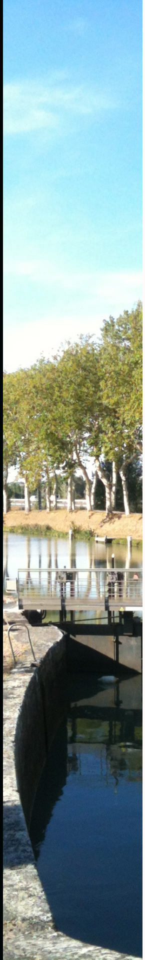


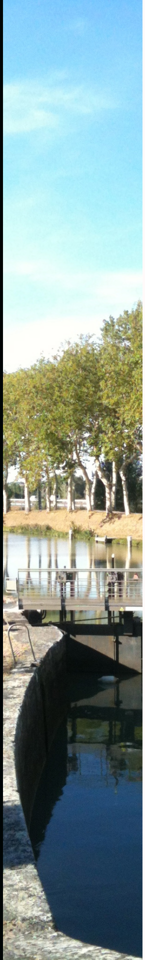
A natural framework for natural language semantics:

many sorted logic and Hilbert operators in type theory

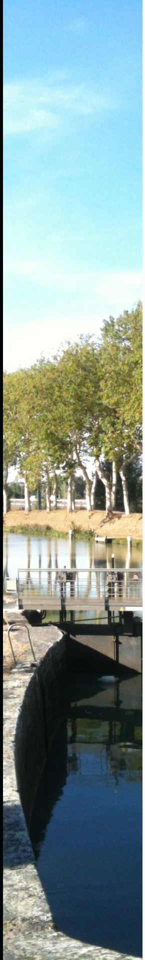
Christian Retoré

Université de Bordeaux & IRIT-CNRS Toulouse in 2012-2013

Logic colloquium 2013 Evora



A Reminder on Montague semantics

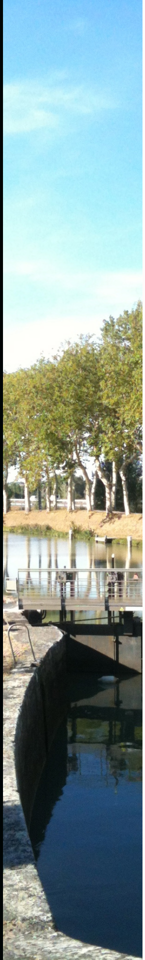


A.1. Representing formulae within lambda calculus — language constants

one two-place predicate	
likes	$\lambda x^{\mathbf{e}} \lambda y^{\mathbf{e}}\ (\underline{likes}^{\mathbf{e}\to(\mathbf{e}\to\mathbf{t})}\ y)\ x$
two one place predicates	
cat	$\lambda x.\underline{cat}^{\mathbf{e}\to\mathbf{t}}$
sleeps	$\lambda x.\underline{sleep}^{\mathbf{e}\to\mathbf{t}}$
two proper names	
Evora	$\underline{Evora} : \mathbf{e}$
Anne−Sophie	$\underline{Anne-Sophie} : \mathbf{e}$

possibly $(\mathbf{e} \to \mathbf{t}) \to \mathbf{t}$

Normal terms (preferably η-long) of type $\mathbf{t}$ are formulae.



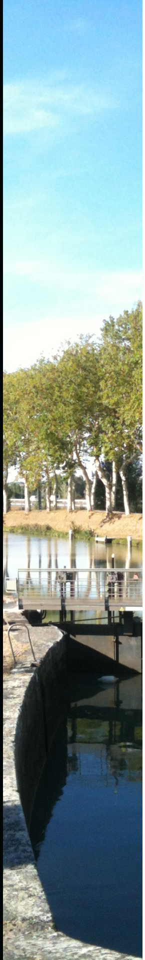
A.2. Ingredients: a parse structure & a lexicon

Syntactical structure

(some (club)) (defeated Leeds)

Semantical lexicon:

word	***semantics : λ-term of type (sent. cat.)**** *x^v the variable or constant x is of type v*
some	$(e \to t) \to ((e \to t) \to t)$ $\lambda P^{e\to t}\, \lambda Q^{e\to t}\, (\exists^{(e\to t)\to t}\, (\lambda x^e (\wedge^{t\to(t\to t)} (P\ x)(Q\ x))))$
club	$e \to t$ $\lambda x^e (\mathtt{club}^{e\to t}\ x)$
defeated	$e \to (e \to t)$ $\lambda y^e\, \lambda x^e\, ((\mathtt{defeated}^{e\to(e\to t)}\ x) y)$
Leeds	e Leeds



A.3. Computing the semantic representation

1) Insert the semantics terms into the parse structure

2) β reduce the resulting term

$$\Big(\big(\lambda P^{e\to t}\,\lambda Q^{e\to t}\,(\exists^{(e\to t)\to t}\,(\lambda x^{e}(\wedge(P\;x)(Q\;x)))))\big)\big(\lambda x^{e}(\mathtt{club}^{e\to t}\;x)\big)\Big)$$

$$\Big(\big(\lambda y^{e}\,\lambda x^{e}\,((\mathtt{defeated}^{e\to(e\to t)}\;x)y)\big)\;Leeds^{e}\Big)$$

$$\downarrow\beta$$

$$\big(\lambda Q^{e\to t}\,(\exists^{(e\to t)\to t}\,(\lambda x^{e}(\wedge^{t\to(t\to t)}(\mathtt{club}^{e\to t}\;x)(Q\;x)))))\big)$$

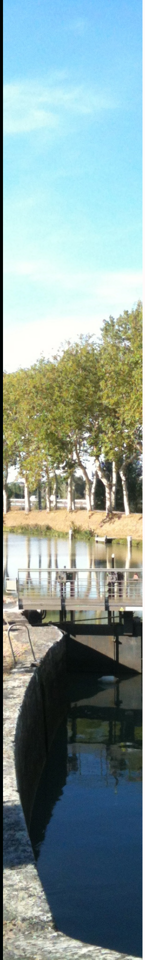
$$\big(\lambda x^{e}\,((\mathtt{defeated}^{e\to(e\to t)}\;x)Leeds^{e})\big)$$

$$\downarrow\beta$$

$$\big(\exists^{(e\to t)\to t}\,(\lambda x^{e}(\wedge(\mathtt{club}^{e\to t}\;x)((\mathtt{defeated}^{e\to(e\to t)}\;x)Leeds^{e}))))\big)$$

Usually human beings prefer to write it like this:

$$\exists x : e\;(\mathtt{club}(x)\;\wedge\;\mathtt{defeated}(x, Leeds))$$



A.4. Montague: good architecture / limits

Good trick (Church):

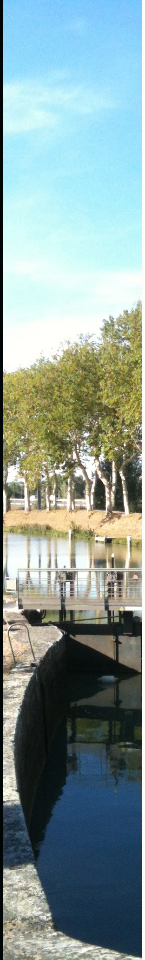
a propositional logic for meaning assembly (proofs/λ-terms)
computes
formulae of another logic H/F OL (formulae/meaning; no proofs)

The dictionary says ”barks” requires a subject of type ”animal”.
How could we block:

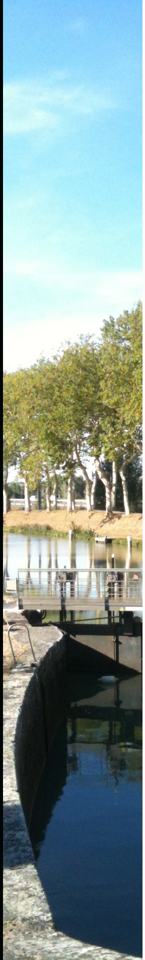
(1) * The chair barked.

By type mismatch, $(f^{A \to X}(u^B))$ hence **many types** are needed.

Description with few operators
$\longrightarrow$ **factorise** similar operations acting on terms/types
$\longrightarrow$ **quantification over types**



B ΛTy_n: system F tuned for semantics



B.1. System F

Types:

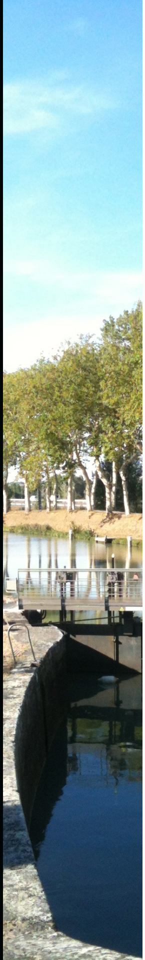
- $\mathbf{t}$ (prop)
- many entity types $\mathbf{e}_i$
- type variables $\alpha, \beta, \ldots$
- $\Pi\alpha.\ T$
- $T_1 \rightarrow T_2$

Terms

- Constants and variables for each type
- $(f^{T\rightarrow U} a^T) : U$
- $(\lambda x^T.\ u^U) : T \rightarrow U$
- $t^{(\Lambda\alpha.\ T)}\{U\} : T[U/\alpha]$
- $\Lambda\alpha.u^T : \Pi\alpha.T$ — no free α in a free variable of u.

The reduction is defined as follows:

- $(\Lambda\alpha.\tau)\{U\}$ reduces to $\tau[U/\alpha]$
  (remember that α and U are types).
- $(\lambda x.\tau)u$ reduces to $\tau[u/x]$ (usual reduction).



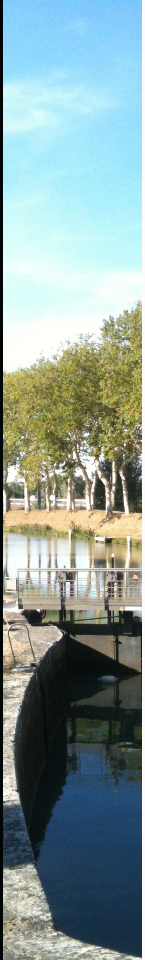
B.2. Basic facts on system F

We do not really need system F but any type system with quantification over types. F is syntactically the simplest.

Confluence and strong normalisation — requires the comprehension axiom for all formulae of HA_2. (Girard 1971)

A concrete categorical interpretation with coherence spaces that shows that there are distinct functions from A to B.

Terms of type $\mathbf{t}$ with constants of mutisorted FOL (resp. HOL) correspond to multisorted formulae of FOL (resp. HOL)



B.3. Examples of second order usefulness

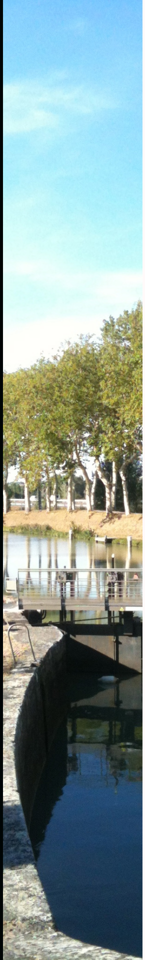
Arbitrary modifiers: $\Lambda\alpha\lambda x^{A} y^{\alpha} f^{\alpha\to R}.((\mathsf{read}^{A\to R\to t}\ x)\ (f\ y))$

Polymorphic conjunction:

> Given predicates $P^{\alpha\to\mathbf{t}}$, $Q^{\beta\to\mathbf{t}}$ over respective types α, β,
> given any type ξ with two morphisms from ξ to α and to β
> we can coordinate the properties P, Q
> of (the two images of) an entity of type ξ:

The polymorphic conjunction $\&^{\Pi}$ is defined as the term

$$\begin{aligned}\&^{\Pi} = \Lambda\alpha\Lambda\beta\lambda P^{\alpha\to\mathbf{t}}\lambda Q^{\beta\to\mathbf{t}} \\ \Lambda\xi\lambda x^{\xi}\lambda f^{\xi\to\alpha}\lambda g^{\xi\to\beta}. \\ (\mathrm{and}^{\mathbf{t}\to\mathbf{t}\to\mathbf{t}}\ (P\ (f\ x))(Q\ (g\ x)))\end{aligned}$$



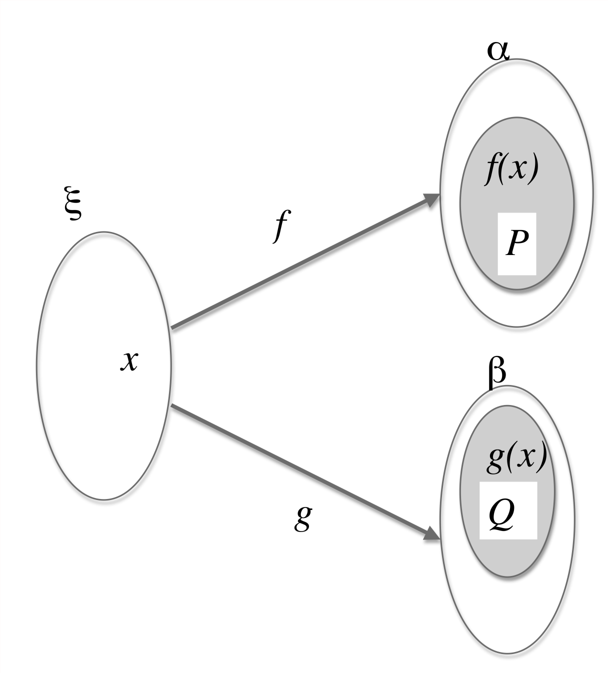
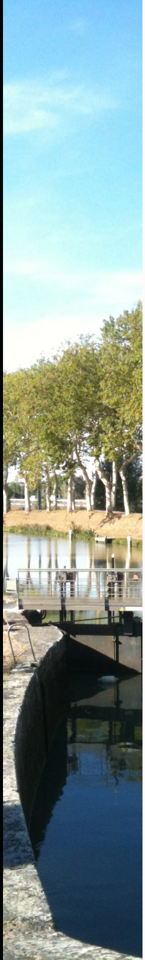
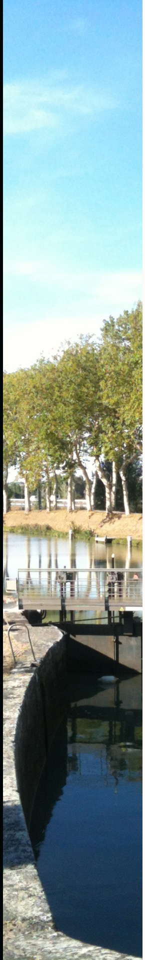


Figure 1: Polymorphic conjunction: $P(f(x))\&Q(g(x))$
with $x:\xi$, $f:\xi\to\alpha$, $g:\xi\to\beta$.



C System F based semantics and pragmatics



C.1. Examples

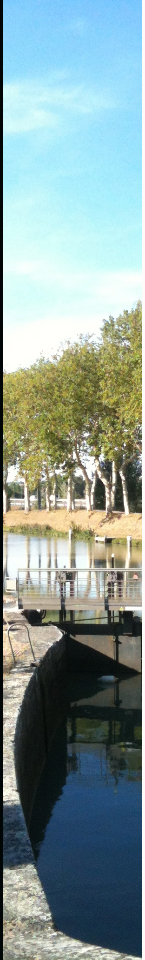
(2) Dinner was delicious but took ages.
(event / food)

(3) * The salmon we had for lunch was lightning fast.
(animal / food)

(4) I carried the books from the shelf to the attic.
Indeed, I already read them all.
(phys. / info — think of possible multiple copies of a book)

(5) Liverpool is a big place and voted last Sunday.
(geographic / people)

(6) * Liverpool is a big place and won last Sunday.
(geographic / football club)



C.2. The Terms: principal or optional

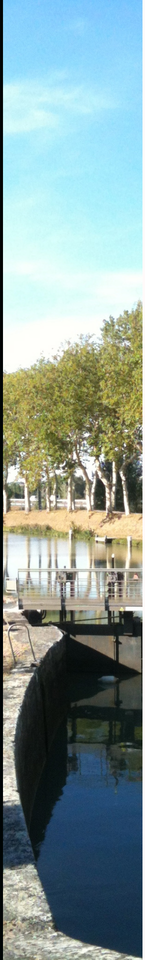
A standard λ-term attached to the main sense:

- Used for compositional purposes
- Comprising detailed typing information (restrictions of selection)

Some optional λ-terms (none is possible)

- Used, or not, for adaptation purposes
- Each associated with a constraint : *rigid,* $\emptyset$

Both function and argument may contribute to meaning transfers.



C.3. RIGID vs FLEXIBLE use of optional terms

RIGID

Such a transformation is exclusive:

the other aspects of the same word are not used.

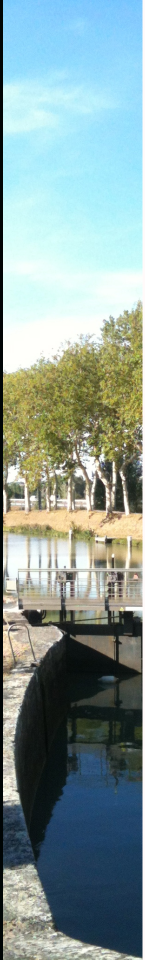
Each time we refer to the word it is with the same aspect.

FLEXIBLE

There is no constraint.

Any subset of the flexible transformation can be used:

different aspects of the words can be simultaneously used.

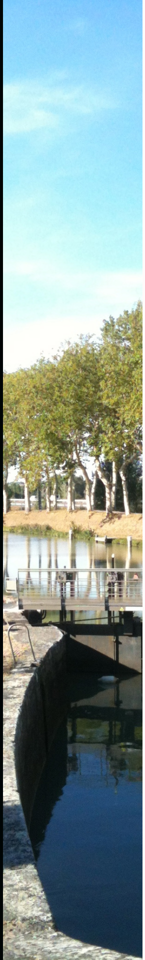


C.4. Correct copredication

word	principal λ-term	optional λ-terms	rigid/flexible
Liverpool	$liverpool^T$	$Id_T : T \rightarrow T$	(F)
		$t_1 : T \rightarrow F$	(R)
		$t_2 : T \rightarrow P$	(F)
		$t_3 : T \rightarrow Pl$	(F)
is_a_big_place	$big\_place : Pl \rightarrow \mathbf{t}$		
voted	$voted : P \rightarrow \mathbf{t}$		
won	$won : F \rightarrow \mathbf{t}$		

where the base types are defined as follows:

T	Town
F	football club
P	people
Pl	place



C.5. Meaning transfers

(7) Liverpool is a big place.

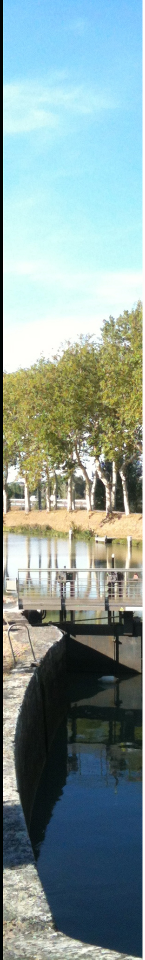
(8) Liverpool won.

(9) Liverpool voted.

$$big\_place^{Place \rightarrow \mathbf{t}}\, Liverpool^{Town}$$

Type mismatch, use the appropriate optional term.

$$big\_place^{Place \rightarrow \mathbf{t}}(t_3^{Town \rightarrow Place}\, Liverpool^{Town})$$

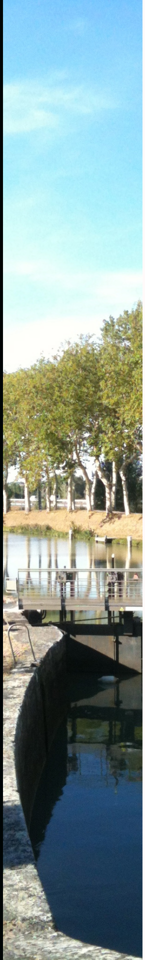


C.6. (In)felicitous copredications

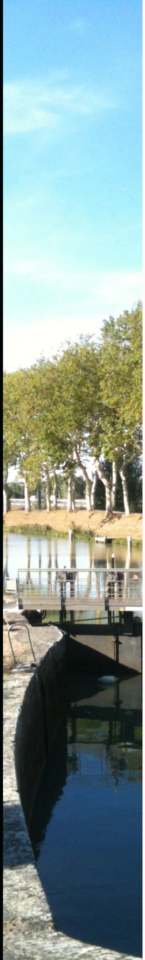
Use polymorphic ”and”... specialised to the appropriate types:

(10) Liverpool is a big place and voted.
Town $\rightarrow$ *Place* and *Town* $\rightarrow$ *People*
fine

(11) * Liverpool won and voted.
Town $\rightarrow$ *FootballClub* and *Town* $\rightarrow$ *People*
blocked because the first transformation is **rigid**.
(sole interpretation: *football* team or committee voted)



D Integrating other aspects



D.1. Quantifier: critics of the standard solution

Syntactical structure of the sentence $\neq$ logical form.

(12) Keith played some Beatles songs.

(13) syntax (Keith (played (some (Beatles songs))))

(14) semantics: (some (Beatles songs)) (λx. Keith played x)

Asymmetry class / predicate

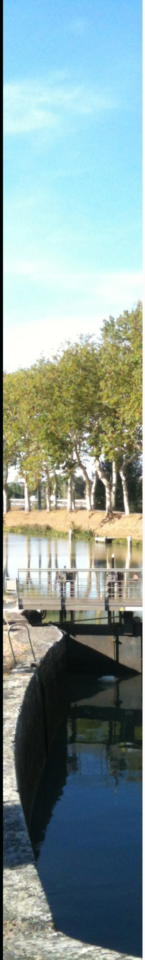
(15) Some politicians are crooks

(16) ? Some crooks are politicians

(17) $\exists x.\ crook(x) \& politician(x)$

There can be a reference before the predicate arrives (if any):

(18) Un luth, une mandore, une viole, que Michel-Ange... (M. Énard)



D.2. A solution: Hilbert's epsilon

$\varepsilon : \Lambda\alpha(\alpha \to \mathbf{t}) \to \alpha$ with $F(\varepsilon_x F) \equiv \exists x.\ F(x)$.

A cat. cat$^{animal \to \mathbf{t}}$ $(\varepsilon\{animal\}cat^{animal \to \mathbf{t}}) : animal$

Presupposition $F(\varepsilon_x F)$ is added: $cat(\varepsilon\{animal\}cat^{animal \to \mathbf{t}})$

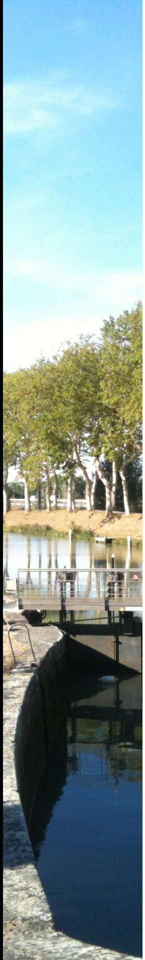
$\varepsilon_x F$: individual. Follows syntactical structure. Asymmetry subject/predicate.

ε lso solves the so-called E-type pronouns interpretation:

(19) A man came in. He sat dow.

(20) $''He'' = ''A\ man'' = (\varepsilon_x\ M(x))$.

For applying ε to a type say cat,
any type has a predicative counterpart cat (type) $\widehat{cat} : \mathbf{e} \to \mathbf{t}$.
(domains can be restrained / extended)



D.3. Remarks on ε

Hilbert's work: fine! (Grundlagen der Mathematik, with P. Bernays)

Rule 1: From $P(x)$ with x generic infer $P(\varepsilon_x.\neg P(x)) \equiv P(\tau_x.P(x)) \equiv \forall x\ P(x)$

Rule 2: From $P(t)$ infer $P(\varepsilon_x P(x)) \equiv \exists x\ P(x)$

ε-elimination (1st & 2nd ε-theorems), proof of Herbrand theorem.

Little else is known (extra formulae, proofs, models), erroneous results.

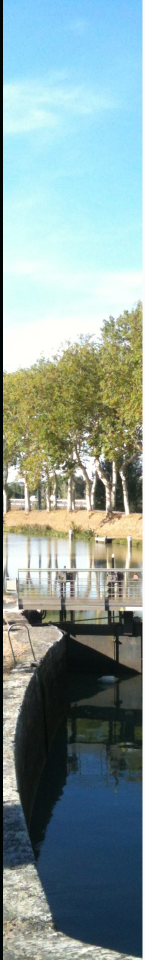
$$Sleeps(\varepsilon_x Cat(x)) \equiv ???$$
$$(Cat(\varepsilon_x Cat(x)) \& Sleeps(\varepsilon_x Cat(x))) \equiv \exists x\ Cat(x) \& Sleeps(x)$$

Heavy notation: $\forall x \exists y P(x,y)$ is $\quad P(\tau_x P(x, \varepsilon_y P(\tau_x P(x,y),y)), \varepsilon_y P(\tau_x P(x,y),y))$

von Heusinger interpretations differ for different occurrences of $\varepsilon_x F(x)$.

(21) a. A tall man went in. A blonde man went out.

b. *Not the same F but necessarily different interpretations.*



D.4. Coercive subtyping for F (Luo, Soloviev for MTT)

Key property: at most one coercion between any two types.
Given coercions between base types.
Propagates through type hierarchy (unique possible restoration).

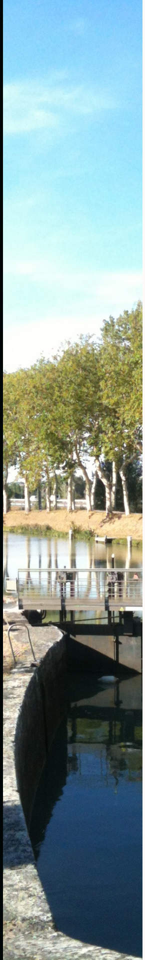
coercive application $\dfrac{f : A \to B \quad u : A_0 \quad A_0 < A}{(f\ \ a) : B}$

$$\frac{A < B \quad C < D}{B \to A < C \to D} \qquad \frac{A < B}{X \to A < X \to B} \qquad \frac{A < B}{B \to X < A \to X}$$

$$\frac{S[X] < T[X]}{\Pi X.S[X] < \Pi X.T[X]} \qquad \frac{U < T[X]}{U < \Pi X.T[X]}\ \textit{no free } X \textit{ in } U \qquad \frac{S[W] < U}{\Pi X.S[X] < U}$$

$$\frac{U < \Pi X.T[X]}{U < T[A]} \qquad \frac{\Pi X.S[X] < U}{S[A] < U}$$

Key lemma: transitivity of $<$ is unnecessary.



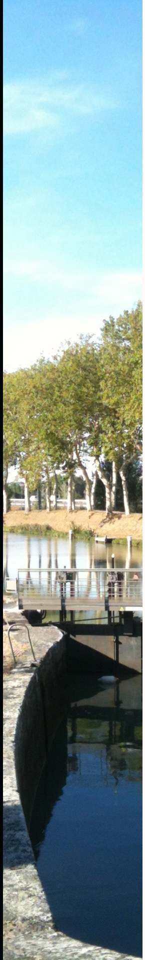
D.5. Other applications in natural language semantics

Generalised quantifiers ("*most*") with generic elements.
The Brits love France.

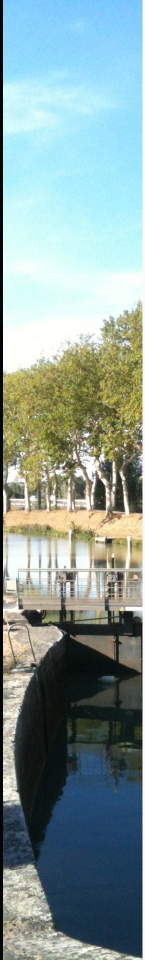
Plurals: collective / distributive readings (with Moot)
The players from Benfica won although they had the flu.

Virtual traveller / fictive motion (with Moot & Prévot)
"*The road does down for twenty minutes*"

Deverbals: meanings copredications (with Livy Real):
"*A assinatura atrasou três dias / * e estava ilegìvel.*"



E Conclusion



E.1. What we have seen so far

A **general framework** for

the logical syntax of **compositional semantics**
some **lexical semantics/pragmatics** phenomena

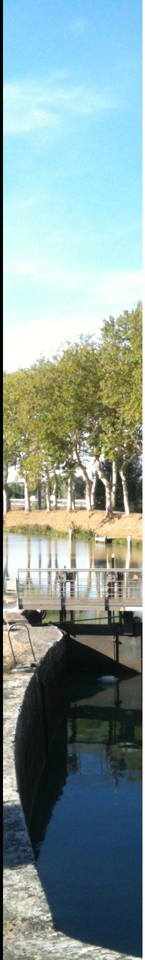
Guidelines:

Terms: semantics, instructions for computing references
Types: pragmatics, defined from the context
Idiosyncratic meaning transfers word-driven (not type-driven)

(22) Mon vélo est crevé. /??? My bike is flat.

(23) Classe $\rightarrow$ room promotion $\not\rightarrow$ room

Practically: implemented in Grail, Moot's wide coverage categorial parser, with hand-typed semantic lexica (with λ-DRT instead of HOL in λ-calculus).
Questions: Base types? Acquisition? Sublte copredication constraints?



E.2. Logical perspectives

Cohabitation of types and formulae of first/higher order logic:

Typing ($\sim$ presupposition) is irrefutable $sleeps(x : cat)$

Type to Formula:

type cat mirrored as a predicate $\widehat{cat} : \mathbf{e} \rightarrow \mathbf{t}$

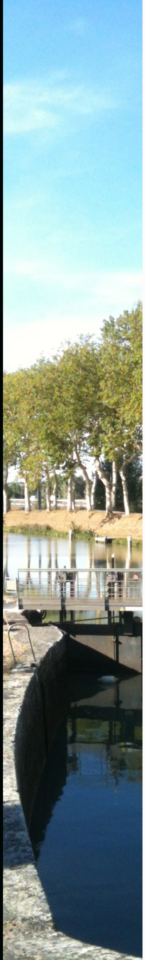
Formula to Type?

Formula with a single free variable $\sim$ type?

$cat(x) \wedge belong(x, john) \wedge sleeps(x) \sim$ type?

At least it is not a natural class.

Quantification, generics in this typed setting with Hilbert operators



Any question?