Pregroup Semantics in compact closed monoidal categories with biproducts

Anne Preller
LIRMM-CNRS, Montpellier, France

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Pregroup grammar, a compact type-logical grammar

Syntactic Calculus	Pregroup Calculus	(Lambek)
higher order logic	compact bilinear logic	(Lambek)
cartesian closed categories	compact 2-categories	(Lambek)
$\lambda -$ term	syntactic morphism $+$ semantic morphism	
topoi	compact closed categories with biproducts	
Montague semantics	functions in two-sorted first order logic	

from logic to categories

- syntactical analysis — proof in compact bilinear logic
- proofs — morphisms in compact 2-category freely generated by basic types
- meanings — morphisms in compact closed category with biproducts freely generated by lexical category

Semantics adds
biproducts (counting, disambiguation)

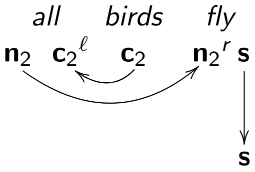
pregroup lexicon, syntax

pregroup dictionary

all	: $\mathbf{n}_2\, \mathbf{c}_2{}^{\ell}$	*birds*	: $\mathbf{c}_2$
some	: $\mathbf{n}_2\, \mathbf{c}_2{}^{\ell}$	*fly*	: $\mathbf{n}_2{}^{r}\, \mathbf{s}$

syntactical analysis:
concatenate types, find proof (reduction to sentence type)

$$\mathbf{n}_2\, \mathbf{c}_2{}^{\ell}\, \mathbf{c}_2\, \mathbf{n}_2{}^{r}\, \mathbf{s} \vdash \mathbf{s} =$$



pregroup lexicon, functional semantics

Pregroup lexicon, functional version

		meaning			meaning
all	$: \mathbf{n}_2\, \mathbf{c}_2{}^{\ell}$ ::	$\mathtt{all} : E \to E$	*birds*	$: \mathbf{c}_2$ ::	$\mathtt{bird} \subseteq E$
some	$: \mathbf{n}_2\, \mathbf{c}_2{}^{\ell}$ ::	$\mathtt{some} : E \to E$	*fly*	$: \mathbf{n}_2{}^{r}\, \mathbf{s}$ ::	$\mathtt{fly} : E \to S$

E set of 'individuals', $S = \{\top, \bot\}$ set of 'truth-values', $\mathtt{all}, \ldots, \mathtt{fly}$ 2-sorted functions

meaning of sentences

$$m(\textit{all birds fly}) = \mathtt{fly}(\mathtt{all}(\mathtt{bird}))$$
$$m(\textit{some birds fly}) = \mathtt{fly}(\mathtt{some}(\mathtt{bird}))$$

meta-theory

2FOL two-sorted first order logic

- two sorts: elements and sets

$$a, \quad A, \quad a \in A, \quad a = b, \quad A = B, \quad \forall_x, \exists_x, \quad \forall_X, \exists_X$$

- operators

2-sorted union	2-sorted product
$a \cup b \quad = \{a, b\}$	$a \times b \quad = \langle a, b \rangle$
$a \cup B \quad = \{a\} \cup B$	$a \times B \quad = \{a\} \times B$
$A \cup b \quad = A \cup \{b\}$	$A \times b \quad = A \times \{b\}$
$A \cup B \quad = A \cup B$	$A \times B \quad = A \times B$

2-sorted functions

a two-sorted function from A to B is a function g such that

- $$\begin{array}{rl} x \in A \Rightarrow & g(x) \in B \text{ or } g(x) \subseteq B \\ X \subseteq A \Rightarrow & g(X) \in B \text{ or } g(X) \subseteq B \end{array}$$
- $$\begin{array}{rcl} g(\emptyset) & = & \emptyset \\ g(\{a\}) & = & g(a) \text{ for } a \in A \\ g(X \cup Y) & = & g(X) \cup g(Y) \text{ for } X, Y \subseteq A \end{array}$$

examples, properties of 2-sorted functions

- examples

  $A \neq \emptyset,\ A \subseteq E$

  point $\ulcorner A \urcorner : \{*\} \to E \quad \ulcorner A \urcorner(*) = A$

  partial identity on A $\quad q_A(a) = \begin{cases} a & \text{if } a \in A \\ q(a) = \emptyset & \text{else} \end{cases}$

  identity $\quad id(a) = a$

  partial identities $\quad q(a) = a \text{ or } = \emptyset$

- $\Longrightarrow$
  - $q_A \circ q_B = q_B \circ q_A = q_A,$ for $A \subseteq B$
  - $q_A(X) = X \cap A \quad id(X) = X \quad q(X) \subseteq X,$ for X, A finite sets

predicates

- A 2-sorted function with values in S is a predicate if it maps elements to elements
- $\Longrightarrow$ Fundamental property
  For every predicate $g : A \to B$, $b \in B$, $X \subseteq A$

$$g(X) = b \Leftrightarrow X \neq \emptyset \text{ and } g(x) = b, \text{ for all } x \in X$$

- $\Longrightarrow$

$$g(X) = \emptyset \Leftrightarrow X = \emptyset$$

$$X \neq \emptyset \Longrightarrow g(X) = \top \text{ or } g(X) = \bot \text{ or } g(X) = \{\top, \bot\}$$

truth of sentences

- Definition

  A statement $w_1 \ldots w_n$ is holds in the interior logic if

  $$m(w_1 \ldots w_n) = \top \text{ is derivable}$$

- NB

  $$m(w_1 \ldots w_n) \neq \top \Rightarrow \begin{array}{l} m(w_1 \ldots w_n) = \bot \\ \text{or} \\ m(w_1 \ldots w_n) = \{\top, \bot\} \end{array}$$

universal quantifier

Logical content of the word *all* is captured by

$$\forall_X(\mathtt{all}(X) = X) \qquad \text{(U)}$$

$$\Longrightarrow$$

$$\mathtt{all} = id$$
$$\mathtt{fly(all(bird))} = \mathtt{fly(bird)}$$

Fundamental Property $\Longrightarrow$

$$\mathtt{fly(all(bird))} = \top \Leftrightarrow \mathtt{bird} \neq \emptyset \wedge \forall_x(x \in \mathtt{bird} \Rightarrow \mathtt{fly}(x) = \top)$$

existential quantifier

Logical content of the word *some* is captured by

$$\forall_X(\texttt{some}(X) \subseteq X) \qquad \text{(E)}$$

$\Longleftrightarrow$ `some` is a partial identity Property (E) does not determine the function `some` uniquely

For every partial identity q

$$\begin{aligned} \texttt{fly}(q(\texttt{bird})) = \top \Leftrightarrow \quad & q(\texttt{bird}) \neq \emptyset \wedge \forall_x(x \in q(\texttt{bird}) \Rightarrow \texttt{fly}(x) = \top) \\ \Rightarrow \quad & \exists_x(x \in \texttt{bird} \wedge \texttt{fly}(x) = \top) \\ \Rightarrow \quad & \exists_X(X \subseteq \texttt{bird} \wedge \texttt{fly}(X) = \top) \end{aligned}$$

witnesses instead of explicit existential quantifiers

$\texttt{some(bird)}$ is a witness for

$$\exists_X(X \subseteq \texttt{bird} \wedge \texttt{fly}(X))$$

It provides a referent in discourse representation

The meaning of

Some birds do not fly. They have no wings.

is

$$\texttt{not}_S(\texttt{fly}(\texttt{some}(\texttt{bird}))) \wedge \texttt{not}_S(\texttt{have}(\texttt{they}, \texttt{wing})) \\ \wedge \texttt{they} = \texttt{some}(\texttt{bird})$$

i.e.

$$\texttt{not}_S(\texttt{fly}(\texttt{some}(\texttt{bird}))) \wedge \texttt{not}_S(\texttt{have}(\texttt{some}(\texttt{bird}), \texttt{wing}))$$

category of 2-sorted functions is compact closed

- adjoint of A $\quad A^* = A$
  adjoint of $f : A \to B$ $\quad f^* : B \to A$ $\quad$ ‘inverse image of f’

- forward and backward name of f $\quad$ ‘graph of f’

$$\ulcorner f \urcorner^{\rightarrow} = \begin{array}{c} I \\ \overset{f}{\frown} \\ A \times B \end{array} \qquad \ulcorner f \urcorner^{\leftarrow} = \begin{array}{c} I \\ \overset{f}{\frown} \\ B \times A \end{array}$$

- forward and backward coname

$$\llcorner f \lrcorner_{\rightarrow} = \begin{array}{c} A \times B \\ \underset{f}{\smile} \\ I \end{array} \qquad \llcorner f \lrcorner_{\leftarrow} = \begin{array}{c} B \times A \\ \underset{f}{\smile} \\ I \end{array}$$

category of 2-sorted functions is compact closed

Definition of $f^*, \ulcorner f \urcorner^{\rightarrow}, \ulcorner f \urcorner^{\leftarrow}, \llcorner f \lrcorner_{\rightarrow}, \llcorner f \lrcorner_{\leftarrow}$

$$
\begin{aligned}
f^*(b) &= \{a \in A : \ \Phi(f, a, b)\} \\
\ulcorner f \urcorner^{\rightarrow}(*) &= \{< a, b >: \ \Phi(f, a, b)\} \\
\ulcorner f \urcorner^{\leftarrow}(*) &= \{< b, a >: \ \Phi(f, a, b)\}
\end{aligned}
$$

$$
\llcorner f \lrcorner_{\rightarrow}(a, b) = \llcorner f \lrcorner_{\leftarrow}(b, a) = \begin{cases} * & \text{if } \Phi(f, a, b) \\ \emptyset & \text{else} \end{cases} ,
$$

where

$$
\Phi(f, a, b) \iff f(a) = b \text{ or } b \in f(a)
$$

categorical meanings, morphisms

Pregroup lexicon, categorical version

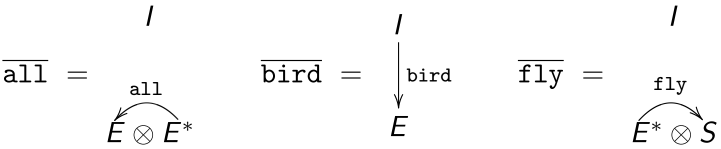
		meaning			meaning
all	$: \mathbf{n}_2\, \mathbf{c}_2{}^{\ell}$ ::	$I \xrightarrow{\overline{\mathtt{all}}} E \otimes E^*$	*birds*	$: \mathbf{c}_2$ ::	$I \xrightarrow{\overline{\mathtt{bird}}} E$
some	$: \mathbf{n}_2\, \mathbf{c}_2{}^{\ell}$ ::	$I \xrightarrow{\overline{\mathtt{some}}} E \otimes E^*$	*fly*	$: \mathbf{n}_2{}^r\, \mathbf{s}$ ::	$I \xrightarrow{\overline{\mathtt{fly}}} E^* \otimes S$

where $I = \{*\}$
remember the functional version

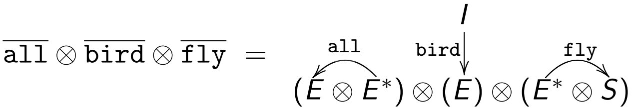
		meaning			meaning
all	$: \mathbf{n}_2\, \mathbf{c}_2{}^{\ell}$ ::	$\mathtt{all} : E \to E$	*birds*	$: \mathbf{c}_2$ ::	$\mathtt{bird} \subseteq E$
some	$: \mathbf{n}_2\, \mathbf{c}_2{}^{\ell}$ ::	$\mathtt{some} : E \to E$	*fly*	$: \mathbf{n}_2{}^r\, \mathbf{s}$ ::	$\mathtt{fly} : E \to S$

categorical meanings, graphs

- replace the morphisms by their graphs



- concatenate graphs: semantical morphism

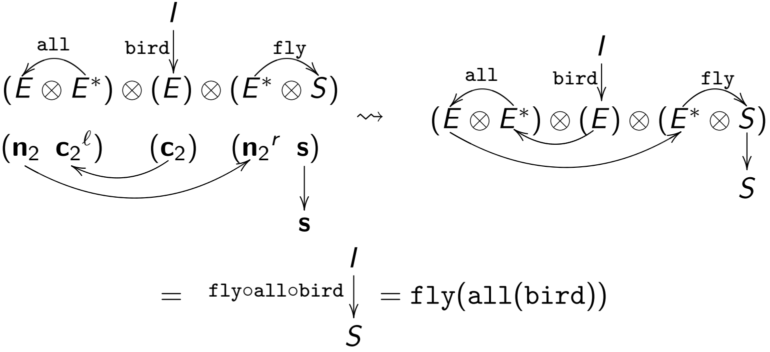


meaning of sentences

syntactical morphism + semantical morphism

- compose product of meanings with reduction

$$m(\textit{all birds fly}) = \textit{reduction} \circ (\overline{\texttt{all}} \otimes \overline{\texttt{bird}} \otimes \overline{\texttt{fly}}) =$$

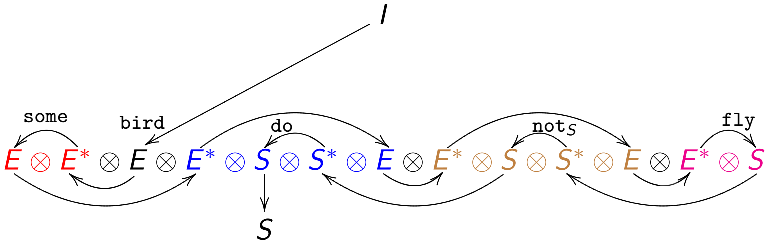


Some birds do not fly

$$do\ :\ \mathbf{n_2}^r \mathbf{s} \mathbf{i}^\ell \mathbf{d}\ ::\ I \xrightarrow{\overline{\mathtt{do}}} E^* \otimes S \otimes S^* \otimes E$$

$$not\ :\ \mathbf{d}^r\, \mathbf{i}\, \mathbf{i}^\ell\, \mathbf{d}\ ::\ I \xrightarrow{\overline{\mathtt{not}}} E^* \otimes S \otimes S^* \otimes E$$

$$r \circ (\overline{\mathtt{some}} \otimes \overline{\mathtt{bird}} \otimes \overline{\mathtt{do}} \otimes \overline{\mathtt{not}} \otimes \overline{\mathtt{fly}}) =$$



$$= \mathtt{not}_S(\mathtt{fly}(\mathtt{some}(\mathtt{bird})))$$

categorical notation and parameters

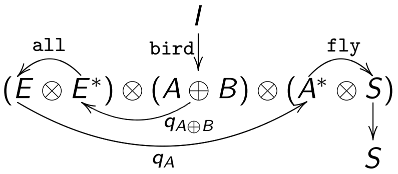
The lexical category $\mathcal{L} = \mathcal{L}(I, \mathcal{O})$ depends on

- unit I
  - $I = \{*\}$
    $\Longrightarrow$ semantical category : 2-sorted functions
  - $I = [0, 1]_{\leq}$
    $\Longrightarrow$ semantical category : distributional vector models
    (semi-modules over lattice I)
- ontology $\mathcal{O}$
  - $\mathcal{O} = \{E\}$
  - $\mathcal{O} = \{\ldots, A \subseteq E, B \subseteq E, \ldots, \}$

ontology and disambiguation

$A =$ *animal with wings, girl, shuttlecock*,
$B =$ *dismissal from employment*
$A \subseteq E, B \subseteq E$

$$\begin{array}{llll} \textit{birds} & : \mathbf{c}_2 & :: & \texttt{bird} \subseteq A \oplus B \\ \textit{fly} & : \mathbf{n}_2{}^r\,\mathbf{s} & :: & \texttt{fly} : A \to S \end{array}$$



$= \texttt{fly} \circ q_A \circ \texttt{all} \circ q_{A \oplus B}(\texttt{bird})$

$= \texttt{fly} \circ q_A \circ q_{A \oplus B}(\texttt{bird})$
$= \texttt{fly} \circ q_A(\texttt{bird})$