

**UM. Autumn 2020. Homework 2 to the course «Information theory».**  
[ should be returned by Sep 22 to be counted in *contrôle continu* ]

**Problem 1.** We are given  $n = 14$  coins, and one of them is fake. All genuine coins have the same weights, the fake one is heavier or lighter. We can use balance scales to compare weights of any two groups of coins. In the class we proved that the fake coin can be found in at most 4 weighings. Prove that 3 operations are not enough.

**Problem 2.** Let  $\alpha$  and  $\beta$  be two jointly distributed random variables. Prove that

$$H(\alpha, \beta) = H(\alpha) + H(\beta)$$

if and only if  $\alpha$  and  $\beta$  are independent.

**Problem 3.** I choose a random element from  $X = \{x_1, \dots, x_k\}$  with probabilities  $p_1, \dots, p_k$  respectively (each  $p_i$  is a non-negative number, and  $\sum_{i=1}^k p_i = 1$ ). You have to guess which number is chosen by asking questions admitting answers *yes* and *no*. Suggest a strategy that allow to find the chosen  $x_i$  in less than

$$\left( \sum_{i=1}^k p_i \log_2 p_i \right) + 1$$

questions *on average*.