**UM. Autumn 2019. Homework 4 to the course «Information theory».**
[ should be returned by Oct 7 to be counted in *contrôle continu* ]

**Problem 1.** Construct prefix code with minimal average length for the following probability distributions :
(a) 0.6, 0.4
(b) 0.4, 0.3, 0.3
(c) 0.4, 0.3, 0.2, 0.1

**Problem 2.** Show that for every triple of non-negative real numbers $h_1, h_2, h_3$ there exists a pair of jointly distributed random variables $(\alpha, \beta)$ such that

$$\begin{array}{rcl} H(\alpha) & = & h_1 + h_2 \\ H(\beta) & = & h_1 + h_3 \\ H(\alpha, \beta) & = & h_1 + h_2 + h_3 \end{array}$$

**Problem 3.** We are given $n = 13$ coins, and one of them is fake. All genuine coins have the same weight, the fake one can be heavier or lighter. The position of the fake coin (between 1 and 13) and its relative weight (whether it is heavier or lighter than the genuine coins) are chosen at random, and all $2 \times 13 = 26$ variants have the same probability $1/26$. We can use balance scales to compare weights of any two groups of coins.

(i) Find a strategy that discovers the fake coin with minimal *on average* number of weighings.

(ii) Compute Shannon's entropy of each weighings that can be used in an optimal strategy (at least for the 1st and or the 2nd weighing in each branch of the strategy).

*Hint 1 :* There are several different optimal strategies, but each of them uses 3 operation.

*Hint 2 :* It is helpful to start the solution with question (ii), and construct an optimal strategy by choosing on each stage the weighing that brings the maximal possible value of entropy.