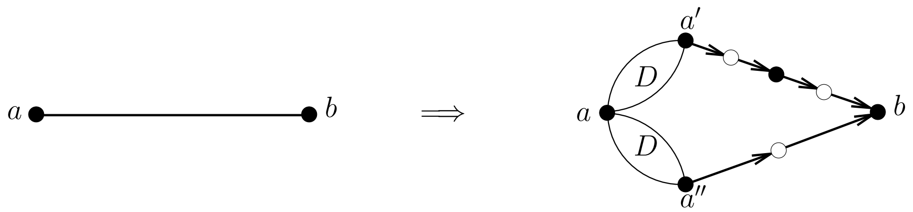

Figure 9: The edge gadget.

Let us show that G satisfies the conditions of Theorem 5. Recall that the girth g of H satisfies $8 \leqslant g \leqslant 15$. The distance between x_1 and x_2 in H' is at least $g - 2$. The distance between z and z' in D is at least $g - 2$ in the case $S = \{3\}$ and is 6 in the case $S \cap \{2, 4\} \neq \emptyset$. So, the distance between z and z' in D is at least 6. Moreover, D contains no cycle of length strictly smaller than g. The distance between old vertices in G is at least $6 + 2 = 8$. Thus the shortest cycles of G that are not contained in a copy of H' have length at least $2 \times 8 = 16$. So G contains no cycle of length strictly smaller than g. Finally, suppose that H is bipartite. Then H' is bipartite and the distance between x_1 and x_2 in H' is even. Also, D is bipartite and the distance between z and z' is even. Thus the distance between old vertices is even, which implies that G is bipartite.

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