UM. Autumn 2020. Homework 4 to the course «Information theory»
[recommended to return by octobre 6]

Problem 1. Construct an optimal prefix code for the probability distribution

$$(0.33, 0.32, 0.2, 0.1, 0.5).$$

What is the average length of a codeword of this code (for the given distribution)? *Hint:* Use Huffman's construction of an optimal code.

Problem 2. In Lecture 3 (on September 22) we discussed a construction of a prefix code for a distribution of probabilities $(p_1, \ldots, p_k)$ with codewords of length $\lceil \log \frac{1}{p_i} \rceil$, $i = 1, \ldots, k$. Find an example of a distribution for which this construction gives a code that is *not* optimal. *Hint:* Compair this code with Huffman's code, which is known to be optimal.

Problem 3. (a) Prove that

$$1 + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{n} = an\sqrt{n} + b\sqrt{n} + O(1)$$

for some constant a, b (that do not depend on n).

(b) Find the values of a and b in this forumal.

(c) Find and prove a similar formula for the sum

$$1 + \sqrt[3]{2} + \sqrt[3]{3} + \ldots + \sqrt[3]{n}.$$

Hint: Use the ideas from the proof of the simplified Stirling formula proven in class.