“Chaque vin a sa lie” vs. ‘Toute nuit a un jour”

Does the difference in the human processing of “chaque” and “tout” match the difference between the proof rules for conjunction and quantification?

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Plan

Universal quantification in French

The meaning and uses of tout / chaque

The meaning

Prescriptive and descriptive generalizations

Universal quantification from a logical point of view

Universal quantification in French

Tout / chaque: singular morphology on the noun.

(1) a. Tout étudiant a rendu son devoir.
TOUT student has returned his homework.
b. Chaque étudiant a rendu son devoir.
CHAQUE student has returned his homework.

Tous les: plural morpholoy on the noun.

(2) Tous les étudiants ont rendu leur devoir.
TOUS LES students have returned their homework.

NB Les is not a quantifier, but maximality, and non trivial relation with *tous les*, see Corblin, 2008.

(3) Les étudiants ont rendu leur devoir.
The students have returned their homework.

Scope and goal

Semantics:

- Empirical scope narrowed down to the pair tout / chaque.
- The semantics of tout / chaque.
- Considering the discourses and the types of utterances. Link the type of the utterance to the semantics.

Modelling

- Establish a connection between two types of quantifiers and discourses and two types of universal quantification in logic.

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Universal quantification in French

The meaning and uses of tout / chaque

- The meaning
- Prescriptive and descriptive generalizations

Universal quantification from a logical point of view

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Generic statements

Tout only can be used in generic sentences.

(4) a. Tout homme est mortel.
TOUT man is mortal.
b. #Chaque homme est mortel.
CHAQUE man is mortal.

$\rightarrow$ *Tout* unrestricted generality.

Generic statements

Tout only can be used if no elements of the class. Which s typical of generic sentences.
If no student got an A:

(5) a. Tout étudiant ayant eu un A a un prix.
TOUT student who got an A has a price.
b. #Chaque étudiant qui a eu un A a un prix.
CHAQUE student who got an A has a price.

Restricting the domain

(6) a. Chaque plante de mon jardin est verte.
CHAQUE plant in my garden is green.
b. Toute plante de mon jardin est verte.
TOUT plant in my garden is green.

The b. sentence is about the types of plants that are allowed in my garden, not necessarily about the actual plants which are in my garden.
$\rightarrow$ *Tout* creates a type reading when restricted.

Restricting the domain

(7) a. #Tout homme sur terre est mortel.
TOUT man on earth is mortal. (sounds odd)
b. Chaque homme sur terre est mortel.
CHAQUE man on earth is mortal. (sounds as a weak generalization, but true)

The restriction has the effect of narrowing the domain for *chaque*. This restriction is needed for *chaque* but it not with *tout*. A subtype is created with *tout* and hence the oddness, as the a. sentence suggests that there are types on non earthy men who are not mortal.

Tout as a free choice?

It has been suggested that *tout* is a free choice item (FCI), by Jayez and Tovena (2006), like *n'importe lequel/laquelle*. We disagree.

Generic again

(8) a. Tout homme est mortel.
TOUT man is mortal.
b. ??N'importe quel homme est mortel.
FCI man is mortal.

The FCI version is odd: a restriction for the FCI needs to be accommodated.

But ...

When the restriction is implicit, FCI *n'importe quoi* is fine, *tout* is not.

(9) a. #Prends toute carte.
Pick TOUTE card.
b. Prends n'importe quelle carte.
Pick FCI card.

Here there is an implicit restricted set of card FCI is fine, *tout* is not.

But ...

An overt restriction must be added. (The phenomenon whereby a restriction is added to have a FCi to go through is called subtrigging. But subtrigging is needed for FCI in episodic statements, here the phenomenon is different, we need to constrain the absolute generality of *tout*).

(10) a. Prends toute carte qui te fasse gagner.
Pick TOUTE card that allows you to win.
b. Prends n'importe quelle carte qui te fasse gagner
Pick FCI card that allows you to win.

FCI require a domain restriction of some sort, *tout* does not.

Note that ...

Tout / *chaque* in the same context

(11) a. Prends toute carte qui te fasse gagner.
Pick TOUTE card that allows you to win.
b. Prends chaque carte qui te fasse gagner
Pick CHAQUE card that allows you to win.

a. sentence: you do not need to pick all the cards that allows you to win.
b. sentence you must pick all the cards that allow you to win.
On the assumption that the imperative is analyzed as permission modal (Dayal, 1998), we see that in a. the universal scopes above the modal (for every card there is a world in which you pick them, but not necessarily the same one), in the b. sentence the universal scopes below the modal (there is a world in which you pick all the cards).

First generalizations

- *Tout* conveys total generality. Unrestricted quantification. *Chaque* requires a well determined domain of quantification.
- *Tout* compatible with absence of instances *Chaque* is not.
- When restricted, *tout* creates a type reading. *Chaque*, does not.
- *Tout* wide scope (over modals). *Chaque* does not.

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Tout and Prescriptivity

Tout requires an underlying rule:
$\Box P(x) \rightarrow Q(x)$.
Tout is used in prescriptive statements (similarly to indefinite generic statements, Cohen, 2001).

(12) a. Tout chien a un systme nerveux.
TOUT dog has a nervous system.

Having a nervous system is part of being a dog.

Chaque and Descriptivity

Chaque requires that one investigates, one by one, all the members of the class.

Recall: *chaque* requires a closed domain of quantification. It conveys that all the members have been inspected.

There is no rule underlying the use of *chaque*, and any property can be used.

Chaque and Descriptivity

We expect:

- Different distributions of the types of properties they can combine with.
- Different patterns of tolerance to exceptions.

Essential vs. accidental properties

Tout is only compatible with essential properties (subtrigging car rescue, but not always).

(13) a. Tout enfant est joyeux.
TOUT child is happy.
b. #Tout enfant est malade.
TOUT child is sick.

Essential vs. accidental properties

Chaque is compatible with both essential and accidental properties. (recall that *chaque* requires that there is a determined domain of quantification).

(14) a. Chaque enfant est joyeux.
CHAQUE child is happy.
b. Chaque enfant est malade.
CHAQUE child is sick.

Tolerance to exceptions

Tout tolerates exceptions as classes

(15) a. Tout enfant est joyeux, sauf les enfants pauvres.
TOUT child is happy, but the poor ones.

Tolerance to exceptions

Tout can tolerate (not very well) individual exceptions.

(16) a. Tout enfant est joyeux, sauf Jean.
TOUT child is happy, but John.

There is an effect though ....

Tolerance to individual exceptions and prescriptivity

Tout can be used in prescriptive statements; it can provide a rule (similarly to indefinite generic statements, Cohen, 2001).
Can it stand individual exceptions ? A first type of individual exceptions.

(17) a. Tout chien a quatre pattes.
TOUT dog has four legs/a brain.
b. Sauf le mien, il a eu un accident.
All but mine, he had an accident.

With the b. sentence you discard the accident information. My dog is a regular dog with 4 legs (it is accidental that he does not have four).

Tolerance to individual exceptions and prescriptivity

Can it stand individual exceptions ? A second type of individual exceptions with essential properties.

(18) a. Tout chien n'a pas de plumage.
TOUT dog has a brain.
b. Sauf le mien, il avait déchiré un oreiller.
All but mine, he tore a pillow.
c. Sauf Fido, il est né ainsi.
All but Fido, he was born so.

Observe that Fido, cannot count as a dog to begin with, especially for , where he never was a real dog. I can reply (19):

(19) Tu rigoles, Fido n'est pas un chien !
You're kidding, Fido is not a dog.

Tolerance exceptions and descriptivity

Chaque is intolerant to exceptions of any kind.

(20) a. Chaque enfant est joyeux, #sauf les pauvres.
CHAQUE child is happy, but the poor ones.
b. Chaque enfant est joyeux, #sauf Jean.
CHAQUE child is happy, but John.

Summarizing ...

Tout

- Compatible with an infinite domain.
- Requires the existence of a law (hence compatible with absence of instances)
- Only compatible with essential properties
- In discourse: it is used prescriptively.

Chaque

- Compatible with both essential and accidental properties
- It requires a well determined domain of quantification (hence incompatible with absence of instances and infinite domains).
- In discourse: it is used descriptively.

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Before Frege: Ancient logic, Scholastics

Aristotle's view of logic (III BC):

> *analyse and extract the rigour of mathematical reasoning (proofs)*
>
> *and apply it to any other science (that's the reason why Organon is to be studied first, and first the Categories).*

Maths since Thales, Pythagoras (VI BC) and Euclide is contemporary to Aristotle (although little is known about Euclide).

By that time, no models nor interpretations, but rules and axioms.

Quantification before Frege

Strangely enough first logic is not propositional logic (Stoics) but (restricted) quantified formulae:

A All A are B

E Some A are B

I No A is B

O Not all A are B / some A are not B.

Principles before Frege

Rules, patterns (axioms schemes, syllogisms) but no models.

Identity: All A are A
Non contradiction NOT (A and NOT A)

> *Avicenna: Every person refuting the principle of non contradiction should be beaten and burnt until he admits that being beaten is not the same has not being beaten and that being being burnt is not the same has not being burnt.*

Excluded middle : A ou NON A (tertium non datur)

Principles before Frege

Rules, patterns (axioms schemes, syllogisms) but no models.
Syllogisms, e.g. bArOcO

ASSUME All A are B	*A*
ASSUME Not all C are B	*O*
THEREFORE Not all C are A.	*O*

Why quantified sentences firstly? one cannot check that a property holds for each number/triangle (one must forget the figure and reason on the idea of a generic triangle)

British Algebraic Logic XIX: Boole, De Morgan, Pierce, ...

Boole:

$$\forall x.(I(x) \Rightarrow F(x) \vee M(x))$$
$$\Rightarrow (\forall x.(I(x) \Rightarrow F(x)) \vee (\forall x.(I(x) \Rightarrow M(x))))$$

???

Frege: proofs

Proof system: Begriffsschrift
Proof rules (admittedly obscure)
Proofs are finite, tree-like.
Idea of mechanised reasoning (finite and partly computable).
Unique sort: $\forall \mathsf{x}{:}\mathsf{A}.\ \mathsf{B}(\mathsf{x}) \equiv \forall \mathsf{x}.\ \mathsf{A}(\mathsf{x}) \Rightarrow \mathsf{B}(\mathsf{x})$

$$symmetrically\ \exists \mathsf{x}{:}\mathsf{A}.\ \mathsf{B}(\mathsf{x}) \equiv \forall \mathsf{x}.\ \mathsf{A}(\mathsf{x})\&\mathsf{B}(\mathsf{x}))$$

what about "most"?

$$\text{most}\, \mathsf{x}{:}\mathsf{A}.\ \mathsf{B}(\mathsf{x}) \not\equiv \text{most}\, \mathsf{x}.\ \mathsf{A}(\mathsf{x}) \Rightarrow \mathsf{B}(\mathsf{x})$$

Frege: Sinn / Bedeutung

The proofs (there can be several non equivalent proofs) of a formula can be seen as its sense (Sinn) cf. e.g. Dummet.

Models , compositional/inductive interpretation of a (logical) sentence, this is commonly viewed as the denotation (Bedeutung) of the sentence. Cf. later.

Hilbert proof systems

A proof is a finite tree starting from axioms, and yielding via a finite set of rules (patterns) to the conclusion.

Deduction theorem: $A \vdash B$ iff $A \Rightarrow B$.

Proof systems : formalisation of mathematical proofs in order to obtain consistency of arithmetics or of analysis etc. by combinatorial arguments on the proofs: if there would be a proof of a contradiction, then every thing would be provable, and there would exists a *normal* proof of $0 = 1$, but there cannot be such a proof. (this method fails for proper theories including arithmetic: cf. Gödel incompleteness theorem)

Hilbert, Gentzen: rules for quantification

Hilbert's rules for quantification:

- $(\phi \Rightarrow \psi) \Rightarrow (\phi \Rightarrow \forall x \psi)$ (no free x in ϕ)
- $\forall x(\phi \Rightarrow \psi) \Rightarrow ((\forall x \phi) \Rightarrow (\forall x \psi))$
- $(\forall x \phi) \Rightarrow \phi[x := t]$

Sequents: $X, Y, Z \vdash C$ conclusion C under assumptions X, Y, Z

First rule better stated with sequents:

$$\frac{\Gamma \vdash A(x)}{\Gamma \vdash \forall x.\ A(x)} \ \textit{no free } x \textit{ in } \Gamma$$

Hilbert's τ operator, rules

For any formula $F[x]$ there is a term $\tau_x.F[x]$ (and a term $\epsilon_x F[x] = \tau_x \neg F[x]$)
Rules:

$$\frac{\Gamma \vdash A[x]}{\Gamma \vdash A[\tau_x A[x]]} \textit{ no free } x \textit{ in } \Gamma \qquad\qquad \frac{\Gamma \vdash A[\tau_x A[x]]}{\Gamma \vdash A[t]}$$

Hilbert's τ calculus: properties

$\tau_x.F$ enjoys the property F iff everything enjoys F:
$F[\tau_x.F[x]] \equiv \forall x.\ F[x]$

$\epsilon_x.F$ enjoys the property F iff something enjoys F:
$F[\epsilon_x.F[x]] \equiv \exists x.\ F[x]$

Overbinding, in situ quantification: a term inside a predicate of a large formula may have scope over the whole formula.

More formulae than usual: $P(\tau_x Q(x))$ is not equivalent to any usual formula (first or higher order)

First proofs of quantifier elimination and of Herbrand theorem.

Remark: epsilon $\neq$ choice function:
the language (constants, functions, predicates,...) is not extended
this binder is enough for for all formulae at once.

Models (due to Frege, then Löwenheim, Skolem, Gödel)

Model: (family of) situations: set of individuals and interpretation of the constants (individuals, functions, predicates)

Nothing is finite nor computable: even checking the truth of a given formula in a given model can be an infinite process.

As for the propositional calculus the interpretation is flat:

$\forall x P(x)$ is true in M if for all x in M the interpretation of $P(x)$ is true.

Denotation (Bedeutung) of a formula in a model (truth value) , in a family of models (the models in which it is true), etc.

Completeness (Gödel, 1929)

Completeness (Gödel, 1929)
However there is a link between the two:

- F is provable if and only if it is true in any model.
- F can be proved from T if and only if any model that satisfies T satisfies F as well.

Observe that completeness is quite particular for first order logic (classical, and modal intuitionistic with Kripke models)
It fails for second order logic unless one use not-so-natural Henkin models, where predicate vary among definable subsets.

Halfway models/proofs Gentzen ω-rule

Known domain e.g integers for arithmetic which appears as constants in the logical language:

$$\dfrac{\begin{array}{c}\vdots\,\delta(0)\\ \Gamma\vdash A(0)\end{array}\quad\begin{array}{c}\vdots\,\delta(1)\\ \Gamma\vdash A(1)\end{array}\quad\begin{array}{c}\vdots\,\delta(2)\\ \Gamma\vdash A(2)\end{array}\quad\begin{array}{c}\vdots\,\delta(3)\\ \Gamma\vdash A(3)\end{array}\quad\cdots}{\Gamma\vdash\forall x.\ A(x)}\ \omega$$

Very different from the standard rule:

- infinite proof although every branch is finite
- the profos $\delta(i)$ are not necessarily uniforms
- no finite description of the proof (unless there is a description of the proof $\delta(n)$ from the previous $\delta(i)$ with $i \leq n$).
- $\forall x.\ A(x)$ looks like $\&_{i:integer} A(i)$ but this is not a first order formula.

Gentzen ω-rule versus standard rule

<u>$\forall \geq \omega$ Standard rule $\rightarrow \omega$-rule</u> Observe that if one has a proof $\delta(x)$ with a generic element x (a variable not free in any hypothesis) of $P(x)$ ie. when the classical rules works, one can have an omega version (provided the integers are constants of the language) by specialising $\delta(x)$ to each/every number to get $\delta(0)$, $\delta(1)$, $\delta(2)$, $\delta(3)$, ...

<u>$\forall \not\leq \omega$ ω-rule $\not\rightarrow$ standard rule</u> The converse does not hold, unless all the $\delta(i)$ are uniform, have the same shape and do not make use of any particularity of i.

A way to express the ω rule is to assert that $\forall \leq \omega$.

As we want to distinguish the two, we may write $\mathrm{chaque}(x : D)\ A(x)$, instead of $\forall x.A(x)$ as the conclusion of the ω-rule, that is a mere shorthand for $\&_{i:D} A(i)$ this presupposes that the basis of the (possible) world(s) is known.

TOUT: standard rule (generalisation)

When can we correctly assert a TOUT sentence?
As said above TOUT matches rather well the standard rule.
Indeed, TOUT has to be established by reasoning:

- its domain can be a sort, an infinite collection or not so well defined collection, that cannot be throughly examined
- it has a sempiternal nature, it is a rule

What about exceptions: even in maths we intend to make such mistakes and to correct them afterwards:

for instance we can wrongly derive $\forall n.1/n \leq 1$ *when* n *is an integer, one often forgets that* n *cannot be* 0 *and then fix it afterwards:* $\forall n.n \neq 0 \Rightarrow 1/n \leq 1$

this works as well (or even better, since we needed to refer to an element) when the exception corresponds to a property: $\forall n.\ 1/(nmod2) = 1$ *is fixed as* $\forall n.\ Odd(n) \Rightarrow 1/(nmod2) = 1$

Chaque: models or ω inspired rule

$\mathrm{chaque}(x : D)\ A(x)$ can be asserted when the domain is known and when for any x in D one has $A(x)$ hence it is a mere shorthand for $\&_{i:D} A(i)$ this presupposes that the basis of the (possible) world(s) is known.
Indeed, CHAQUE rather correspond to a thorough inspection of every element in the domain of quantification,$\&_{i \in D} A(i)$ (which is not a first order formula) [The model approach corresponding to can be supported, but it is a different framework.]

Now if we think at the situation in which one can assert CHAQUE it is because we have a proof or evidence for every entity x in the domain that $A(x)$ holds, (hence the form of the rule is similar to the one of the omega rule).

Comparing the assertion conditions of TOUT and CHAQUE

One often uses CHAQUE while TOUT can be asserted. This is fairly normal: if one is able to say TOUT, if there is a rule, than the conjunction for a precise domain at a precise moment can be deduced from the generic proof, by specialisation, as said above about proofs.

Refutation of universal quantification

In order to test this correspondence,
how do we refute CHAQUE and TOUT
in our opinion:

- Refutation of CHAQUE : find a counter example and that's all! The asserter needs to accept or redefine the domain
- Refutation of TOUT show that a subclass A (we remain with properties and generic associated with these properties) and add this as a restriction so one obtains the provable formula $\forall x. A(x) \Rightarrow P(x)$ — the counter example like the previous condition $n \neq 0$ is a particular case, that's a class with one element.

Forhtcoming experiments

2 groups of students, spring 2016
Web questionaries (with limited time per question)

- situation with precise and imprecise domains, with finite and infinite domains,
- preferred way to express a situation
- true or not in a situation
- preferred refutations of a given quantified sentence

DataBase postgresql/php to stock the information on the subjects and the results, and do statistics

Ideally we'd like to test with particular subjects, eg. dyslexic children as some experiments by Delfitto and Vender had interesting results on negation processing with the A E I O statements.

Epilog: “Chaque vin a sa lie.” vs. “Toute nuit a un jour.”

Same structure, same verb but one includes a possessive related to the singular quantifier.

First observe that it is a matter of *preference* and not a yes/no answer. For instance, when swapping the two quantifiers the two resulting variants of the proverbs sound not that bad. Also observe that “sa” goes well with “chaque”, and less well with “tout”

Nevertheless, before experiments are made, an intuitive analysis at those two proverbs supports our claim:

- the wine is much more concrete, and one can think of each of them as the content of a barrel, because each of them as its lie (which lies in the barrel).
- night/day are even more metaphorical and abstract, more infinite, they seem to be essences that are constant in time.