

UM. Autumn 2019. Homework 2 to the course «Information theory».
[should be returned by Sep 24 to be counted in *contrôle continu*]

Problem 1. We are given a heap of n stones, and we can use balance scales to compare weights of any two stones. We want to find in the heap two stones : the one with the maximal weight and the one with the minimal weight. How many weighing do we need (in the worst case)? Propose a search algorithm and prove that it is optimal.

Problem 2. Prove that for every non-negative real number h there exists a random variable (distribution) α such that the Shannon entropy of α is equal to h .

Problem 3. Let α and β be two jointly distributed random variables. Prove that

$$H(\alpha, \beta) = H(\alpha) + H(\beta)$$

if and only if α and β are independent.

Problem 4. Let us have a rooted binary tree with n leaves. Let l_i denote the distances from the root to the i -th leaf, $i = 1, \dots, n$. Prove that

$$\sum_{i=1}^n 2^{-l_i} \leq 1.$$