UM. Autumn 2020. Homework to the course «Information theory».
[should be returned by Dec 8 to be counted in *contrôle continu*]

Problem 1. Let (α, β) be a pair of jointly distributed random variables. Prove that $I(\alpha : \beta) = H(\alpha)$ if and only if α is a deterministic function of β.

Problem 2. Let (α, β, γ) be a triple of jointly distributed random variables. The conditional mutual information is defined as $I(\alpha : \beta|\gamma) = H(\beta|\gamma) - H(\beta|\alpha, \gamma)$. Prove that

$$\begin{array}{lrcl} (a) & I(\alpha : \beta|\gamma) & = & H(\alpha|\gamma) + H(\beta|\gamma) - H(\alpha, \beta|\gamma), \\ (b) & I(\alpha : \beta|\gamma) & = & H(\alpha, \gamma) + H(\beta, \gamma) - H(\alpha, \beta, \gamma) - H(\gamma), \\ (c) & I(\alpha : \beta|\gamma) & = & I(\beta : \alpha|\gamma). \end{array}$$

Problem 3. Construct a joint distribution (α, β, γ) such that

$$\begin{array}{l} H(\alpha) = H(\beta) = H(\gamma) = 1, \\ H(\alpha, \beta) = H(\beta, \gamma) = H(\alpha, \gamma) = 2, \\ H(\alpha, \beta, \gamma) = 2. \end{array}$$

Draw a diagram representing the entropy values of this distribution. Find the values $I(\alpha : \beta)$ and $I(\alpha : \beta|\gamma)$.