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On interval representations of graphs

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ABSTRACT

The interval number $i(G)$ of a graph G is the least integer i such that G is the intersection graph of sets of at most i intervals of the real line. The local track number $l(G)$ is the least integer l such that G is the intersection graph of sets of at most l intervals of the real line and such that two intervals of the same vertex belong to different components of the interval representation. The track number $t(G)$ is the least integer t such that $E(G)$ is the union of t interval graphs. We show that the local track number of a planar graph with girth at least 7 is at most 2. We also answer a question of West and Shmoys in 1984 by showing that the recognition of 2-degenerate planar graphs with maximum degree 5 and interval number at most 2 is NP-complete.

1. Introduction

Let G be a graph. A *d-interval representation* of G is a mapping that assigns a set of at most d intervals of the real line to every vertex of G, such two vertices u and v are adjacent if and only if some interval for u intersects some interval for v. The *interval number* $i(G)$ of a graph G is the least integer d such that G has a d-interval representation. A *component* of a d-interval representation is a maximal subset S of the real line such that every point in S is contained in an interval of the representation. A *d-local representation* of G is a d-interval representation of G with the additional requirement that two intervals for the same vertex belong to distinct components. The *local track number* $l(G)$ has been recently introduced by Knauer and Ueckerdt [9] as the least integer d such that G has a d-local representation. The *track number* $t(G)$ is the least integer t such that there exist t interval graphs G_j on the same vertex set as G that satisfy $E(g) = \bigcup_{1 \leqslant j \leqslant t} E(G_j)$. such that $E(G)$ is the union of. If $t(G) \leqslant d$, then a *d-track representation* of G is given by the union of the d (1-interval) representations of the graphs G_j for $1 \leqslant j \leqslant d$. The d real lines supporting the representations of the graphs G_j are called *tracks*. Obviously, we have $i(G) \leqslant l(G) \leqslant t(G)$. We say that G is *d-interval* (resp. *d-local*, *d-track*) if $i(G) \leqslant d$ (resp. $l(G) \leqslant d$, $t(G) \leqslant d$).

For every planar graph G, Scheinerman and West [10] obtained that $i(G) \leqslant 3$ and Gonçalves [5] obtained that $t(G) \leqslant 4$. Both bounds are best possible [10,6].

The *girth* of a graph is the length of a shortest cycle. We denote by $\mathcal{P}_g$ the class of planar graphs with girth at least g (note that $\mathcal{P}_3$ is simply the class of planar graphs). In this paper, we obtain the following results (see Fig. 1).

Theorem 1. *The local track number of a graph in $\mathcal{P}_7$ is at most* 2.

West and Shmoys [11] have shown that recognizing d-interval graphs is NP-complete for every fixed $d \geqslant 2$. Gyárfás and West [7] obtained that recognizing 2-track graphs is NP-complete. This result has been extended by Jiang [8] who proved that recognizing d-track graphs is NP-complete for every fixed $d \geqslant 2$. We generalize these results as follows.

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Graph class	track number	local track number	interval number
$\mathcal{P}_3$	4 UB: [5]	3-4	3 UB: [10]
$\mathcal{P}_4$	4 LB: [6]	3 UB: [7]	3 LB: [10]
$\mathcal{P}_5$	3-4	2-3	2-3
$\mathcal{P}_6$	3 LB: [6] UB: [6]	2-3	2-3
$\mathcal{P}_7$	2-3	2 UB: Th. 1	2
$\mathcal{P}_8$	2-3	2	2
$\mathcal{P}_9$	2-3	2	2
$\mathcal{P}_{10}$	2 UB: [6]	2	2

Fig. 1. Table of results.

Theorem 2. *Let $d \geqslant 2$ be a fixed integer. Given a graph G such that $t(G) \leqslant d$ if and only if $i(G) \leqslant d$, determining whether $t(G) \leqslant d$ is NP-complete, even if G is $(K_4, 2K_3)$-free, alternately orientable, a Meyniel graph, and a string graph.*

Recall that $i(G) \leqslant l(G) \leqslant t(G)$ holds unconditionally. Since Theorem 2 holds for graphs such that $t(G) \leqslant d$ if and only if $i(G) \leqslant d$, it uses only one reduction to show that recognizing d-track graphs, d-local graphs, and d-interval graphs is NP-complete for every fixed $d \geqslant 2$.

West and Shmoys [11] asked in 1984 whether it is NP-complete to determine the interval number of a planar graph. The following result implies that both the interval number and the local track number of a planar graph are NP-complete to determine.

Theorem 3. *Given a 2-degenerate planar graph G with maximum degree 5 such that $l(G) \leqslant 2$ if and only if $i(G) \leqslant 2$, determining whether $l(G) \leqslant 2$ is NP-complete.*

Concerning the track number of planar graphs, Gonçalves and Ochem [6] have proved that recognizing d-track bipartite planar graphs is NP-complete for $d = 2$ and $d = 3$.

2. Preliminaries

A k-vertex is a vertex of degree k. A k-clause is a clause of size k. A pair of adjacent vertices x and y are *true twins* if $N[x] = N[y]$. Non-adjacent vertices x and y are *false twins* if $N(x) = N(y)$. In an interval representation, we say that an interval is *displayed* if some part of this interval intersects no other interval. Similarly, the extremity of an interval is *displayed* if it intersects no other interval. By convention, an interval representation does not contain an interval that does not intersect another interval. A vertex is displayed if it is represented by strictly less intervals than what is allowed by the considered representation, or if one of its interval is displayed. We also say that an interval *a covers* an interval b if b is contained in a. The maximum average degree mad(G) of a graph G is defined by $\text{mad}(G) = \max_{H \subseteq G} \{2|E(H)|/|V(H)|\}$. It is well-known that for every graph $G \in \mathcal{P}_g$, we have $\text{mad}(G) \leqslant 2g/(g-2)$ [2].

3. Proof of Theorem 1

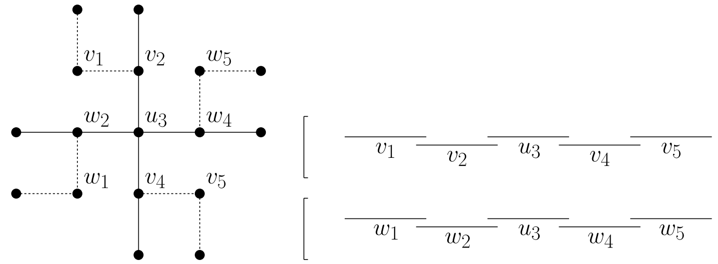
We define a *good representation* as a 2-local representation such that every vertex is displayed. We prove that every graph with girth at least 7 and maximum average degree strictly smaller than $\frac{14}{5}$ admits a good representation. Let G be a hypothetical counter-example to this statement that is minimal for the subgraph order. That is, G has girth at least 7 and maximum average degree strictly smaller than $\frac{14}{5}$, G does not admit a good representation, and every proper subgraph of G admits a good representation. The graph G must be connected, since otherwise we would obtain a good representation of G by gathering the good representations of its connected components.

First, suppose for contradiction that G contains a 1-vertex a adjacent to a vertex b. By minimality, $G \backslash a$ admits a good representation. In this representation, b is represented by at most one interval or one interval of b is displayed. In both cases, we can extend the representation into a good representation of G. In the former case, we add two new intersecting intervals in a new component: one interval for a and one interval for b. In the latter case, we add a new interval for a intersecting only an existing displayed interval of b. This contradicts the fact that G has no good representation, thus G contains no 1-vertex.

A vertex is said *weak* if it is a 2-vertex or a 3-vertex adjacent to a 2-vertex.

Now, suppose for contradiction that G contains an induced path $P = \{v_1, \ldots, v_k\}$ on $k \geqslant 2$ vertices such that every v_i has exactly one neighbor in $G \backslash P$. Consider a good representation of the graph G' obtained from G by deleting the edges contained in P. Every vertex v_i has degree 1 in G' and is thus represented by only one interval. So we can extend the good representation of G' to G by representing the edges of P on a new component, which is a contradiction. The case $k = 2$ means that G does not contain two adjacent 2-vertices. The case $3 \leqslant k \leqslant 5$ means that a 3-vertex in G is adjacent to at most one weak vertex.

Finally, suppose for contradiction that G contains a 4-vertex u_3 adjacent to four weak vertices as shown in Fig. 2(a). The vertex v_1 (resp. w_1, v_5, w_5) exists if and only if v_2 (resp. w_2, v_4, w_4) is a weak 3-vertex. The fact that the girth of G is at least 7 ensures that no two of the vertices depicted in Fig. 2(a) can be identified, that is, Fig. 2(a) describes all the possible



(a) Neighborhood of the 4-vertex.

(b) Good representation of P' and P''.

Fig. 2. A 4-vertex adjacent to four weak vertices.

cases such that a 4-vertex is adjacent to four weak vertices. We consider only the case such that u_3 is adjacent to four weak 3-vertices, since the other cases are similar. Let $P' = \{v_1, v_2, u_3, v_4, v_5\}$ and $P'' = \{w_1, w_2, u_3, w_4, w_5\}$ be induced paths of G. Consider a good representation of the graph G' obtained from G by deleting the edges contained in P' and P''. The vertices v_i and w_i (for $i \in \{1, 2, 4, 5\}$) have degree 1 in G' and are thus represented by only one interval. So we can extend the good representation of G' to G by representing the edges of P' and P'' on two new components, as shown in Fig. 2(b), which is a contradiction.

We show that the maximum average degree of G is at least $\frac{14}{5}$ using the following discharging procedure. The initial charge of every vertex is its degree and we apply the following rules:

R1: Every 3^+-vertex gives $\frac{2}{5}$ to each adjacent 2-vertex.
R2: Every non-weak vertex gives $\frac{1}{5}$ to each adjacent weak 3-vertex.

Let us check that the final charge $ch(v)$ of every vertex v is at least $\frac{14}{5}$.

- if $d(v) = 2$ then $ch(v) = 2 + 2 \times \frac{2}{5} = \frac{14}{5}$ by R1.
- if $d(v) = 3$ and v is weak, then v has two non-weak neighbors and one 2-neighbor, so $ch(v) = 3 + 2 \times \frac{1}{5} - \frac{2}{5} = 3$ by R2 and R1.
- if $d(v) = 3$ and v is not weak, then v is adjacent to at most one weak 3-vertex, so $ch(v) \geqslant 3 - \frac{1}{5} = \frac{14}{5}$ by R2.
- if $d(v) = 4$ then v is adjacent to at most three weak vertices, so $ch(v) \geqslant 4 - 3 \times \frac{2}{5} = \frac{14}{5}$.
- if $d(v) = k \geqslant 5$ then $ch(v) \geqslant k - k \times \frac{2}{5} = \frac{3k}{5} \geqslant 3$.

We have shown that every graph with girth at least 7 and maximum average degree strictly smaller than $\frac{14}{5}$ admits a good representation. Since graphs in $\mathcal{P}_g$ have maximum average degree strictly smaller than $2g/(g-2)$, every graph in $\mathcal{P}_7$ has a good representation.

4. NP-completeness results

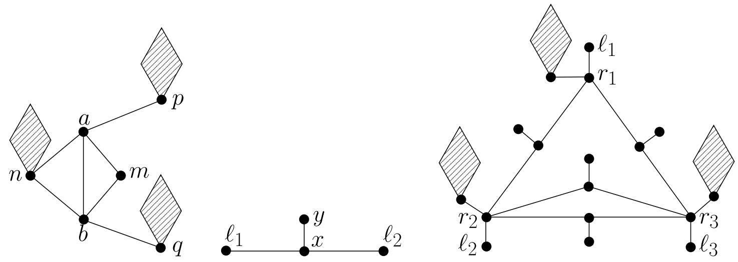
4.1. Proof of Theorem 2

We consider the graph Q obtained from $K_{d,2d^2}$ by adding a 1-vertex adjacent to every d-vertex. In a d-interval representation of Q, there are at most $2d^2$ extremities for the intervals corresponding to the $2d^2$-vertices, since there are d such vertices, each represented by at most d intervals. Each of the d-vertices must have an interval that is not covered by an interval of a $2d^2$-vertex since otherwise we cannot represent the adjacent 1-vertex. This means that all the $2d^2$ mentioned extremities are covered, and none of the $2d^2$-vertices has a displayed extremity. On the other hand, it is easy to see that Q admits a d-track representation.

We reduce the NP-complete problem [6] of determining whether the track number of a bipartite planar graph with girth 6 is at most 2. Let H be an instance of this problem. We construct an instance G of the problem in Theorem 2 from H as follows. We take H and two copies Q_a and Q_b of Q. We pick a $2d^2$-vertex a in Q_a and a $2d^2$-vertex b in Q_b. For every vertex v_i of H, we add the edges v_ia and v_ib. Also, for every vertex v_i in H, we add $d-2$ copies $Q_{i,j}$ of Q, with $3 \leqslant j \leqslant d$. Finally, for every $Q_{i,j}$, we pick a $2d^2$-vertex $v_{i,j}$ in $Q_{i,j}$ and add the edge $v_iv_{i,j}$.

Suppose that H has a 2-track representation. To obtain a d-track representation of G, we put the representation of one track of H on track 1 of G and cover it by an interval of a. Also, we put the representation of the other track of H on track 2 of G and cover it by an interval of b. Then we can complete the d-track representation of Q_a and Q_b. For every vertex v_i of H, we put an interval of v_i on each track j, for $3 \leqslant j \leqslant d$, such that all of these intervals are disjoint. We cover the interval of v_i on track j by an interval of the vertex $v_{i,j}$. Then we can complete the d-track representation of $Q_{i,j}$.

Now, suppose that G has a d-interval representation. Notice that $d-2$ intervals of a vertex v_i are covered by an interval of the vertices $v_{i,j}$, for $1 \leqslant i \leqslant |V(H)|$ and $3 \leqslant j \leqslant d$. Thus, only 2 intervals of v_i can be used to represent an edge of H.



(a) Variable. (b) 2-clause. (c) 3-clause.

Fig. 3. The variable and clause gadgets.

Since v_i is adjacent to a and b and since all the extremities of the intervals of a and b are covered by vertices in Q_a and Q_b, one of the two remaining intervals of v_i is covered by an interval of a and the other remaining interval of v_i is covered by an interval of b. By putting on one track the representation of the intervals of the vertices of H covered by the intervals of a and on another track the representation of the intervals of the vertices of H covered by the intervals of b, all the edges of H are represented and we obtain a 2-track representation of H.

Let us verify that G has the claimed properties. We recall the definition of the considered graph classes [4]. A graph is Meyniel if every cycle of odd length at least 5 has at least 2 chords. A graph is alternately orientable if and only if it admits an orientation in which no chordless cycle of length at least 4 contains a directed P_3. A graph is a string graph if it is the intersection graph of curves in the plane. These classes, as well as K_4-free graphs, are closed under 1-sum. We have to show that G is in the class $\mathcal{C}$ of graphs that are $(K_4, 2K_3)$-free, alternately orientable, Meyniel, and string. The 2-connected components of G are isomorphic to either K_2, $K_{d,2d^2}$, or a bipartite planar graph dominated by two non-adjacent vertices. Since K_2 and $K_{d,2d^2}$ are complete bipartite graphs, they are in $\mathcal{C}$. We denote by BP^{++} the last 2-connected component. Every bipartite planar graph is in $\mathcal{C}$. Notice that the graph obtained by adding a universal vertex to a bipartite graph in $\mathcal{C}$ is also in $\mathcal{C}$. Moreover, $\mathcal{C}$ is closed under the operation of adding a false twin to a vertex. Thus, BP^{++} is in $\mathcal{C}$. Now, notice that $\mathcal{C}$ is almost closed under 1-sum, except that the resulting graph may contain an induced $2K_3$. However, since G has only one 2-connected component that contains triangles, then G is $2K_3$-free and thus G is in $\mathcal{C}$. Finally, notice that G is perfect since it is Meyniel, and that we get a bipartite graph by removing the vertices a and b from G.

4.2. Proof of Theorem 3

We reduce the NP-complete problem [3] RESTRICTED PLANAR 3-SAT. This variant of SAT is such that:

- each clause has size 2 or 3,
- each variable appears exactly twice positively and once negatively, and
- the variable–clause incidence graph is planar.

We consider an instance I of RESTRICTED PLANAR 3-SAT and construct a graph G corresponding to I as follows. For every variable (resp. 2-clause, 3-clause) of I, we take a copy of the graph depicted in Fig. 3(a) (resp. Fig. 3(b), Fig. 3(c)). The gray diamond shape used in the construction of the variable gadget and the 3-clause gadget is the graph F depicted in Fig. 4. The vertex n in a variable gadget corresponds to the negative literal and the vertices p and q correspond to the two positive literals of the corresponding variable. Also, the vertices ℓ_i in a clause gadget correspond to the literals of the corresponding clause. To obtain G, we consider every literal ℓ of I and identify the vertex corresponding to ℓ in the variable gadget with the vertex corresponding to ℓ in the clause gadget.

Let us describe the properties of F. Consider a 2-interval representation of F. The graph F has eight 5-vertices, so at most 32 interval extremities correspond to the 5-vertices. Since F is triangle-free, each of these extremities intersects at most one other interval. Every representation of an edge linking 5-vertices is such that either one interval is contained in the other or two intervals overlap. In both cases, two extremities are covered. Since every edge is represented at least once and the graph induced by the 5-vertices has 12 edges, there are at most $32 - 12 \times 2 = 8$ such extremities that are not covered by an interval corresponding to a 5-vertex. Moreover, F has eight 3-vertices that are each adjacent to two 5-vertices. A 3-vertex covers at least one extremity of a 5-vertex, since otherwise it would not be possible to represent its adjacent 1-vertex. Thus, every 3-vertex covers exactly one extremity of a 5-vertex. That is, for every 3-vertex, one of its intervals is contained in the interval of a 5-vertex and its other interval has one covered extremity, so that at most one extremity is displayed. A 2-local representation of F is given by the coloring of the edges in Fig. 4: each color class induces a caterpillar and corresponds to a component, and every vertex is incident to at most 2 colors.

Now, consider any 2-interval representation of the variable gadget. The copies of F force that for every vertex in $\{n, p, q\}$, one interval is completely covered and the other has a displayed extremity in F. Suppose for contradiction that there exists a 2-interval representation of the variable gadget that both n and p have a displayed extremity. Then a is not displayed, since one interval of a is covered by an interval of n and the other is covered by an interval of p. So we cannot represent m, which

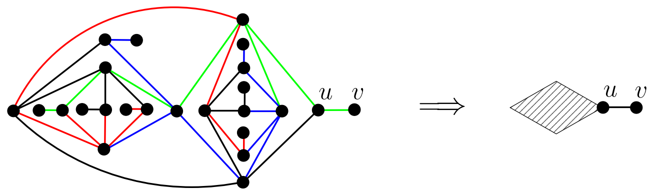


Fig. 4. The forcing graph F. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

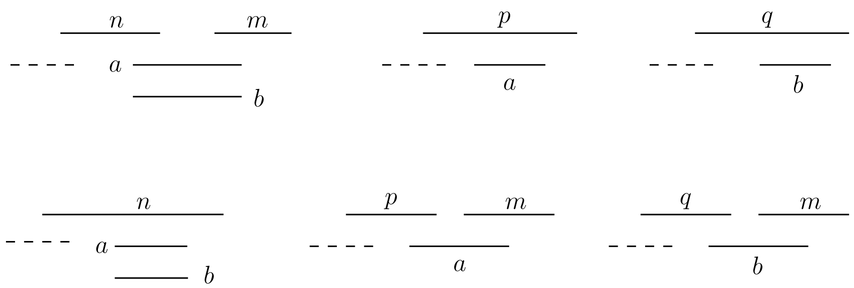


Fig. 5. 2-local representations of the variable gadget.

is adjacent to a but is adjacent neither to n nor p. Thus, no 2-interval representation of the variable gadget is such that both n and p (resp. n and q) have a displayed extremity. On the other hand, there exist 2-local representations of the variable gadget such that:

- both p and q have a displayed extremity (and thus n has no displayed extremity).
- n has a displayed extremity (and thus neither p nor q has a displayed extremity).

These two representations are given at the top and at the bottom of Fig. 5. The interval of n, p, or q that is covered by an interval of a vertex in F is not represented. The displayed interval of n, p, or q has one extremity that is covered by an interval of a vertex in F, which is represented by a dashed interval. In both representations, the 6 intervals for n, p, and q lie on different components.

In the reduction, a literal is true if and only if the corresponding vertex has a displayed extremity in the representation of the variable gadget. A variable is false if and only if its vertex n has a displayed extremity in the representation of the variable gadget. Let us show that a clause gadget has a 2-local representation if the corresponding clause is satisfied, and has no 2-interval representation otherwise. The case of a 2-clause is easy: the displayed intervals of the literals ℓ_1 and ℓ_2 lie on distinct components, so we can extend the 2-local representation to the vertices x and y of the 2-clause gadget if at least one of these intervals has a displayed extremity, and otherwise we cannot extend it into a 2-interval representation. Let us consider a 3-clause. One of the four extremities of the intervals of a vertex r_i is covered by its attached copy of F. If the literal ℓ_i is false, then an interval of r_i is completely covered by an interval of ℓ_i, so that there remains only one displayed extremity for r_i. If the literal ℓ_i is true, then we can represent r_i such that one interval of r_i has an extremity covered by F in one component and the other interval of r_i has an extremity covered by ℓ_i in another component, so that there remain two displayed extremities in distinct components for r_i. If the clause is not satisfied, then there remain three displayed extremities for r_1, r_2, and r_3, and thus we cannot extend a 2-local representation to the four 3-vertices of the clause gadget in order to get a 2-interval representation. If the clause is satisfied, then there remain at least 4 displayed extremities for r_1, r_2, and r_3 that all lie in distinct components, and it is easy to check that we can indeed extend a 2-local representation to the 3-vertices of the clause in every case.

Let us summarize the reduction.

1. Given an assignment satisfying I, we obtain a 2-local representation of G using the constructions described above.
2. Given a 2-interval representation of G, we obtain an assignment satisfying I by setting a variable to false if and only if the corresponding vertex n has a displayed extremity in the representation induced by the vertices of the variable gadget.

To finish the proof, observe that G is planar, 2-degenerate, with maximum degree 5, and can be constructed from I in polynomial time.

5. 2-track graphs vs edge-disjoint interval graphs

Gyárfás and West [7] ask whether there exist d-track graphs that are not the union of d pairwise edge-disjoint interval graphs. In this section, we give a positive answer with $d = 2$. Let W be the graph depicted in Fig. 6.

Theorem 4. *The graph W is a 2-track graph and the edge AA' is represented twice in every 2-interval representation.*

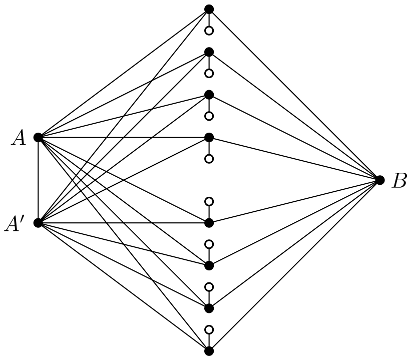


Fig. 6. The graph W.

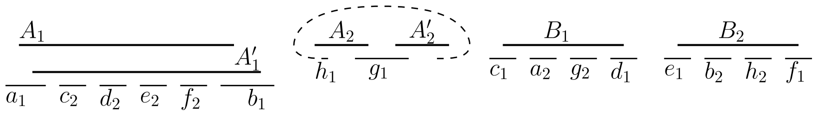


Fig. 7. The graph W has no 2-interval representation such that AA' appears only once.

Proof. Notice that $W \setminus \{A'\}$ corresponds to the graph Q for $d = 2$ in the construction of Theorem 2. So $W \setminus \{A'\}$ is a 2-track graph. Moreover, 2-track graphs are closed by the addition of a true twin. So W is a 2-track graph.

Now, assume for contradiction that there exists a 2-interval representation of W such that the edge AA' is represented once. For every vertex x, we denote by x_1 and x_2 the intervals representing x. Without loss of generality we assume that A_1 intersects A'_1. Since AA' is already represented and B is adjacent to neither A nor A', then the intervals $A_1 \cup A'_1$, A_2, A'_2, B_1, and B_2 do not intersect.

Let I be the set of intervals representing vertices of degree at least 2. Every 4-vertex x intersects at least two disjoint intervals in I. Due to the 1-vertex adjacent to x, we assume without loss of generality that x_1 is displayed in I. This implies that x_1 covers an extremity of an interval in I. If both intervals of x are displayed, then we can assume that at most one of them intersects $A_1 \cup A'_1$. If both intervals of x are displayed and one of them intersects $A_1 \cup A'_1$, then we assume without loss of generality that x_1 intersects $A_1 \cup A'_1$. We partition the sets of 4-vertices into the set V_1 of 4-vertices x such that x_1 intersects $A_1 \cap A'_1$, B_1 or B_2 and the set V_2 of 4-vertices x such that x_1 intersects neither $A_1 \cap A'_1$, B_1, nor B_2. We define $n_1 = |V_1|$ and $n_2 = |V_2|$. Each of the intervals $A_1 \cup A'_1$, B_1, and B_2 has two extremities, so there are at most six vertices in V_1.

$$n_1 \leqslant 6. \tag{1}$$

The sets V_1 and V_2 form a partition of the eight 4-vertices.

$$n_1 + n_2 = 8. \tag{2}$$

Every vertex x in V_1 is such that x_1 covers at least one extremity of I since x_1 is displayed. Every vertex x in V_2 covers at least 2 extremities of distinct intervals in I since x_1 and x_2 have to intersect three disjoint intervals. Moreover, the 5 intervals $A_1 \cup A'_1$, A_2, A'_2, B_1, and B_2 have 10 extremities in total, so we have:

$$n_1 + 2n_2 \leqslant 10. \tag{3}$$

From Eqs. (1) to (2), we have that $n_2 \geqslant 2$. From Eqs. (2) to (3), we have that $n_2 \leqslant 2$. Hence $n_2 = 2$ and $n_1 = 6$. Therefore, all the extremities of $A_1 \cup A'_1$, B_1, and B_2 are covered by the vertices in V_1. Recall that a vertex in V_2 has to cover the extremities of two distinct intervals of I. Hence, both vertices in V_2 must cover one extremity of A_2 and one extremity of A'_2. Since this is impossible, we have a contradiction (see Fig. 7). □

6. Conclusion

Balogh et al. [1] proved that the interval number of a planar graph with maximum degree at most 4 is at most 2. The degree bound is optimal because Theorem 3 implies that there exist 2-degenerate planar graphs with maximum degree 5 and interval number 3. By taking two copies of $F \setminus \{v\}$ and identifying their vertex u, we even obtain a bipartite 2-degenerate planar graph with maximum degree 5 and interval number 3. However, we have not been able to restrict Theorem 3 to bipartite graphs, and maybe it is impossible to do so. Indeed, the complexity of recognizing triangle-free 2-interval graphs has been asked in [7,8,11] and remains an open problem.

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