Continuation semantics for generalized Lambek calculus

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Logic, categories, semantics
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Abstract

Lambek's syntactic calculi, both the associative and the non-associative variant, are strictly contextfree. A well-tried strategy to overcome this expressive limitation has been to extend the calculi with unary modalities allowing for controlled forms of associativity/commutativity, cf the use of exponentials in linear logic.

Here we pursue an alternative strategy, exploiting the symmetries between residuated and Galois connected families of connectives, and between these and their duals. Communication between these families takes the form of linear, structure-preserving distributivity principles. Compositional interpretation takes the form of a continuation-passing-style translation, mapping syntactic derivations to terms of the linear lambda calculus.

Background reading:

Moortgat 2009, Symmetric categorial grammar. JPL, 38 (6) 681-710.

Moortgat 2010, Symmetric categorial grammar: residuation and Galois connections. Linguistic Analysis. Special issue dedicated to Jim Lambek, 36(1–4), 2010. arXiv CoRR 1008.0170.

1. Background: the Lambek calculi

A landscape of logics The type logics deriving from Lambek '58, '61 form a hierarchy, reflecting different views on

- the nature of the assumptions in judgements $\Gamma \vdash A$:
  - sound patterns, phrases, meanings, . . .
- their structure and its effect on well-formedness

LOGIC	Γ	ASSOCIATIVE	COMMUTATIVE
LP (=MILL)	multiset	✓	✓
L	string	✓	-
NL	tree	-	-

Linguistic resources What aspects of grammaticality do we want to capture with these different views? How do we want to relate them?

2. Interpreting Lambek derivations

Compositionality Montague's view: homomorphism relating source to target algebra. For Lambek categorial grammar:

$\mathbf{(N)L}^{\{n,np,s\}}_{/,\backslash}$	$\xrightarrow{\quad(\cdot)'\quad}$	$\mathbf{LP}^{\{e,t\}}_{\rightarrow}$ (MILL)
syntactic calculus	homomorphism	semantic calculus

- target: MILL/**LP**; linear λ calculus
- source: **(N)L**, structured assumptions (strings, trees); directional linear λ calculus

Types $np' = e\ ;\ s' = t\ ;\ n' = e \rightarrow t\ ;\ (A\backslash B)' = (B/A)' = A' \rightarrow B'$

Terms $x' = \widetilde{x}\ ;\ (\lambda^l x.M)' = (\lambda^r x.M)' = \lambda\widetilde{x}.M'\ ;\ (N \triangleright M)' = (M \triangleleft N)' = (M'\ N')$

Remark Derivational versus lexical semantics. Lexicon: extra finestructure; no linearity restriction.

3. Lost in translation

$$(\Lambda_{\mathbf{NL}})' \subset (\Lambda_{\mathbf{L}})' \subset \Lambda_{\mathbf{LP}}$$

$\rightsquigarrow$ desirable programs are often unobtainable as image of **(N)L** proofs — compare:

Argument lowering valid in **NL**, hence also in **L**, **LP** OK...

$$\begin{array}{rl} \mathbf{NL}: & z : (B/(A\backslash B))\backslash C \vdash \lambda^l x.((\lambda^r y.(x \triangleright y)) \triangleright z) : A\backslash C \\ (\cdot)': & \widetilde{z} \vdash \lambda\widetilde{x}.(\widetilde{z}\ \lambda\widetilde{y}.(\widetilde{y}\ \widetilde{x})) \end{array}$$

Function composition invalid in **NL**, limited in **L**, valid in **LP** but...

$$\begin{array}{rl} \mathbf{L}: & y : A\backslash B, z : B\backslash C \vdash \lambda^l x.((x \triangleright y) \triangleright z) : A\backslash C \\ (\cdot)': & \widetilde{y}, \widetilde{z} \vdash \lambda\widetilde{x}.(\widetilde{z}\ (\widetilde{y}\ \widetilde{x})) \end{array}$$

Argument raising valid only in **LP** BUT...

$$\mathbf{LP}: \quad x : A \to (B \to C) \vdash \lambda w.\lambda z.(w\ \lambda y.((x\ y)\ z)) : ((A \to C) \to C) \to (B \to C)$$

4. Meeting the challenge

Expressive limitations Problematic are discontinuous dependencies:

- Extraction. Who _ stole the tarts? vs What did Alice find _ there?
  - only peripheral assumptions are accessible for / or \ Introduction
- Infixation. Alice thinks someone is cheating local vs non-local interpretation.
  - non-local reading ('there is x s.t. A thinks x is cheating') unavailable

Stragegies for bringing form/meaning composition in sync

- **NL** $+ \Diamond, \Box$: controlled structural options, embedding translations; $\sim$ LL !,?
- Lambek-Grishin calculus **LG**, after Grishin 1983
  - symmetry: residuated, Galois connected operations and their duals
  - structural rules $\rightsquigarrow$ logical distributivity principles
  - continuation semantics: relieves the burden on syntactic source calculus

5. Lambek-Grishin calculus

Source V.N. Grishin, Об одном обобщении системы Айдукевича–Ламбека

- In A.I. Mikhailov, editor, *Studies in Nonclassical Logics and Formal Systems*, pages 315–334. Nauka, Moscow, 1983
- On a generalization of the Ajdukiewicz-Lambek system, in Abrusci and Casadio (eds.) Proceedings 5th Roma Workshop, Bulzoni Editore, Roma, 2002
- Corrected translation at http://symcg.pbworks.com/

Related work

- Lambek 1993. From categorial to bilinear logic. In Došen & Schröder-Heister, eds, *Substructural Logics*, OUP.
- Abrusci 2002. Classical conservative extensions of Lambek calculus. St Logica 71.
- de Groote & Lamarche 2002. Classical non-associative Lambek calculus. *ibid*

6. LG: the core system

Types $A, B \quad ::= \quad p \mid A \otimes B \mid A \backslash B \mid B / A \mid {}^{\mathbf{0}}A \mid A^{\mathbf{0}} \mid$
$A \oplus B \mid A \oslash B \mid B \obslash A \mid A^{\mathbf{1}} \mid {}^{\mathbf{1}}A$

Derivability arrows Preorder laws:

$$A \rightarrow A \qquad \frac{A \rightarrow B \quad B \rightarrow C}{A \rightarrow C}$$

(Co)residuated triples, fusion versus fission:

$$A \rightarrow C/B \quad \Leftrightarrow \quad A \otimes B \rightarrow C \quad \Leftrightarrow \quad B \rightarrow A \backslash C$$
$$B \obslash C \rightarrow A \quad \Leftrightarrow \quad C \rightarrow B \oplus A \quad \Leftrightarrow \quad C \oslash A \rightarrow B$$

(Co)Galois connected pairs:

$$B \rightarrow A^{\mathbf{0}} \quad \Leftrightarrow \quad A \rightarrow {}^{\mathbf{0}}B \quad ; \quad {}^{\mathbf{1}}B \rightarrow A \quad \Leftrightarrow \quad A^{\mathbf{1}} \rightarrow B$$

Models Generalized Kripke frames (Gehrke ea); phase semantics (Bastenhof 2010)

7. Through the Looking Glass

Two symmetries To the left-right symmetry $\cdot^{\bowtie}$ of **(N)L**, **LG** adds an arrow reversal symmetry $\cdot^{\infty}$

$$A^{\bowtie} \xrightarrow{f^{\bowtie}} B^{\bowtie} \quad \Leftrightarrow \quad A \xrightarrow{f} B \quad \Leftrightarrow \quad B^{\infty} \xrightarrow{f^{\infty}} A^{\infty}$$

$\bowtie$, ∞, together with identity and composition ($\bowtie\infty$): Klein group.

'Translation tables' $(C/D)^{\bowtie} = D^{\bowtie}\backslash C^{\bowtie}$, $(D\backslash C)^{\bowtie} = C^{\bowtie}/D^{\bowtie}$ etc

$$\bowtie \quad \frac{C/D \quad A \otimes B \quad B \oplus A \quad D \obslash C \quad {}^{\mathbf{0}}A \quad A^{\mathbf{1}}}{D\backslash C \quad B \otimes A \quad A \oplus B \quad C \oslash D \quad A^{\mathbf{0}} \quad {}^{\mathbf{1}}A}$$

$$\infty \quad \frac{C/B \quad A \otimes B \quad A\backslash C \quad {}^{\mathbf{0}}A \quad A^{\mathbf{0}}}{B \obslash C \quad B \oplus A \quad C \oslash A \quad A^{\mathbf{1}} \quad {}^{\mathbf{1}}A}$$

$\leadsto$ theorems form quartets

8. Compositions, tonicity

Adjoint pairs ϵ and η arrows

$$\begin{array}{ccc}
A \otimes (A\backslash B) \to B \to A\backslash(A \otimes B) & \longleftrightarrow & (B/A) \otimes A \to B \to (B \otimes A)/A \\
\updownarrow & & \updownarrow \infty \\
(B \oplus A) \oslash A \to B \to (B \oslash A) \oplus A & \underset{\bowtie}{\longleftrightarrow} & A \obslash (A \oplus B) \to B \to A \oplus (A \obslash B)
\end{array}$$

Galois connected pairs closure and interior operations

$$\begin{array}{ccc}
A \to {}^{\mathbf{0}}(A^{\mathbf{0}}) & \longleftrightarrow & A \to ({}^{\mathbf{0}}A)^{\mathbf{0}} \\
\updownarrow & & \updownarrow \infty \\
({}^{\mathbf{1}}A)^{\mathbf{1}} \to A & \underset{\bowtie}{\longleftrightarrow} & {}^{\mathbf{1}}(A^{\mathbf{1}}) \to A
\end{array}$$

Tonicity $(\uparrow \otimes \uparrow), (\uparrow / \downarrow), (\downarrow \backslash \uparrow), (\uparrow \oplus \uparrow), (\uparrow \oslash \downarrow), (\downarrow \obslash \uparrow);\quad {}^{\mathbf{0}}(\downarrow), (\downarrow)^{\mathbf{0}}, {}^{\mathbf{1}}(\downarrow), (\downarrow)^{\mathbf{1}}$

9. Distributivity

Interaction fusion, fission Grishin considers two groups of distributivity principles

- respecting resources, cf weak/linear distributivities Cockett-Seely, de Paiva
- respecting structure: non-associativity/commutativity $\otimes / \oplus$

Option A Recipe: select a $\otimes / \oplus$ factor in the premise; simultaneously introduce the residual operations for the remaining two in the conclusion. Note: $\cdot^{\bowtie}$ symmetry.

$$\frac{A \otimes B \to C \oplus D}{C \obslash A \to D / B} \qquad \frac{A \otimes B \to C \oplus D}{B \oslash D \to A \backslash C}$$

$$\frac{A \otimes B \to C \oplus D}{C \obslash B \to A \backslash D} \qquad \frac{A \otimes B \to C \oplus D}{A \oslash D \to C / B}$$

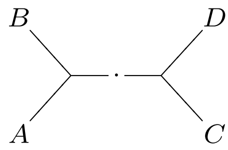
Option B Converses of A. Characteristic theorems: $(A \oplus B) \otimes C \to A \oplus (B \otimes C)$ etc

Conservativity Adding A or B to the pure residuation logic is conservative; with A+B structure-preservation is lost.

10. Distributivity: graphically

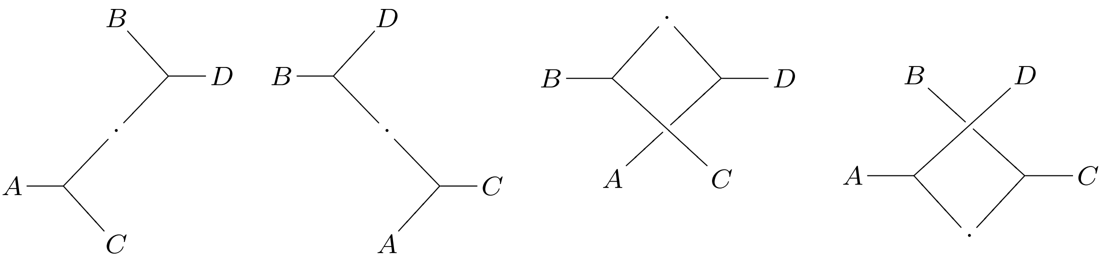
The distributivity principles in the proof net format of Moot 2007.

(fusion)



(fission)

$$\frac{A \otimes B \to C \oplus D}{C \obslash A \to D \,/\, B} \qquad \frac{A \otimes B \to C \oplus D}{B \oslash D \to A \,\backslash\, C} \qquad \frac{A \otimes B \to C \oplus D}{C \obslash B \to A \,\backslash\, D} \qquad \frac{A \otimes B \to C \oplus D}{A \oslash D \to C \,/\, B}$$



11. Display sequent calculus

Motivation At first sight, it looks like the Grishin distributivity laws could be absorbed in Lambek's logical sequent rules. Compare ($\Delta[B\backslash A]$: $B\backslash A$ in a structural $\oplus$ context):

$$\textbf{NL}: \quad \frac{(B,\Gamma) \vdash A}{\Gamma \vdash B\backslash A}\ \backslash R \qquad \overset{?}{\leadsto} \qquad \textbf{LG}: \quad \frac{(B,\Gamma) \vdash \Delta[A]}{\Gamma \vdash \Delta[B\backslash A]}\ \backslash R$$

But, this **LG** rule (and the $\bowtie, \infty$ duals) is incomplete: no cut-free derivations below.

$$(a, (c \oslash ((a\backslash b) \otimes c))) \vdash b \qquad b \vdash (((c/(b \oslash a))\backslash c), a)$$

Display sequent calculus Goré 1999, MM 2007.

- structural punctuation for every logical connective
- (dual) residuation, Galois principles: structural rules, display equivalences
- Grishin's distributivity laws: structural too

12. LG display calculus: structural rules

Sequents Arrows $A \to B$ to sequents $X \vdash Y$, with X (Y) input (output) structures.

$$\begin{array}{lcl}
\mathcal{I} & ::= & x : A \mid \mathcal{I} \cdot\otimes\cdot \mathcal{I} \mid \mathcal{I} \cdot\oslash\cdot \mathcal{O} \mid \mathcal{O} \cdot\obslash\cdot \mathcal{I} \mid {}^{1\cdot}\mathcal{O} \mid \mathcal{O}^{\cdot 1} \\
\mathcal{O} & ::= & \alpha : A \mid \mathcal{O} \cdot\oplus\cdot \mathcal{O} \mid \mathcal{I} \cdot\backslash\cdot \mathcal{O} \mid \mathcal{O} \cdot/\cdot \mathcal{I} \mid \mathcal{I}^{\cdot 0} \mid {}^{0\cdot}\mathcal{I}
\end{array}$$

Axiom, formula cut

$$A \vdash A \qquad \frac{X \vdash A \quad A \vdash Y}{X \vdash Y}$$

Residuation, Galois laws Display equivalences. For example:

$$\frac{A \to C/B}{A \otimes B \to C} \;\rightsquigarrow\; \frac{X \vdash Z \cdot/\cdot Y}{X \cdot\otimes\cdot Y \vdash Z} \quad ; \quad \frac{{}^{\mathbf{1}}B \to A}{A^{\mathbf{1}} \to B} \;\rightsquigarrow\; \frac{{}^{1\cdot}Y \to X}{X^{\cdot 1} \to Y}$$

Distributivity laws All operations are structural. For example:

$$\frac{A \otimes B \to C \oplus D}{C \obslash A \to D/B} \;\rightsquigarrow\; \frac{X \cdot\otimes\cdot Y \vdash Z \cdot\oplus\cdot W}{Z \cdot\obslash\cdot X \vdash W \cdot/\cdot Y}$$

13. LG display calculus: logical rules

Each connective has a left and a right introduction rule. They fall in two groups.

Rewrite rules Reversible. Toggle between logical, structural operation.

$$\frac{A \cdot \oslash \cdot B \vdash Y}{A \oslash B \vdash Y} \oslash L \qquad\qquad \frac{A^{\cdot 1} \vdash Y}{A^{\mathbf{1}} \vdash Y} \cdot^{\mathbf{1}} L$$

Monotonicity rules

$$\frac{X \vdash A \qquad B \vdash Y}{X \cdot \oslash \cdot Y \vdash A \oslash B} \oslash R \qquad\qquad \frac{A \vdash Y}{Y^{\cdot 1} \vdash A^{\mathbf{1}}} \cdot^{\mathbf{1}} R$$

Complete the picture using the $\bowtie$ and ∞ symmetries (EXERCISE).

Cut elimination MM 2007.

14. Cut-free derivation

Display property Any formula component of a sequent can be displayed as the sole antecedent or succedent part, where the logical rules are applicable.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{a \vdash a} \qquad \dfrac{}{b \vdash b}}{b \cdot \oslash \cdot a \vdash b \oslash a}\ \oslash R \qquad \dfrac{}{c \vdash c}}{c/(b \oslash a) \vdash c \cdot / \cdot (b \cdot \oslash \cdot a)}\ /L}{b \cdot \oslash \cdot a \vdash (c/(b \oslash a)) \cdot \backslash \cdot c}\ r}{b \cdot \oslash \cdot a \vdash (c/(b \oslash a))\backslash c}\ \backslash R}{b \vdash ((c/(b \oslash a))\backslash c) \cdot \oplus \cdot a}\ dr$$

Observe This sequent is problematic for a standard Gentzen presentation, with only structural punctuation for $\otimes$ and $\oplus$.

15. The syntax-semantics mapping

Continuation semantics More balanced division of labour between source and target:

- distinction values vs continuations: functions from values to answer type
- evaluation context made explicit part of interpretation process

Antecedents

- Girard, 1991. A new constructive logic: classical logic; Lafont, Reus, Streichter, 1993. Continuation semantics or expressing $\rightarrow$ by $\neg$; Hofmann & Streicher, 1997. Cont models are $\forall$ for $\lambda\mu$ calculus.
- Selinger, 2001. Control categories and duality: on the categorical semantics of the $\lambda\mu$ calculus; Curien & Herbelin, 2000. The duality of computation.

LG and continuations

- Bernardi & MM, 2007, LNCS 4576; 2010 Inf & Computation 208(5):397–416
- Bastenhof 2010. Polarized Montagovian semantics for LG. Proc FG10.

16. LG: continuation semantics

$$\mathbf{LG}^{\mathcal{A}}_{/,\backslash,\oslash,\varobslash,\cdot 1,1\cdot,\cdot 0,0\cdot} \xrightarrow{\lceil\cdot\rceil} \mathbf{LP}^{\mathcal{A}\cup\{\bot\}}_{\rightarrow} \xrightarrow{[\![\cdot]\!]} \mathbf{IL}^{\{e,t\}}_{\rightarrow}$$

Two-step interpretation

- $\lceil\cdot\rceil$: double-negation/continuation-passing-style translation
  - maps multiple conclusion source logic to intuitionistic linear logic
  - introduces special response type $\bot$
- $[\![\cdot]\!]$: combining lexical with derivational semantics
  - atomic types: $[\![np]\!] = e$, $[\![s]\!] = [\![\bot]\!] = t$
  - terms: nonlinearity restricted to constants;

$$[\![(M\ N)]\!] = ([\![M]\!]\ [\![N]\!]) \quad ; \quad [\![\lambda x.M]\!] = \lambda\widetilde{x}.[\![M]\!]$$

- target interpretation: composition $[\![\cdot]\!] \circ \lceil\cdot\rceil$

17. CPS translation

Bernardi & MM 2010. Below the call-by-value $\lceil\cdot\rceil$ version. Call by name: $\lfloor A \rfloor = \lceil A^{\infty} \rceil$.

CPS mapping

- Source: **LG** display sequent calculus.
- Target: fragment of natural deduction **LP** (MILL) with response type $\bot$; all functions have head type $\bot$. Notation: $A^{\bot} \triangleq A \rightarrow \bot$.

Types For source types A, the target calculus distinguishes values: $\lceil A \rceil$, continuations: $\lceil A \rceil^{\bot}$, and computations: $\lceil A \rceil^{\bot\bot}$. For p atomic, $\lceil p \rceil = p$.

- Target is non-directional: $\lceil A^{\bowtie} \rceil = \lceil A \rceil$.
- Duality (co)implication: $\lceil A \backslash B \rceil = \lceil B \rceil^{\bot} \rightarrow \lceil A \rceil^{\bot} \quad ; \quad \lceil A \oslash B \rceil = \lceil A \backslash B \rceil^{\bot}$
- Negations: $\lceil A^{\mathbf{0}} \rceil = \lceil {}^{\mathbf{1}}A \rceil = \lceil A \rceil^{\bot}$

18. Translation (cont'd)

Structures **LG** structures are translated into MILL linear typing environments:

- Atomic structures: $\lceil x : A \rceil = \{\widetilde{x} : \lceil A \rceil\}$; $\lceil \alpha : A \rceil = \{\widetilde{\alpha} : \lceil A \rceil^\perp\}$
- Composite, for n-place structure building operations f:

$$\lceil f(X_1, \ldots, X_n) \rceil = \bigcup_{i=1}^{n} \lceil X_i \rceil$$

Sequents Neutral (commands); active output (terms) or input (contexts) formula.

Invariants of the translation

source: $\mathbf{LG}^{\mathcal{A}}_{/,\backslash,\oslash,\obslash,\cdot\mathbf{1},\mathbf{1}\cdot,\cdot\mathbf{0},\mathbf{0}\cdot}$		$\xrightarrow[\text{CPS}]{\lceil\cdot\rceil}$	target: $\mathbf{LP}^{\mathcal{A}\cup\{\perp\}}_{\rightarrow}$		
terms	$X \vdash B$		$\lceil X \rceil$	$\vdash$	$M : \lceil B \rceil^{\perp\perp}$
contexts	$A \vdash Y$		$\lceil Y \rceil$	$\vdash$	$K : \lceil A \rceil^{\perp}$
commands	$X \vdash Y$		$\lceil X \rceil \cup \lceil Y \rceil$	$\vdash$	$S : \perp$

19. Translation (cont'd)

Identity Axiom, co-axiom; cut.

$$\frac{}{x : A \vdash A}\ \mathrm{Ax} \qquad \frac{X \vdash A \quad A \vdash Y}{X \vdash Y}\ \mathrm{Cut} \qquad \frac{}{A \vdash \alpha : A}\ \text{Co-Ax}$$

$$\lceil \mathrm{Ax} \rceil = \lambda k.(k\ \widetilde{x}) : \lceil A \rceil^{\perp\perp} \qquad \lceil \mathrm{Cut} \rceil = (M^{\lceil A \rceil^{\perp\perp}}\ K^{\lceil A \rceil^{\perp}}) : \perp \qquad \lceil \text{Co-Ax} \rceil = \widetilde{\alpha} : \lceil A \rceil^{\perp}$$

Activate a passive formula New wrt the neutral sequent presentation.

$$\frac{X \vdash \alpha : A}{X \vdash A}\ \mu \qquad \frac{x : A \vdash Y}{A \vdash Y}\ \widetilde{\mu}$$

$$\lceil \mu \rceil = \lambda \widetilde{\alpha}.S^{\perp} : \lceil A \rceil^{\perp\perp} \qquad \lceil \widetilde{\mu} \rceil = \lambda \widetilde{x}.S^{\perp} : \lceil A \rceil^{\perp}$$

Deactivate cuts with a (co)axiom as premise.

20. LG: logical rules

Monotonicity

$$\frac{X \vdash A \quad B \vdash Y}{A\backslash B \vdash X \cdot \backslash \cdot Y} \backslash L \qquad \frac{X \vdash A \quad B \vdash Y}{X \cdot \oslash \cdot Y \vdash A \oslash B} \oslash R$$

$$\begin{array}{lcll} \lceil \backslash L \rceil & = & \lambda u.(M^{\lceil A \rceil^{\perp\perp}}(u\ K^{\lceil B \rceil^{\perp}})) & : \lceil A\backslash B \rceil^{\perp} \\ \lceil \oslash R \rceil & = & \lambda k.(k\ \lceil \backslash L \rceil) & : \lceil A \oslash B \rceil^{\perp\perp} \end{array}$$

Rewrites reversible

$$\frac{X \vdash x : A \cdot \backslash \cdot \beta : B}{X \vdash A\backslash B} \backslash R \qquad \frac{x : A \cdot \oslash \cdot \beta : B \vdash X}{A \oslash B \vdash X} \oslash L$$

$$\lceil \oslash L \rceil = \lceil \backslash R \rceil = \lambda h.(h\ \lambda \widetilde{\beta} \lambda \widetilde{x}.S^{\perp}) : \lceil A\backslash B \rceil^{\perp\perp} = \lceil A \oslash B \rceil^{\perp}$$

21. Logical rules: negations

Galois

$$\frac{X \vdash {}^{0\cdot}(x : A)}{X \vdash {}^{\mathbf{0}}A} \; {}^{\mathbf{0}}\cdot R \qquad \lambda k.(k\ \lambda\widetilde{x}.S^{\perp}) : \lceil {}^{\mathbf{0}}A \rceil^{\perp\perp}$$

$$\frac{X \vdash A}{{}^{\mathbf{0}}A \vdash {}^{0\cdot}X} \; {}^{\mathbf{0}}\cdot L \qquad M : \lceil {}^{\mathbf{0}}A \rceil^{\perp} = \lceil A \rceil^{\perp\perp}$$

Dual Galois

$$\frac{A \vdash Y}{Y^{\cdot 1} \vdash A^{\mathbf{1}}} \; \cdot^{\mathbf{1}}R \qquad \lambda k.(k\ K^{\lceil A \rceil^{\perp}}) : \lceil A^{\mathbf{1}} \rceil^{\perp\perp}$$

$$\frac{(\alpha : A)^{\cdot 1} \vdash Y}{A^{\mathbf{1}} \vdash Y} \; \cdot^{\mathbf{1}}L \qquad \lambda\widetilde{\alpha}.S^{\perp} : \lceil A^{\mathbf{1}} \rceil^{\perp}$$

22. Exploiting continuations

Balance source/target The continuation semantics optimizes the division of labour between syntactic source and semantic target.

- Continuations in the interpretation of the **LG** core system
  - illustration: scope ambiguities in terms closure/interior operations
  - new expressivity already for Lambek fragment
- Grishin's distributivity laws can establish discontinuous dependencies
  - structural deformations under which interpretations are stable
  - illustration: modeling binding $q(A, B, C)$ as $(B \oslash C) \obackslash A$

23. Scope ambiguity in base logic

We use the interior operation ${}^{\mathbf{1}}(\ \cdot\ {}^{\mathbf{1}})$ to obtain scope ambiguities.

Compare: two derivations/readings for ‘everyone met somebody’.

$$\dfrac{np \vdash \cdot\, np\, \cdot}{np^{\cdot 1} \vdash np^{\mathbf{1}}}\ \cdot^{\mathbf{1}} R$$

$$\dfrac{np^{\cdot 1} \vdash np^{\mathbf{1}}}{{}^{1\cdot}(np^{\mathbf{1}}) \vdash np}\ \Rightarrow$$

$$\dfrac{{}^{1\cdot}(np^{\mathbf{1}}) \vdash np \qquad s \overset{\alpha}{\vdash} \cdot\, s\, \cdot}{np\backslash s \vdash {}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s}\ \backslash L$$

$$\dfrac{np\backslash s \vdash {}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s \qquad \cdot\, np\, \cdot \overset{y}{\vdash} np}{(np\backslash s)/np \vdash ({}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s) \cdot/\cdot np}\ /L$$

$$\dfrac{(np\backslash s)/np \vdash ({}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s) \cdot/\cdot np}{\color{red}{np} \vdash (np\backslash s)/np \cdot\backslash\cdot ({}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s)}\ \Leftarrow$$

$$\dfrac{\color{red}{np} \vdash (np\backslash s)/np \cdot\backslash\cdot ({}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s)}{((np\backslash s)/np \cdot\backslash\cdot ({}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s))^{\cdot 1} \vdash \color{red}{np^{\mathbf{1}}}}\ \cdot^{\mathbf{1}} R \quad \text{(direct object)}$$

$$\dfrac{((np\backslash s)/np \cdot\backslash\cdot ({}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s))^{\cdot 1} \vdash np^{\mathbf{1}}}{{}^{\mathbf{1}}(np^{\mathbf{1}}) \vdash (np\backslash s)/np \cdot\backslash\cdot ({}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s)}\ {}^{\mathbf{1}}\cdot L$$

$$\dfrac{{}^{\mathbf{1}}(np^{\mathbf{1}}) \vdash (np\backslash s)/np \cdot\backslash\cdot ({}^{1\cdot}(np^{\mathbf{1}}) \cdot\backslash\cdot s)}{{}^{\mathbf{1}}(np^{\mathbf{1}}) \vdash s \cdot/\cdot ((np\backslash s)/np \cdot\otimes\cdot {}^{\mathbf{1}}(np^{\mathbf{1}}))}\ {}^{\mathbf{1}}\cdot L$$

$$\dfrac{{}^{\mathbf{1}}(np^{\mathbf{1}}) \vdash s \cdot/\cdot ((np\backslash s)/np \cdot\otimes\cdot {}^{\mathbf{1}}(np^{\mathbf{1}}))}{\underbrace{{}^{\mathbf{1}}(np^{\mathbf{1}})}_{\text{su}} \cdot\otimes\cdot (\underbrace{(np\backslash s)/np}_{\text{tv}} \cdot\otimes\cdot \underbrace{{}^{\mathbf{1}}(np^{\mathbf{1}})}_{\text{do}}) \vdash s}\ \leftrightharpoons$$

$\lceil\cdot\rceil$ translation: $\lambda\widetilde{\alpha}.(\color{red}{\text{do}}\ \lambda\widetilde{y}.((\color{blue}{\text{tv}}\ \lambda u.(\color{blue}{\text{su}}\ (u\ \widetilde{\alpha})))\ \widetilde{y}))$

24. Object wide scope: step by step

$$
\begin{array}{rrcl}
\cdot^{\mathbf{1}}R & \lambda k.(k\ \widetilde{\beta}) & : & \lceil np^{\mathbf{1}}\rceil^{\perp\perp} \\
\Rightarrow & \lambda\widetilde{\beta}.(\widetilde{\gamma}\ \widetilde{\beta}) = \widetilde{\gamma} & : & \lceil np^{\mathbf{1}}\rceil^{\perp} = \lceil np\rceil^{\perp\perp} \\
\backslash L & \lambda u.(\widetilde{\gamma}\ (u\ \widetilde{\alpha})) & : & \lceil np\backslash s\rceil^{\perp} \\
/L & \lambda u'.(u'\ \lambda u.(\widetilde{\gamma}\ (u\ \widetilde{\alpha}))\ \widetilde{y}) & : & \lceil (np\backslash s)/np\rceil^{\perp} \\
\Leftarrow & \lambda\widetilde{y}.(\mathrm{tv}\ \lambda u.(\widetilde{\gamma}\ (u\ \widetilde{\alpha}))\ \widetilde{y}) & : & \color{red}{\lceil np\rceil^{\perp}} \\
\cdot^{\mathbf{1}}R & \lambda k.(k\ \lambda\widetilde{y}.(\mathrm{tv}\ \lambda u.(\widetilde{\gamma}\ (u\ \widetilde{\alpha}))\ \widetilde{y})) & : & \color{red}{\lceil np^{\mathbf{1}}\rceil^{\perp\perp}} \\
{}^{\mathbf{1}}\cdot\ L & \lambda\widetilde{\kappa}.(\widetilde{\kappa}\ \lambda\widetilde{y}.(\mathrm{tv}\ \lambda u.(\widetilde{\gamma}\ (u\ \widetilde{\alpha}))\ \widetilde{y})) & : & \lceil{}^{\mathbf{1}}(np^{\mathbf{1}})\rceil^{\perp} \\
{}^{\mathbf{1}}\cdot\ L & \lambda\widetilde{\gamma}.(\mathrm{do}\ \lambda\widetilde{y}.(\mathrm{tv}\ \lambda u.(\widetilde{\gamma}\ (u\ \widetilde{\alpha}))\ \widetilde{y})) & : & \lceil{}^{\mathbf{1}}(np^{\mathbf{1}})\rceil^{\perp} \\
\leftrightharpoons & \lambda\widetilde{\alpha}.(\mathrm{do}\ \lambda\widetilde{y}.((\mathrm{tv}\ \lambda u.(\mathrm{su}\ (u\ \widetilde{\alpha})))\ \widetilde{y})) & : & \lceil s\rceil^{\perp\perp}
\end{array}
$$

25. Subject wide scope

Below an alternative derivation, targeting first the subject, rather than the direct object.

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{\cdot\, np \cdot \overset{x}{\vdash} np \qquad s \overset{\alpha}{\vdash} \cdot\, s \,\cdot}{np\backslash s \vdash np \cdot \backslash \cdot s}\;\backslash L
      \qquad
      \dfrac{\vdots}{{}^{1\cdot}(np^{\mathbf{1}}) \vdash np}\;\Rightarrow
    }{(np\backslash s)/np \vdash (np \cdot \backslash \cdot s) \cdot / \cdot {}^{1\cdot}(np^{\mathbf{1}})}\;/L
  }{\color{red}{np} \vdash s \cdot / \cdot ((np\backslash s)/np \cdot \otimes \cdot {}^{1\cdot}(np^{\mathbf{1}}))}\;\Leftarrow
}{(s \cdot / \cdot ((np\backslash s)/np \cdot \otimes \cdot {}^{1\cdot}(np^{\mathbf{1}})))^{\cdot 1} \vdash \color{red}{np^{\mathbf{1}}}}\;\cdot^{\mathbf{1}}R \quad \text{(subject)}
$$

$$\vdots$$

$\lceil\cdot\rceil$ translation: $\lambda\widetilde{\alpha}.(\color{red}{\mathrm{su}}\ \lambda\widetilde{x}.(\color{blue}{\mathrm{do}}\ (\color{blue}{\mathrm{tv}}\ \lambda u.((u\ \widetilde{\alpha})\ \widetilde{x}))))$

26. Translating the lexical constants

The table below gives the $[\![\lceil \cdot \rceil]\!]$ translation of the constants, for the sample sentence 'everyone saw something', assuming $[\![\lceil np \rceil]\!] = e$, $[\![\lceil s \rceil]\!] = [\![\lceil \bot \rceil]\!] = t$, and a target constant 'see' of type $e \to e \to t$.

source	$[\![\lceil \cdot \rceil]\!]$ translation
everyone : $\lceil np \rceil^{\bot\bot}$	$\forall : (e \to t) \to t$
someone : $\lceil np \rceil^{\bot\bot}$	$\exists : (e \to t) \to t$
saw : $(\lceil s \rceil^{\bot} \to \lceil np \rceil^{\bot})^{\bot} \to \lceil np \rceil^{\bot}$	$\lambda v \lambda y.(v\ \lambda c \lambda x.(c\ ((\mathrm{see}\ y)\ x)))$
	$: (((t \to t) \to e \to t) \to t) \to e \to t$

Final result Composition $[\![\cdot]\!] \circ \lceil \cdot \rceil$, and an evaluation step, providing the identity function $\lambda p.p$ for the abstraction over the parameter c of type $t \to t$.

$$[\![\lambda \widetilde{\alpha}.(\mathrm{do}\ \lambda \widetilde{y}.((\mathrm{tv}\ \lambda u.(\mathrm{su}\ (u\ \widetilde{\alpha})))\ \widetilde{y}))]\!] = \lambda c.(\exists\ \lambda y.(\forall\ \lambda x.(c\ ((\mathrm{see}\ y)\ x))))$$

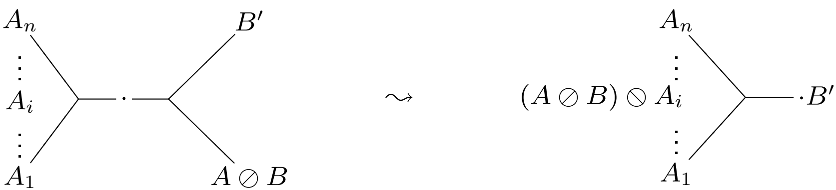
$$[\![\lambda \widetilde{\alpha}.(\mathrm{su}\ \lambda \widetilde{x}.(\mathrm{do}\ (\mathrm{tv}\ \lambda u.((u\ \widetilde{\alpha})\ \widetilde{x}))))]\!] = \lambda c.(\forall\ \lambda x.(\exists\ \lambda y.(c\ ((\mathrm{see}\ y)\ x))))$$

27. Grishin interaction: infixation

Application versus co-application

$$
\dfrac{\dfrac{X \vdash A \quad B \vdash Y}{A \backslash B \vdash X \cdot \backslash \cdot Y}\ \backslash L}{X \cdot \otimes \cdot (A \backslash B) \vdash Y}
\qquad
\dfrac{\dfrac{X \vdash A \quad B \vdash Y}{X \cdot \oslash \cdot Y \vdash A \oslash B}\ \oslash R}{X \vdash (A \oslash B) \cdot \oplus \cdot Y}
$$

Grishin interaction On the right, if X is a $\otimes$ tree with yield $A_1, \ldots, A_n$, the conditions for Grishin interaction are met: $A \oslash B$ can associate with any A_i.



28. Illustration: tense

Infixation Modeling $q(A, B, C)$: a formula $(B \oslash C) \obslash A$

- behaves locally as an A within a context of type B
- acts as a function that transforms B into C

Tense We type an inflected verb as a combination of a tenseless verb stem, and a tense morpheme $vp \oslash tns$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\cdot\, np \,\cdot \overset{\text{john}}{\vdash} np \qquad vp \overset{\alpha_1}{\vdash} \cdot\, vp \,\cdot}{np\backslash vp \vdash np \cdot \backslash \cdot vp}\ \backslash L \qquad \cdot\, np \,\cdot \overset{\text{mary}}{\vdash} np}{(np\backslash vp)/np \vdash (np \cdot \backslash \cdot vp) \cdot / \cdot np}\ /L}{np \cdot \otimes \cdot ((np\backslash vp)/np \cdot \otimes \cdot np) \vdash vp}\ \leftrightharpoons \qquad tns \overset{\alpha_0}{\vdash} \cdot\, tns \,\cdot}{(np \cdot \otimes \cdot ((np\backslash vp)/np \cdot \otimes \cdot np)) \cdot \oslash \cdot tns \vdash vp \oslash tns}\ \oslash R}{\dfrac{(vp \oslash tns) \obslash ((np\backslash vp)/np) \vdash (np \cdot \backslash \cdot tns) \cdot / \cdot np}{\underbrace{np}_{\text{john}} \cdot \otimes \cdot (\underbrace{(vp \oslash tns) \obslash ((np\backslash vp)/np)}_{\text{saw}} \cdot \otimes \cdot \underbrace{np}_{\text{mary}}) \vdash tns}\ \leftrightharpoons}\ \obslash L
$$

29. Interpretation

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdots}{np \cdot \otimes \cdot ((np\backslash vp)/np \cdot \otimes \cdot np) \vdash vp} \quad tns \overset{\alpha_0}{\vdash} \cdot tns \cdot}{(np \cdot \otimes \cdot ((np\backslash vp)/np \cdot \otimes \cdot np)) \cdot \oslash \cdot tns \vdash vp \oslash tns}\ \oslash R}{(vp \oslash tns) \obslash ((np\backslash vp)/np) \vdash (np \cdot \backslash \cdot tns) \cdot / \cdot np}\ \obslash L}{\underbrace{np}_{\text{john}} \cdot \otimes \cdot \underbrace{((vp \oslash tns) \obslash ((np\backslash vp)/np)}_{\text{saw}} \cdot \otimes \cdot \underbrace{np}_{\text{mary}}) \vdash tns}\ \leftrightharpoons \tag{1}$$

$$\lceil(1)\rceil \quad = \quad \lambda\widetilde{\alpha}_0.(\textsf{saw}\ \lambda\widetilde{\beta}.(\lambda\widetilde{z}.(\widetilde{\beta}\ \lambda h.((\widetilde{z}\ \lambda u.((u\ (h\ \widetilde{\alpha}_0))\ \textsf{john}))\ \textsf{mary})))) \tag{2}$$

$$[\![(2)]\!] \quad = \quad \lambda c.(c\ (\textsc{past}\ ((\textsc{see}\ \textsc{mary})\ \textsc{john})))$$

$$[\![\textsf{see}]\!] \quad = \quad \lambda V \lambda y.(V\ \lambda c \lambda x.(c\ ((\textsc{see}^{e\to e\to t}\ y)\ x)))$$

$$[\![\textsf{-ed}]\!] \quad = \quad \lambda c \lambda v.(c\ (\textsc{past}^{t\to t}\ v))$$

$$[\![\textsf{saw}]\!] \quad = \quad \lambda Q.((Q\ \lambda u.(u\ [\![\textsf{-ed}]\!]))\ [\![\textsf{see}]\!])$$

30. Conclusions

The symmetric Lambek-Grishin calculus offers some strategies to tackle the expressive limitations of the original Lambek calculi:

- Form
  - logical distributivity laws relating dual families
  - narrows the options for structural reasoning: preservation properties
- Meaning
  - continuation semantics for multiple-conclusion source calculus
  - optimizes division of labour between syntax and semantics

More to explore ESSLLI 2007 course wiki

http://symcg.pbworks.com/

□